Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.4-Improper/1.1.4.2-c-x-^m-a-x^j+b-x^n-^p

Nasser M. Abbasi

September 20, 2021 Compiled on September 20, 2021 at 6:01am

Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [340]. This is test number [17].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. `IntegrateAlgebraic` under Mathematica 12.3.1 on windows 10. `https://github.com/stblake/algebraic_integration`. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (340)	0.00 (0)
Mathematica	100.00 (340)	0.00 (0)
Maple	85.59 (291)	14.41 (49)
Fricas	75.29 (256)	24.71 (84)
Giac	70.00 (238)	30.00 (102)
IntegrateAlgebraic	61.18 (208)	38.82 (132)
Mupad	51.76 (176)	48.24 (164)
Maxima	45.00 (153)	55.00 (187)
Sympy	33.53 (114)	% 66.47 (226)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

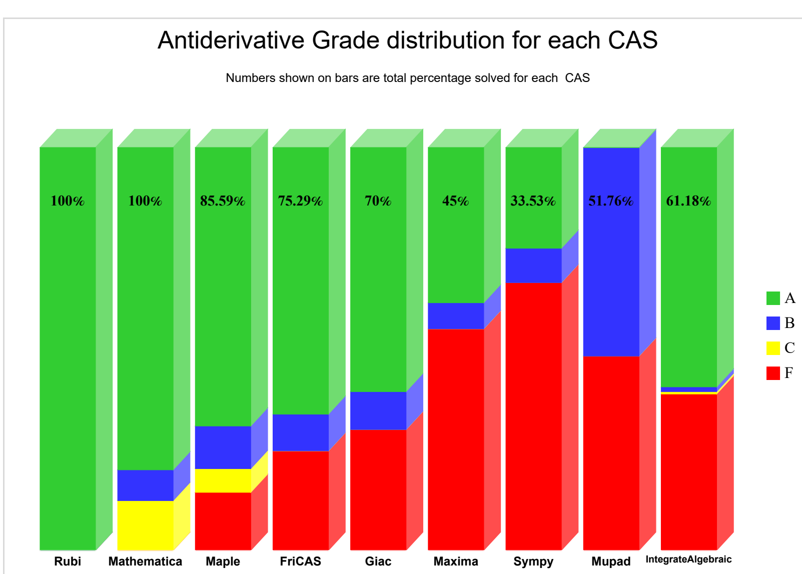
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

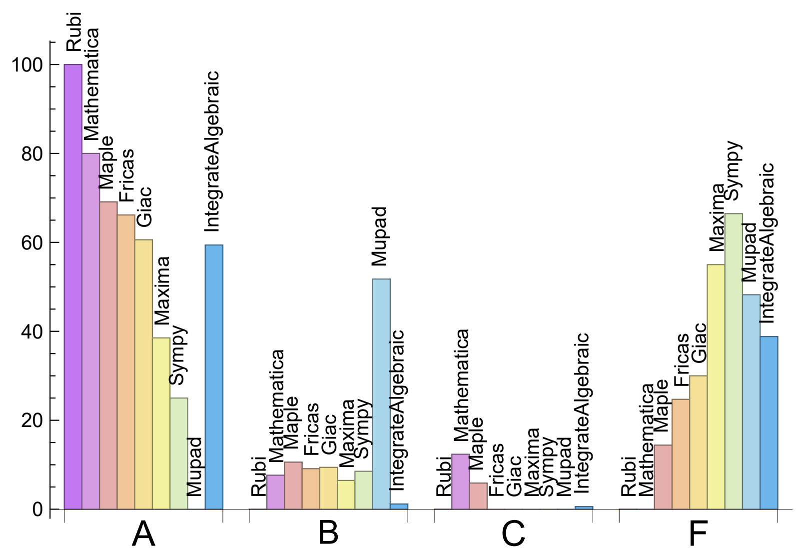
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	80.00	7.65	12.35	0.00
Maple	69.12	10.59	5.88	14.41
Fricas	66.18	9.12	0.00	24.71
Giac	60.59	9.41	0.00	30.00
IntegrateAlgebraic	59.41	1.18	0.59	38.82
Maxima	38.53	6.47	0.00	55.00
Sympy	25.00	8.53	0.00	66.47
Mupad	N/A	51.76	0.00	48.24

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	49	100.00 %	0.00 %	0.00 %
Fricas	84	0.00 %	58.33 %	41.67 %
IntegrateAlgebraic	132	100.00 %	0.00 %	0.00 %
Giac	102	86.27 %	0.00 %	13.73 %
Maxima	187	100.00 %	0.00 %	0.00 %
Sympy	226	83.19 %	16.81 %	0.00 %
Mupad	164	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

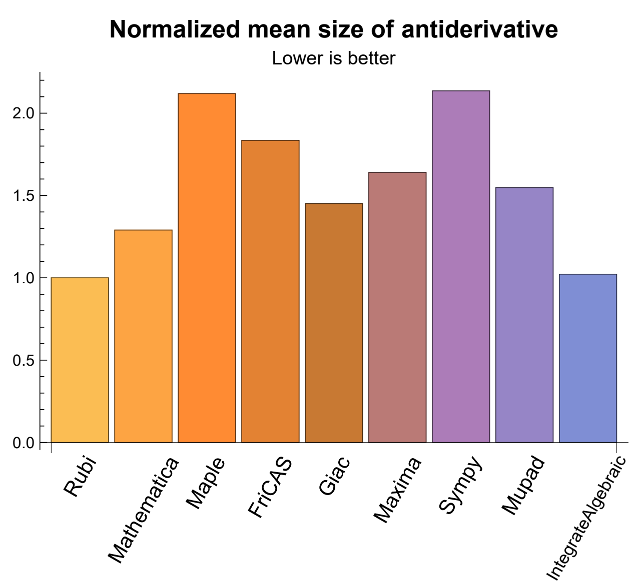
1.3 Performance

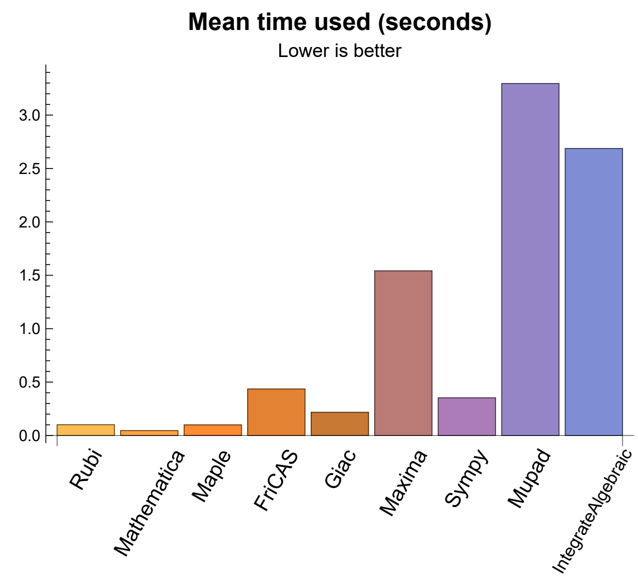
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.10	75.16	1.00	51.00	1.00
Mathematica	0.05	57.87	1.29	48.00	1.00
Maple	0.10	112.31	2.12	52.00	0.94
Maxima	1.54	42.06	1.64	27.00	0.89
Fricas	0.44	78.83	1.83	57.00	1.18
Sympy	0.35	45.86	2.14	26.00	0.88
Giac	0.22	79.02	1.45	47.00	0.97
Mupad	3.30	46.30	1.55	33.50	0.89
IntegrateAlgebraic	2.69	76.63	1.02	69.00	0.96

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.




Mean time used (seconds)
Lower is better
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
IntegrateAlgebraic

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {250}

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

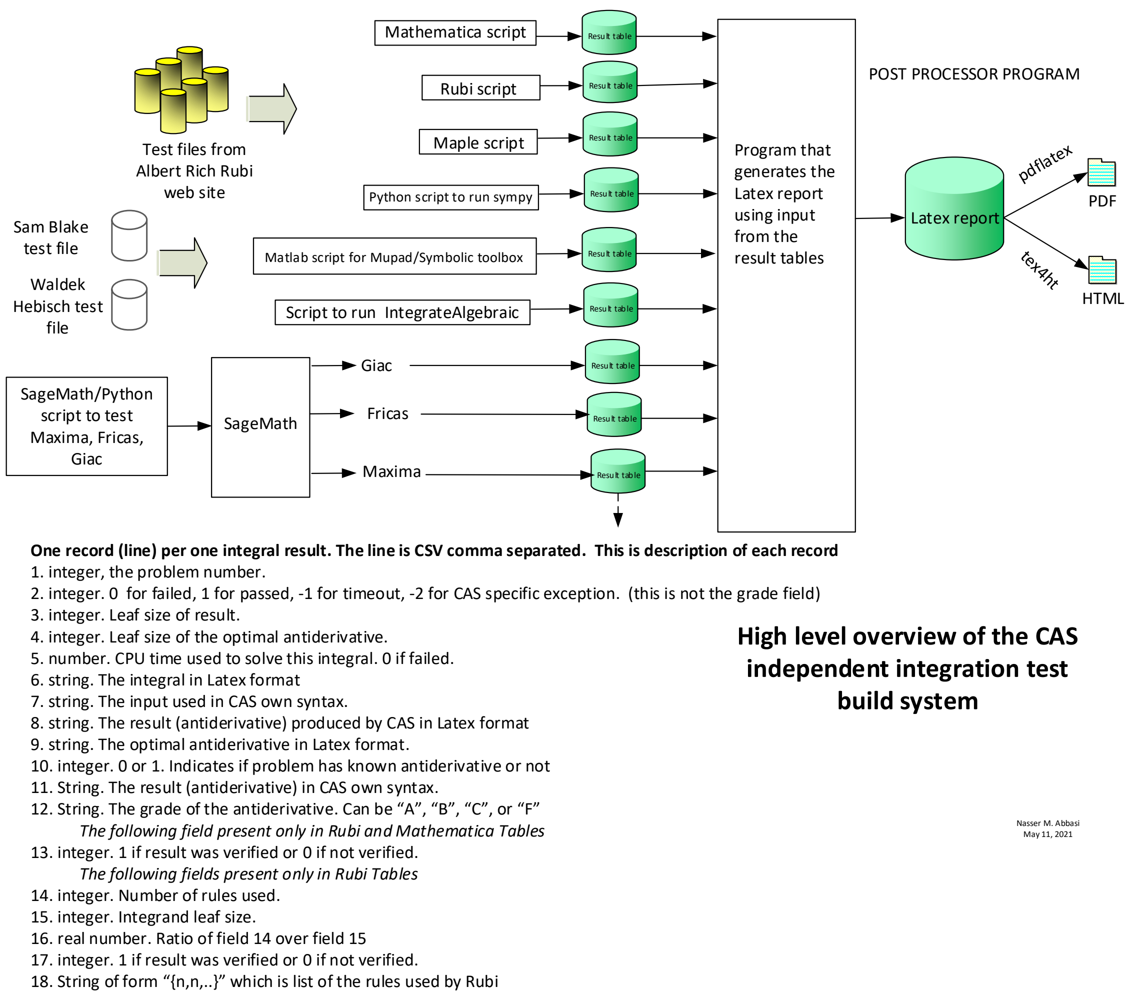
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 29, 31, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 97, 98, 99, 100, 101, 106, 107, 108, 109, 110, 111, 115, 116, 117, 118, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 161, 162, 163, 164, 165, 166, 167, 169, 173, 174, 175, 176, 177, 178, 181, 182, 183, 184, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 234, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 303, 304, 310, 311, 312, 314, 315, 316, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340 }

B grade: { 26, 28, 30, 32, 34, 231, 232, 233, 235, 236, 237, 252, 253, 254, 255, 256, 257, 295, 313, 317, 318, 319, 320, 321, 322, 323 }

C grade: { 49, 51, 53, 67, 68, 69, 81, 82, 93, 94, 95, 96, 102, 103, 104, 105, 112, 113, 114, 119, 120, 121, 122, 123, 159, 160, 168, 170, 171, 172, 179, 180, 185, 186, 187, 188, 189, 305, 306, 307, 308, 309 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 57, 58, 59, 60, 62, 74, 75, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 258, 259, 261, 263, 265, 266, 272, 280, 285, 286, 287, 288, 289, 291, 292, 297, 305, 311, 324, 325, 326, 327, 328, 329, 334, 335, 336 }

B grade: { 28, 32, 49, 61, 67, 68, 69, 76, 77, 81, 82, 83, 84, 193, 216, 231, 232, 233, 234, 235, 236, 237, 238, 239, 251, 252, 253, 254, 255, 256, 257, 260, 262, 264, 290, 315 }

C grade: { 55, 56, 63, 64, 65, 66, 70, 71, 72, 73, 78, 79, 80, 85, 86, 87, 207, 208, 310, 314 }

F grade: { 267, 268, 269, 270, 271, 273, 274, 275, 276, 277, 278, 279, 281, 282, 283, 284, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 306, 307, 308, 309, 312, 313, 316, 317, 318, 319, 320, 321, 322, 323, 330, 331, 332, 333, 337, 338, 339, 340 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 31, 33, 34, 35, 36, 37, 57, 58, 59, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 161, 162, 163, 164, 165, 166, 173, 174, 175, 176, 181, 182, 183, 184, 202, 203, 204, 209, 210, 211, 212, 222, 223, 224, 225, 226, 227, 228, 229, 241, 242, 243, 247, 248, 249, 250, 252, 255, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 272, 280, 286, 290, 297, 305, 326, 327, 328, 329, 334, 335, 336 }

B grade: { 28, 30, 32, 201, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 244, 245, 246, 251, 253, 254, 256 }

C grade: { }

F grade: { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 157, 158, 159, 160, 167, 168, 169, 170, 171, 172, 177, 178, 179, 180, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 205, 206, 207, 208, 213, 214, 215, 216, 217, 218, 219, 220, 221, 267, 268, 269, 270, 271, 273, 274, 275, 276, 277, 278, 279, 281, 282, 283, 284, 285, 287, 288, 289, 291, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 330, 331, 332, 333, 337, 338, 339, 340 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 31, 33, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 63, 64, 65, 66, 70, 71, 72, 73, 78, 79, 80, 85, 86, 87, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 228, 229, 241, 242, 243, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 272, 280, 285, 286, 287, 288, 289, 290, 291, 292, 297, 305, 310, 311, 312, 313, 314, 315, 316, 317, 319, 322, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340 }

B grade: { 28, 30, 32, 34, 42, 47, 84, 201, 226, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 244, 245, 246, 250, 251, 252, 253, 254, 255, 256, 257 }

C grade: { }

F grade: { 60, 61, 62, 67, 68, 69, 74, 75, 76, 77, 81, 82, 83, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 267, 268, 269, 270, 271, 273, 274, 275, 276, 277, 278, 279, 281, 282, 283, 284, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 306, 307, 308, 309, 318, 320, 321, 323 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 16, 18, 20, 22, 23, 24, 25, 27, 29, 31, 33, 34, 35, 36, 37, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 211, 212, 222, 223, 224, 225, 227, 228, 229, 230, 241, 242, 243, 247, 248, 249, 250, 258, 259, 260, 261, 262, 263, 265, 266, 272, 280, 297, 326 }

B grade: { 9, 13, 15, 17, 19, 21, 26, 28, 30, 32, 201, 226, 231, 232, 233, 235, 236, 237, 244, 245, 246, 252, 253, 254, 255, 256, 257, 264, 305 }

C grade: { }

F grade: { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 202, 203, 204, 205, 206, 207, 208, 209, 210, 213, 214, 215, 216, 217, 218, 219, 220, 221, 234, 238, 239, 240, 251, 267, 268, 269, 270, 271, 273, 274, 275, 276, 277, 278, 279, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 29, 31, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 70, 74, 75, 76, 77, 78, 79, 80, 84, 88, 89, 90, 91, 92, 93, 94, 95, 96, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 157, 158, 159, 160, 167, 168, 169, 170, 171, 172, 176, 177, 183, 184, 190, 191, 192, 193, 194, 195, 196, 197, 201, 205, 206, 207, 208, 209, 210, 211, 212, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 252, 255, 257, 263, 264, 285, 287, 289, 290, 291, 311, 315, 325, 326, 327, 328, 329 }

B grade: { 26, 28, 30, 32, 34, 97, 98, 99, 100, 156, 161, 162, 163, 164, 165, 166, 231, 232, 233, 234, 235, 236, 237, 238, 239, 251, 253, 254, 256, 265, 286, 324 }

C grade: { }

F grade: { 67, 68, 71, 72, 73, 81, 82, 83, 85, 86, 87, 173, 174, 175, 178, 179, 180, 181, 182, 185, 186, 187, 188, 189, 198, 199, 200, 202, 203, 204, 213, 214, 228, 229, 230, 240, 258, 259, 260, 261, 262, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 288, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 312, 313, 314, 316, 317, 318, 319, 320, 321, 322, 323, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 57, 58, 59, 62, 70, 91, 99, 110, 119, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 157, 161, 162, 163, 164, 165, 166, 173, 174, 175, 176, 179, 181, 182, 183, 184, 187, 189, 198, 199, 200, 201, 202, 203, 204, 211, 212, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 274, 285, 286, 287, 289, 290, 291, 295, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330 }

C grade: { }

F grade: { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 60, 61, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 156, 158, 159, 160, 167, 168, 169, 170, 171, 172, 177, 178, 180, 185, 186, 188, 190, 191, 192, 193, 194, 195, 196, 197, 205, 206, 207, 208, 209, 210, 213, 214, 215, 216, 267, 268, 269, 270, 271, 272, 273, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 288, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340 }

2.1.9 IntegrateAlgebraic

A grade: { 4, 9, 10, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 127, 128, 132, 133, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 202, 203, 204, 205, 206, 207, 208, 209, 210, 213, 214, 215, 216, 217, 218, 219, 220, 221, 259, 260, 261, 262, 263, 264, 265, 272, 273, 274, 275, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 297, 298, 299, 300, 305, 306, 307, 308, 309, 310, 311, 312, 314, 324, 325, 326, 327, 328, 329, 334, 335, 336 }

B grade: { 234, 238, 239, 251 }

C grade: { 315, 316 }

F grade: { 1, 2, 3, 5, 6, 7, 8, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 124, 125, 126, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 198, 199, 200, 201, 211, 212, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 235, 236, 237, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 256, 257, 258, 266, 267, 268, 269, 270, 271, 276, 277, 278, 279, 293, 294, 295, 296, 301, 302, 303, 304, 313, 317, 318, 319, 320, 321, 322, 323, 330, 331, 332, 333, 337, 338, 339, 340 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA` and `IntegrateAlgebraic` to `I.A.`

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	12	13	13	0
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.00
time (sec)	N/A	0.005	0.002	0.039	1.297	0.352	0.098	0.162	0.022	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	12	13	13	0
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.00
time (sec)	N/A	0.004	0.002	0.041	1.303	0.357	0.085	0.147	0.022	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	12	13	13	0
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.00
time (sec)	N/A	0.003	0.000	0.043	1.349	0.346	0.059	0.146	0.019	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	10	10	8	10	10	12
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83	1.00
time (sec)	N/A	0.003	0.000	0.045	1.338	0.375	0.064	0.146	0.017	0.012

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	12	11	11	10	14	11	0
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	1.08	0.85	0.00
time (sec)	N/A	0.005	0.001	0.052	1.304	0.386	0.110	0.150	0.024	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	26	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80	0.00
time (sec)	N/A	0.015	0.001	0.040	1.307	0.359	0.071	0.148	0.041	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	24	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80	0.00
time (sec)	N/A	0.020	0.001	0.038	1.341	0.340	0.071	0.157	0.033	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	26	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80	0.00
time (sec)	N/A	0.011	0.002	0.043	1.318	0.352	0.073	0.150	0.030	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	25	24	24	24	24	24	27
N.S.	1	1.00	1.00	1.56	1.50	1.50	1.50	1.50	1.50	1.69
time (sec)	N/A	0.007	0.002	0.042	1.292	0.371	0.073	0.197	0.031	0.012

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	22	21	21	22	21	21	25
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84	1.00
time (sec)	N/A	0.012	0.001	0.051	1.296	0.390	0.078	0.152	0.030	0.021

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	46	37	36	36	42	36	36	0
N.S.	1	1.00	1.00	0.80	0.78	0.78	0.91	0.78	0.78	0.00
time (sec)	N/A	0.016	0.002	0.036	1.332	0.347	0.074	0.153	0.039	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	24	23	22	20	24	22	0
N.S.	1	1.00	1.00	0.89	0.85	0.81	0.74	0.89	0.81	0.00
time (sec)	N/A	0.023	0.005	0.043	1.322	0.386	0.152	0.155	0.040	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	27	26	82	56	26	23	0
N.S.	1	1.00	1.00	0.87	0.84	2.65	1.81	0.84	0.74	0.00
time (sec)	N/A	0.016	0.009	0.044	2.942	0.423	0.166	0.148	4.913	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	13	10	14	13	0
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87	0.00
time (sec)	N/A	0.007	0.002	0.044	1.423	0.403	0.128	0.191	4.916	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	16	15	67	53	15	16	0
N.S.	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67	0.00
time (sec)	N/A	0.009	0.004	0.044	2.947	0.409	0.147	0.153	0.045	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	21	20	18	15	24	18	0
N.S.	1	1.00	1.00	0.95	0.91	0.82	0.68	1.09	0.82	0.00
time (sec)	N/A	0.012	0.005	0.046	1.343	0.399	0.211	0.147	0.063	0.001

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	34	30	29	82	65	29	26	0
N.S.	1	1.00	1.00	0.88	0.85	2.41	1.91	0.85	0.76	0.00
time (sec)	N/A	0.017	0.013	0.049	2.962	0.418	0.222	0.153	4.956	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	32	31	33	31	43	31	0
N.S.	1	1.00	1.00	0.91	0.89	0.94	0.89	1.23	0.89	0.00
time (sec)	N/A	0.026	0.007	0.051	1.269	0.402	0.292	0.167	0.060	0.001

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	39	40	106	87	40	37	0
N.S.	1	1.00	1.00	0.91	0.93	2.47	2.02	0.93	0.86	0.00
time (sec)	N/A	0.026	0.023	0.047	2.967	0.420	0.252	0.153	4.937	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	44	44	45	42	57	46	0
N.S.	1	1.00	1.00	0.90	0.90	0.92	0.86	1.16	0.94	0.00
time (sec)	N/A	0.033	0.007	0.054	1.388	0.407	0.335	0.157	0.062	0.001

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	45	36	35	120	78	35	33	0
N.S.	1	1.00	1.00	0.80	0.78	2.67	1.73	0.78	0.73	0.00
time (sec)	N/A	0.015	0.027	0.049	2.984	0.427	0.236	0.169	4.955	0.001

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	33	35	34	47	34	47	34	0
N.S.	1	1.00	0.87	0.92	0.89	1.24	0.89	1.24	0.89	0.00
time (sec)	N/A	0.028	0.018	0.054	1.402	0.410	0.334	0.158	0.049	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	54	46	49	136	92	47	44	0
N.S.	1	1.00	0.95	0.81	0.86	2.39	1.61	0.82	0.77	0.00
time (sec)	N/A	0.018	0.038	0.055	2.961	0.422	0.329	0.152	4.975	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	41	46	50	73	51	51	51	0
N.S.	1	1.00	0.84	0.94	1.02	1.49	1.04	1.04	1.04	0.00
time (sec)	N/A	0.039	0.037	0.054	1.365	0.402	0.411	0.209	0.053	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	67	59	64	172	114	59	58	0
N.S.	1	1.00	0.99	0.87	0.94	2.53	1.68	0.87	0.85	0.00
time (sec)	N/A	0.029	0.040	0.061	2.918	0.412	0.386	0.168	5.026	0.001

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	A	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	29	22	21	21	19	23	11	0
N.S.	1	1.00	2.23	1.69	1.62	1.62	1.46	1.77	0.85	0.00
time (sec)	N/A	0.011	0.005	0.046	1.375	0.402	0.107	0.174	4.977	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	18	19	18	14	14	15	14	0
N.S.	1	1.00	0.90	0.95	0.90	0.70	0.70	0.75	0.70	0.00
time (sec)	N/A	0.015	0.003	0.041	1.322	0.389	0.084	0.151	0.036	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	6	6	22	17	16	16	14	18	6	0
N.S.	1	1.00	3.67	2.83	2.67	2.67	2.33	3.00	1.00	0.00
time (sec)	N/A	0.008	0.003	0.049	1.294	0.394	0.108	0.166	0.058	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	14	13	8	8	15	8	0
N.S.	1	1.00	1.00	1.17	1.08	0.67	0.67	1.25	0.67	0.00
time (sec)	N/A	0.006	0.002	0.046	1.295	0.391	0.099	0.148	0.029	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	2	2	19	3	13	13	12	15	2	0
N.S.	1	1.00	9.50	1.50	6.50	6.50	6.00	7.50	1.00	0.00
time (sec)	N/A	0.004	0.002	0.039	1.279	0.409	0.116	0.168	0.033	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	16	15	11	10	16	11	0
N.S.	1	1.00	1.00	1.07	1.00	0.73	0.67	1.07	0.73	0.00
time (sec)	N/A	0.008	0.003	0.057	1.317	0.393	0.106	0.156	4.960	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	24	19	18	20	15	20	8	0
N.S.	1	1.00	3.00	2.38	2.25	2.50	1.88	2.50	1.00	0.00
time (sec)	N/A	0.008	0.003	0.051	1.327	0.407	0.125	0.153	0.033	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	21	20	24	17	26	16	0
N.S.	1	1.00	1.00	0.95	0.91	1.09	0.77	1.18	0.73	0.00
time (sec)	N/A	0.015	0.004	0.049	1.292	0.395	0.113	0.151	0.033	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	31	24	25	30	24	27	13	0
N.S.	1	1.00	2.07	1.60	1.67	2.00	1.60	1.80	0.87	0.00
time (sec)	N/A	0.010	0.004	0.057	1.263	0.404	0.137	0.148	4.924	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	26	27	30	22	33	23	0
N.S.	1	1.00	1.00	0.90	0.93	1.03	0.76	1.14	0.79	0.00
time (sec)	N/A	0.016	0.004	0.046	1.346	0.381	0.131	0.160	0.033	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	13	12	18	14	0
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	1.20	0.93	0.00
time (sec)	N/A	0.009	0.004	0.045	1.289	0.396	0.150	0.149	4.950	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	16	15	15	12	18	16	0
N.S.	1	1.00	1.00	0.89	0.83	0.83	0.67	1.00	0.89	0.00
time (sec)	N/A	0.010	0.004	0.047	1.262	0.385	0.143	0.148	0.052	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	159	130	212	0	376	0	100	-1	124
N.S.	1	1.00	0.82	1.33	0.00	2.36	0.00	0.63	-0.01	0.78
time (sec)	N/A	0.248	0.213	0.092	0.000	0.449	0.000	0.325	0.000	0.127

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	77	70	0	108	0	80	-1	75
N.S.	1	1.00	0.61	0.56	0.00	0.86	0.00	0.63	-0.01	0.60
time (sec)	N/A	0.200	0.042	0.044	0.000	0.414	0.000	0.214	0.000	0.107

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	120	198	0	348	0	86	-1	110
N.S.	1	1.00	0.92	1.52	0.00	2.68	0.00	0.66	-0.01	0.85
time (sec)	N/A	0.206	0.286	0.078	0.000	0.435	0.000	0.305	0.000	0.115

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	66	59	0	97	0	64	-1	64
N.S.	1	1.00	0.65	0.58	0.00	0.96	0.00	0.63	-0.01	0.63
time (sec)	N/A	0.161	0.034	0.046	0.000	0.412	0.000	0.205	0.000	0.107

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	27	0	61	0	17	-1	32
N.S.	1	1.00	1.00	1.08	0.00	2.44	0.00	0.68	-0.04	1.28
time (sec)	N/A	0.037	0.019	0.048	0.000	0.394	0.000	0.293	0.000	0.105

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	55	48	0	86	0	50	-1	53
N.S.	1	1.00	0.72	0.63	0.00	1.13	0.00	0.66	-0.01	0.70
time (sec)	N/A	0.119	0.028	0.047	0.000	0.412	0.000	0.200	0.000	0.115

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	44	37	0	76	0	29	-1	42
N.S.	1	1.00	0.86	0.73	0.00	1.49	0.00	0.57	-0.02	0.82
time (sec)	N/A	0.075	0.024	0.063	0.000	0.432	0.000	0.291	0.000	0.108

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	44	37	0	75	0	33	-1	35
N.S.	1	1.00	0.86	0.73	0.00	1.47	0.00	0.65	-0.02	0.69
time (sec)	N/A	0.074	0.023	0.058	0.000	0.431	0.000	0.303	0.000	0.634

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	55	48	0	87	0	43	-1	46
N.S.	1	1.00	0.72	0.63	0.00	1.14	0.00	0.57	-0.01	0.61
time (sec)	N/A	0.113	0.023	0.048	0.000	0.412	0.000	0.284	0.000	0.916

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	27	0	63	0	23	-1	25
N.S.	1	1.00	1.00	1.08	0.00	2.52	0.00	0.92	-0.04	1.00
time (sec)	N/A	0.037	0.016	0.042	0.000	0.429	0.000	0.197	0.000	0.501

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	66	59	0	95	0	55	-1	57
N.S.	1	1.00	0.65	0.58	0.00	0.94	0.00	0.54	-0.01	0.56
time (sec)	N/A	0.157	0.025	0.043	0.000	0.401	0.000	0.248	0.000	0.892

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	43	217	0	360	0	114	-1	99
N.S.	1	1.00	0.33	1.67	0.00	2.77	0.00	0.88	-0.01	0.76
time (sec)	N/A	0.203	0.014	0.054	0.000	0.442	0.000	0.267	0.000	1.036

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	77	70	0	110	0	90	-1	68
N.S.	1	1.00	0.61	0.56	0.00	0.87	0.00	0.71	-0.01	0.54
time (sec)	N/A	0.192	0.041	0.049	0.000	0.473	0.000	0.269	0.000	1.347

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	159	44	234	0	396	0	104	-1	113
N.S.	1	1.00	0.28	1.47	0.00	2.49	0.00	0.65	-0.01	0.71
time (sec)	N/A	0.241	0.023	0.057	0.000	0.428	0.000	0.250	0.000	1.949

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	152	88	81	0	121	0	147	-1	79
N.S.	1	1.00	0.58	0.53	0.00	0.80	0.00	0.97	-0.01	0.52
time (sec)	N/A	0.233	0.045	0.045	0.000	0.519	0.000	0.324	0.000	2.792

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	46	247	0	422	0	138	-1	126
N.S.	1	1.00	0.24	1.31	0.00	2.23	0.00	0.73	-0.01	0.67
time (sec)	N/A	0.293	0.027	0.061	0.000	0.428	0.000	0.267	0.000	3.423

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	180	180	99	92	0	132	0	202	-1	90
N.S.	1	1.00	0.55	0.51	0.00	0.73	0.00	1.12	-0.01	0.50
time (sec)	N/A	0.285	0.043	0.046	0.000	0.554	0.000	0.412	0.000	5.047

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	81	997	0	133	0	45	-1	62
N.S.	1	1.00	1.47	18.13	0.00	2.42	0.00	0.82	-0.02	1.13
time (sec)	N/A	0.070	0.050	0.089	0.000	0.565	0.000	0.298	0.000	0.539

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	61	979	0	94	0	23	-1	39
N.S.	1	1.00	1.91	30.59	0.00	2.94	0.00	0.72	-0.03	1.22
time (sec)	N/A	0.032	0.032	0.087	0.000	0.574	0.000	0.210	0.000	0.409

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	27	26	19	0	14	19	23
N.S.	1	1.00	1.00	1.17	1.13	0.83	0.00	0.61	0.83	1.00
time (sec)	N/A	0.034	0.029	0.041	1.461	0.420	0.000	0.222	5.134	0.441

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	31	35	38	29	0	30	27	33
N.S.	1	1.00	0.65	0.73	0.79	0.60	0.00	0.62	0.56	0.69
time (sec)	N/A	0.066	0.028	0.040	1.512	0.418	0.000	0.192	5.129	0.461

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	44	48	50	40	0	47	40	44
N.S.	1	1.00	0.59	0.65	0.68	0.54	0.00	0.64	0.54	0.59
time (sec)	N/A	0.102	0.028	0.046	1.494	0.428	0.000	0.199	5.266	0.505

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	174	174	151	223	0	0	0	111	-1	113
N.S.	1	1.00	0.87	1.28	0.00	0.00	0.00	0.64	-0.01	0.65
time (sec)	N/A	0.151	0.187	0.093	0.000	0.000	0.000	0.288	0.000	0.306

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	129	181	0	0	0	83	-1	95
N.S.	1	1.00	1.11	1.56	0.00	0.00	0.00	0.72	-0.01	0.82
time (sec)	N/A	0.087	0.116	0.051	0.000	0.000	0.000	0.312	0.000	0.247

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	88	83	0	0	0	54	72	65
N.S.	1	1.00	1.57	1.48	0.00	0.00	0.00	0.96	1.29	1.16
time (sec)	N/A	0.060	0.053	0.049	0.000	0.000	0.000	0.285	5.243	0.196

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	159	0	19	0	25	-1	25
N.S.	1	1.00	1.00	6.36	0.00	0.76	0.00	1.00	-0.04	1.00
time (sec)	N/A	0.038	0.009	0.055	0.000	0.855	0.000	0.177	0.000	0.144

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	48	218	0	42	0	84	-1	48
N.S.	1	1.00	0.57	2.60	0.00	0.50	0.00	1.00	-0.01	0.57
time (sec)	N/A	0.117	0.052	0.066	0.000	0.901	0.000	0.234	0.000	0.168

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	142	142	72	262	0	64	0	146	-1	72
N.S.	1	1.00	0.51	1.85	0.00	0.45	0.00	1.03	-0.01	0.51
time (sec)	N/A	0.203	0.054	0.065	0.000	0.704	0.000	0.191	0.000	0.201

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	200	200	96	306	0	86	0	208	-1	96
N.S.	1	1.00	0.48	1.53	0.00	0.43	0.00	1.04	-0.00	0.48
time (sec)	N/A	0.297	0.062	0.068	0.000	0.765	0.000	0.198	0.000	0.228

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	197	197	64	549	0	0	0	0	-1	137
N.S.	1	1.00	0.32	2.79	0.00	0.00	0.00	0.00	-0.01	0.70
time (sec)	N/A	0.175	0.065	0.062	0.000	0.000	0.000	0.000	0.000	0.505

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	64	503	0	0	0	0	-1	119
N.S.	1	1.00	0.46	3.62	0.00	0.00	0.00	0.00	-0.01	0.86
time (sec)	N/A	0.127	0.075	0.064	0.000	0.000	0.000	0.000	0.000	0.520

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	64	236	0	0	0	94	-1	90
N.S.	1	1.00	0.83	3.06	0.00	0.00	0.00	1.22	-0.01	1.17
time (sec)	N/A	0.074	0.043	0.048	0.000	0.000	0.000	0.321	0.000	0.386

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	404	0	36	0	34	40	31
N.S.	1	1.00	1.00	16.16	0.00	1.44	0.00	1.36	1.60	1.24
time (sec)	N/A	0.005	0.019	0.059	0.000	0.771	0.000	0.190	5.426	0.275

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	48	524	0	63	0	0	-1	57
N.S.	1	1.00	0.61	6.63	0.00	0.80	0.00	0.00	-0.01	0.72
time (sec)	N/A	0.122	0.082	0.066	0.000	0.815	0.000	0.000	0.000	0.304

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	72	570	0	87	0	0	-1	81
N.S.	1	1.00	0.53	4.16	0.00	0.64	0.00	0.00	-0.01	0.59
time (sec)	N/A	0.202	0.078	0.064	0.000	1.157	0.000	0.000	0.000	0.328

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	195	195	96	614	0	109	0	0	-1	105
N.S.	1	1.00	0.49	3.15	0.00	0.56	0.00	0.00	-0.01	0.54
time (sec)	N/A	0.300	0.085	0.067	0.000	0.870	0.000	0.000	0.000	0.347

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	204	204	164	245	0	0	0	125	-1	126
N.S.	1	1.00	0.80	1.20	0.00	0.00	0.00	0.61	-0.00	0.62
time (sec)	N/A	0.172	0.264	0.055	0.000	0.000	0.000	0.283	0.000	0.354

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	142	203	0	0	0	97	-1	108
N.S.	1	1.00	0.97	1.39	0.00	0.00	0.00	0.66	-0.01	0.74
time (sec)	N/A	0.120	0.209	0.054	0.000	0.000	0.000	0.406	0.000	0.299

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	102	160	0	0	0	69	-1	86
N.S.	1	1.00	1.17	1.84	0.00	0.00	0.00	0.79	-0.01	0.99
time (sec)	N/A	0.079	0.117	0.051	0.000	0.000	0.000	0.269	0.000	0.275

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	65	136	0	0	0	37	-1	40
N.S.	1	1.00	1.91	4.00	0.00	0.00	0.00	1.09	-0.03	1.18
time (sec)	N/A	0.049	0.074	0.050	0.000	0.000	0.000	0.263	0.000	0.144

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	37	194	0	29	0	53	-1	37
N.S.	1	1.00	0.69	3.59	0.00	0.54	0.00	0.98	-0.02	0.69
time (sec)	N/A	0.074	0.061	0.063	0.000	0.703	0.000	0.189	0.000	0.162

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	59	240	0	50	0	115	-1	59
N.S.	1	1.00	0.53	2.14	0.00	0.45	0.00	1.03	-0.01	0.53
time (sec)	N/A	0.154	0.069	0.060	0.000	0.851	0.000	0.211	0.000	0.183

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	170	170	83	284	0	72	0	177	-1	83
N.S.	1	1.00	0.49	1.67	0.00	0.42	0.00	1.04	-0.01	0.49
time (sec)	N/A	0.245	0.086	0.065	0.000	0.710	0.000	0.236	0.000	0.196

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	171	171	62	527	0	0	0	0	-1	124
N.S.	1	1.00	0.36	3.08	0.00	0.00	0.00	0.00	-0.01	0.73
time (sec)	N/A	0.147	0.083	0.064	0.000	0.000	0.000	0.000	0.000	0.451

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	62	440	0	0	0	0	-1	106
N.S.	1	1.00	0.55	3.89	0.00	0.00	0.00	0.00	-0.01	0.94
time (sec)	N/A	0.103	0.068	0.060	0.000	0.000	0.000	0.000	0.000	0.466

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	79	240	0	0	0	0	-1	76
N.S.	1	1.00	1.32	4.00	0.00	0.00	0.00	0.00	-0.02	1.27
time (sec)	N/A	0.067	0.113	0.049	0.000	0.000	0.000	0.000	0.000	0.303

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	45	111	0	54	0	26	-1	46
N.S.	1	1.00	1.50	3.70	0.00	1.80	0.00	0.87	-0.03	1.53
time (sec)	N/A	0.047	0.042	0.059	0.000	0.629	0.000	0.183	0.000	0.280

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	57	548	0	79	0	0	-1	70
N.S.	1	1.00	0.53	5.12	0.00	0.74	0.00	0.00	-0.01	0.65
time (sec)	N/A	0.156	0.051	0.063	0.000	0.646	0.000	0.000	0.000	0.311

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	165	165	83	592	0	101	0	0	-1	96
N.S.	1	1.00	0.50	3.59	0.00	0.61	0.00	0.00	-0.01	0.58
time (sec)	N/A	0.257	0.069	0.067	0.000	0.579	0.000	0.000	0.000	0.348

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	107	636	0	123	0	0	-1	120
N.S.	1	1.00	0.48	2.85	0.00	0.55	0.00	0.00	-0.00	0.54
time (sec)	N/A	0.353	0.066	0.094	0.000	0.479	0.000	0.000	0.000	0.357

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	371	371	181	156	0	0	0	396	-1	185
N.S.	1	1.00	0.49	0.42	0.00	0.00	0.00	1.07	-0.00	0.50
time (sec)	N/A	0.627	0.184	0.070	0.000	0.000	0.000	0.223	0.000	0.117

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	283	283	144	123	0	0	0	312	-1	133
N.S.	1	1.00	0.51	0.43	0.00	0.00	0.00	1.10	-0.00	0.47
time (sec)	N/A	0.441	0.130	0.048	0.000	0.000	0.000	0.216	0.000	0.102

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	195	195	107	90	0	0	0	228	-1	96
N.S.	1	1.00	0.55	0.46	0.00	0.00	0.00	1.17	-0.01	0.49
time (sec)	N/A	0.273	0.090	0.049	0.000	0.000	0.000	0.194	0.000	0.073

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	70	57	0	0	0	143	40	74
N.S.	1	1.00	0.64	0.52	0.00	0.00	0.00	1.31	0.37	0.68
time (sec)	N/A	0.137	0.047	0.045	0.000	0.000	0.000	0.184	5.192	0.056

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	27	0	0	0	23	-1	23
N.S.	1	1.00	1.00	1.17	0.00	0.00	0.00	1.00	-0.04	1.00
time (sec)	N/A	0.040	0.012	0.039	0.000	0.000	0.000	0.208	0.000	0.050

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	57	80	0	0	0	72	-1	76
N.S.	1	1.00	0.63	0.89	0.00	0.00	0.00	0.80	-0.01	0.84
time (sec)	N/A	0.139	0.055	0.049	0.000	0.000	0.000	0.280	0.000	0.186

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	178	178	57	125	0	0	0	126	-1	112
N.S.	1	1.00	0.32	0.70	0.00	0.00	0.00	0.71	-0.01	0.63
time (sec)	N/A	0.296	0.075	0.061	0.000	0.000	0.000	0.278	0.000	0.258

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	266	266	57	167	0	0	0	177	-1	149
N.S.	1	1.00	0.21	0.63	0.00	0.00	0.00	0.67	-0.00	0.56
time (sec)	N/A	0.475	0.050	0.059	0.000	0.000	0.000	0.352	0.000	0.291

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	354	354	57	209	0	0	0	228	-1	186
N.S.	1	1.00	0.16	0.59	0.00	0.00	0.00	0.64	-0.00	0.53
time (sec)	N/A	0.660	0.052	0.061	0.000	0.000	0.000	0.432	0.000	0.391

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	343	343	172	145	0	0	0	770	-1	198
N.S.	1	1.00	0.50	0.42	0.00	0.00	0.00	2.24	-0.00	0.58
time (sec)	N/A	0.616	0.161	0.048	0.000	0.000	0.000	0.308	0.000	4.514

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	135	112	0	0	0	602	-1	161
N.S.	1	1.00	0.53	0.44	0.00	0.00	0.00	2.36	-0.00	0.63
time (sec)	N/A	0.422	0.112	0.051	0.000	0.000	0.000	0.303	0.000	4.497

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	98	79	0	0	0	434	40	98
N.S.	1	1.00	0.58	0.47	0.00	0.00	0.00	2.57	0.24	0.58
time (sec)	N/A	0.249	0.070	0.049	0.000	0.000	0.000	0.230	5.141	4.467

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	63	48	0	0	0	265	-1	87
N.S.	1	1.00	0.75	0.57	0.00	0.00	0.00	3.15	-0.01	1.04
time (sec)	N/A	0.139	0.080	0.051	0.000	0.000	0.000	0.211	0.000	4.539

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	88	69	0	0	0	83	-1	90
N.S.	1	1.00	1.13	0.88	0.00	0.00	0.00	1.06	-0.01	1.15
time (sec)	N/A	0.137	0.104	0.051	0.000	0.000	0.000	0.231	0.000	10.825

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	61	93	0	0	0	92	-1	117
N.S.	1	1.00	0.54	0.82	0.00	0.00	0.00	0.81	-0.01	1.04
time (sec)	N/A	0.184	0.074	0.061	0.000	0.000	0.000	0.261	0.000	16.118

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	203	203	61	139	0	0	0	143	-1	152
N.S.	1	1.00	0.30	0.68	0.00	0.00	0.00	0.70	-0.00	0.75
time (sec)	N/A	0.340	0.054	0.062	0.000	0.000	0.000	0.368	0.000	17.126

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	F(-1)	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	291	291	61	181	0	0	0	194	-1	189
N.S.	1	1.00	0.21	0.62	0.00	0.00	0.00	0.67	-0.00	0.65
time (sec)	N/A	0.522	0.060	0.059	0.000	0.000	0.000	0.383	0.000	18.450

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	F(-1)	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	379	379	61	223	0	0	0	245	-1	226
N.S.	1	1.00	0.16	0.59	0.00	0.00	0.00	0.65	-0.00	0.60
time (sec)	N/A	0.718	0.096	0.069	0.000	0.000	0.000	0.472	0.000	19.894

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	401	401	185	167	0	0	0	206	-1	185
N.S.	1	1.00	0.46	0.42	0.00	0.00	0.00	0.51	-0.00	0.46
time (sec)	N/A	0.728	0.229	0.051	0.000	0.000	0.000	0.299	0.000	0.131

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	313	313	148	134	0	0	0	164	-1	148
N.S.	1	1.00	0.47	0.43	0.00	0.00	0.00	0.52	-0.00	0.47
time (sec)	N/A	0.531	0.141	0.044	0.000	0.000	0.000	0.188	0.000	0.105

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	225	225	111	101	0	0	0	122	-1	111
N.S.	1	1.00	0.49	0.45	0.00	0.00	0.00	0.54	-0.00	0.49
time (sec)	N/A	0.346	0.128	0.046	0.000	0.000	0.000	0.269	0.000	0.101

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	74	68	0	0	0	80	-1	74
N.S.	1	1.00	0.54	0.50	0.00	0.00	0.00	0.58	-0.01	0.54
time (sec)	N/A	0.180	0.084	0.049	0.000	0.000	0.000	0.176	0.000	0.065

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	36	36	0	0	0	36	40	36
N.S.	1	1.00	0.77	0.77	0.00	0.00	0.00	0.77	0.85	0.77
time (sec)	N/A	0.050	0.038	0.044	0.000	0.000	0.000	0.488	5.219	0.043

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	90	61	0	0	0	51	-1	61
N.S.	1	1.00	1.48	1.00	0.00	0.00	0.00	0.84	-0.02	1.00
time (sec)	N/A	0.093	0.130	0.050	0.000	0.000	0.000	0.217	0.000	0.137

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	48	126	0	0	0	109	-1	101
N.S.	1	1.00	0.31	0.82	0.00	0.00	0.00	0.71	-0.01	0.66
time (sec)	N/A	0.239	0.063	0.048	0.000	0.000	0.000	0.260	0.000	0.206

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	241	241	48	188	0	0	0	160	-1	138
N.S.	1	1.00	0.20	0.78	0.00	0.00	0.00	0.66	-0.00	0.57
time (sec)	N/A	0.408	0.079	0.050	0.000	0.000	0.000	0.310	0.000	0.218

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	329	329	48	248	0	0	0	211	-1	175
N.S.	1	1.00	0.15	0.75	0.00	0.00	0.00	0.64	-0.00	0.53
time (sec)	N/A	0.577	0.071	0.053	0.000	0.000	0.000	0.359	0.000	0.247

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	336	336	161	143	0	0	0	214	-1	165
N.S.	1	1.00	0.48	0.43	0.00	0.00	0.00	0.64	-0.00	0.49
time (sec)	N/A	0.599	0.192	0.050	0.000	0.000	0.000	0.245	0.000	3.985

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	248	248	122	110	0	0	0	163	-1	128
N.S.	1	1.00	0.49	0.44	0.00	0.00	0.00	0.66	-0.00	0.52
time (sec)	N/A	0.414	0.127	0.052	0.000	0.000	0.000	0.215	0.000	3.937

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	85	77	0	0	0	112	-1	91
N.S.	1	1.00	0.53	0.48	0.00	0.00	0.00	0.70	-0.01	0.57
time (sec)	N/A	0.242	0.100	0.052	0.000	0.000	0.000	0.228	0.000	4.183

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	60	45	0	0	0	60	-1	54
N.S.	1	1.00	0.88	0.66	0.00	0.00	0.00	0.88	-0.01	0.79
time (sec)	N/A	0.084	0.063	0.047	0.000	0.000	0.000	0.219	0.000	3.974

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	F(-1)	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	45	56	0	0	0	71	40	71
N.S.	1	1.00	0.75	0.93	0.00	0.00	0.00	1.18	0.67	1.18
time (sec)	N/A	0.056	0.042	0.051	0.000	0.000	0.000	0.212	5.358	2.651

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	48	88	0	0	0	105	-1	128
N.S.	1	1.00	0.33	0.60	0.00	0.00	0.00	0.72	-0.01	0.88
time (sec)	N/A	0.241	0.088	0.057	0.000	0.000	0.000	0.259	0.000	4.342

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	236	236	48	126	0	0	0	156	-1	165
N.S.	1	1.00	0.20	0.53	0.00	0.00	0.00	0.66	-0.00	0.70
time (sec)	N/A	0.410	0.076	0.064	0.000	0.000	0.000	0.391	0.000	12.491

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	324	324	48	159	0	0	0	207	-1	202
N.S.	1	1.00	0.15	0.49	0.00	0.00	0.00	0.64	-0.00	0.62
time (sec)	N/A	0.597	0.075	0.069	0.000	0.000	0.000	0.376	0.000	17.211

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	412	412	48	192	0	0	0	258	-1	239
N.S.	1	1.00	0.12	0.47	0.00	0.00	0.00	0.63	-0.00	0.58
time (sec)	N/A	0.840	0.089	0.079	0.000	0.000	0.000	0.452	0.000	18.971

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	12	13	13	0
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.00
time (sec)	N/A	0.009	0.001	0.037	1.334	0.397	0.061	0.187	0.022	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	12	13	13	0
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.00
time (sec)	N/A	0.008	0.001	0.040	1.337	0.440	0.061	0.150	0.022	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	12	13	13	0
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.00
time (sec)	N/A	0.003	0.000	0.045	1.337	0.505	0.067	0.161	0.022	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	12	13	13	15
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.88
time (sec)	N/A	0.006	0.001	0.055	1.363	0.614	0.062	0.147	0.020	0.016

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	10	10	8	10	10	12
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83	1.00
time (sec)	N/A	0.004	0.001	0.046	1.344	0.501	0.062	0.146	0.017	0.013

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	24	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80	0.00
time (sec)	N/A	0.031	0.002	0.045	1.354	0.345	0.070	0.150	0.040	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	26	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80	0.00
time (sec)	N/A	0.017	0.002	0.056	1.205	0.336	0.070	0.148	0.033	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	24	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80	0.00
time (sec)	N/A	0.013	0.002	0.043	1.332	0.353	0.070	0.160	0.031	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	26	24	24	30
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80	1.00
time (sec)	N/A	0.017	0.002	0.038	1.270	0.387	0.072	0.166	0.034	0.024

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	24	24	24	30
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80	1.00
time (sec)	N/A	0.016	0.002	0.046	1.326	0.383	0.070	0.152	0.032	0.021

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	57	52	52	52	49	53	51	0
N.S.	1	1.00	1.00	0.91	0.91	0.91	0.86	0.93	0.89	0.00
time (sec)	N/A	0.036	0.028	0.043	1.274	0.390	0.143	0.151	5.093	0.001

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	44	41	42	41	37	43	40	0
N.S.	1	1.00	1.00	0.93	0.95	0.93	0.84	0.98	0.91	0.00
time (sec)	N/A	0.027	0.004	0.043	1.305	0.396	0.131	0.140	0.042	0.001

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	30	29	29	26	30	29	0
N.S.	1	1.00	1.00	0.97	0.94	0.94	0.84	0.97	0.94	0.00
time (sec)	N/A	0.021	0.004	0.045	1.366	0.387	0.120	0.191	0.041	0.001

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	19	18	17	14	19	18	0
N.S.	1	1.00	1.00	1.06	1.00	0.94	0.78	1.06	1.00	0.00
time (sec)	N/A	0.016	0.003	0.049	1.300	0.376	0.114	0.148	0.036	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	11	10	10	7	11	10	0
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.10	1.00	0.00
time (sec)	N/A	0.007	0.001	0.042	1.323	0.376	0.071	0.146	0.022	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	19	18	16	10	20	15	0
N.S.	1	1.00	1.00	1.06	1.00	0.89	0.56	1.11	0.83	0.00
time (sec)	N/A	0.007	0.004	0.047	1.297	0.385	0.164	0.160	5.120	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	29	28	26	19	30	25	0
N.S.	1	1.00	1.00	1.04	1.00	0.93	0.68	1.07	0.89	0.00
time (sec)	N/A	0.015	0.005	0.050	1.357	0.396	0.198	0.153	0.052	0.001

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	41	40	41	31	45	38	0
N.S.	1	1.00	1.00	0.98	0.95	0.98	0.74	1.07	0.90	0.00
time (sec)	N/A	0.021	0.032	0.053	1.366	0.395	0.214	0.153	0.059	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	56	53	51	54	44	56	48	0
N.S.	1	1.00	1.00	0.95	0.91	0.96	0.79	1.00	0.86	0.00
time (sec)	N/A	0.027	0.006	0.053	1.332	0.385	0.235	0.161	0.061	0.001

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	54	57	59	73	54	62	62	0
N.S.	1	1.00	0.93	0.98	1.02	1.26	0.93	1.07	1.07	0.00
time (sec)	N/A	0.040	0.023	0.052	1.307	0.380	0.210	0.172	0.038	0.001

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	43	45	47	62	44	48	50	0
N.S.	1	1.00	0.93	0.98	1.02	1.35	0.96	1.04	1.09	0.00
time (sec)	N/A	0.030	0.015	0.048	1.317	0.385	0.207	0.155	0.045	0.001

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	29	34	36	47	31	34	36	0
N.S.	1	1.00	0.88	1.03	1.09	1.42	0.94	1.03	1.09	0.00
time (sec)	N/A	0.024	0.014	0.050	1.277	0.387	0.183	0.150	0.040	0.001

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	20	24	26	28	20	24	23	0
N.S.	1	1.00	0.87	1.04	1.13	1.22	0.87	1.04	1.00	0.00
time (sec)	N/A	0.018	0.007	0.049	1.330	0.383	0.141	0.154	0.037	0.001

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	13	13	13	10	12	12	0
N.S.	1	1.00	1.00	1.08	1.08	1.08	0.83	1.00	1.00	0.00
time (sec)	N/A	0.007	0.003	0.049	1.301	0.378	0.141	0.146	5.169	0.001

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	24	30	28	39	22	31	26	0
N.S.	1	1.00	0.83	1.03	0.97	1.34	0.76	1.07	0.90	0.00
time (sec)	N/A	0.019	0.013	0.065	1.291	0.379	0.219	0.150	0.045	0.001

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	35	43	45	63	37	45	41	0
N.S.	1	1.00	0.83	1.02	1.07	1.50	0.88	1.07	0.98	0.00
time (sec)	N/A	0.024	0.051	0.051	1.306	0.399	0.272	0.155	5.343	0.001

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	53	57	64	86	54	64	57	0
N.S.	1	1.00	0.91	0.98	1.10	1.48	0.93	1.10	0.98	0.00
time (sec)	N/A	0.032	0.089	0.059	1.316	0.402	0.311	0.152	5.315	0.001

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	66	68	73	95	66	73	69	0
N.S.	1	1.00	0.96	0.99	1.06	1.38	0.96	1.06	1.00	0.00
time (sec)	N/A	0.039	0.070	0.051	1.343	0.394	0.330	0.147	0.071	0.001

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	79	79	86	108	80	86	79	0
N.S.	1	1.00	0.94	0.94	1.02	1.29	0.95	1.02	0.94	0.00
time (sec)	N/A	0.052	0.047	0.054	1.366	0.398	0.362	0.153	0.077	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	53	57	53	62	0	131	62	66
N.S.	1	1.00	0.50	0.54	0.50	0.59	0.00	1.25	0.59	0.63
time (sec)	N/A	0.120	0.042	0.043	1.459	0.381	0.000	0.163	5.487	0.048

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	42	46	42	51	0	108	51	55
N.S.	1	1.00	0.52	0.58	0.52	0.64	0.00	1.35	0.64	0.69
time (sec)	N/A	0.074	0.044	0.046	1.444	0.393	0.000	0.157	5.475	0.039

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	31	35	30	39	0	81	39	43
N.S.	1	1.00	0.60	0.67	0.58	0.75	0.00	1.56	0.75	0.83
time (sec)	N/A	0.043	0.024	0.049	1.417	0.382	0.000	0.180	5.302	0.025

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	23	27	12	26	0	50	-1	25
N.S.	1	1.00	0.92	1.08	0.48	1.04	0.00	2.00	-0.04	1.00
time (sec)	N/A	0.036	0.011	0.041	1.426	0.402	0.000	0.155	0.000	0.034

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	53	51	0	111	0	67	73	51
N.S.	1	1.00	1.04	1.00	0.00	2.18	0.00	1.31	1.43	1.00
time (sec)	N/A	0.049	0.033	0.043	0.000	0.411	0.000	0.165	5.357	0.070

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	48	56	0	127	0	45	-1	52
N.S.	1	1.00	0.92	1.08	0.00	2.44	0.00	0.87	-0.02	1.00
time (sec)	N/A	0.052	0.054	0.053	0.000	0.409	0.000	0.257	0.000	0.067

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	42	73	0	149	0	72	-1	69
N.S.	1	1.00	0.50	0.87	0.00	1.77	0.00	0.86	-0.01	0.82
time (sec)	N/A	0.092	0.013	0.055	0.000	0.419	0.000	0.209	0.000	0.073

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	42	89	0	175	0	92	-1	80
N.S.	1	1.00	0.38	0.79	0.00	1.56	0.00	0.82	-0.01	0.71
time (sec)	N/A	0.138	0.014	0.056	0.000	0.407	0.000	0.268	0.000	0.077

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	80	79	86	95	0	282	80	87
N.S.	1	1.00	0.50	0.49	0.53	0.59	0.00	1.75	0.50	0.54
time (sec)	N/A	0.231	0.048	0.048	1.520	0.399	0.000	0.179	5.238	4.699

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	69	68	75	84	0	246	69	75
N.S.	1	1.00	0.51	0.50	0.55	0.62	0.00	1.81	0.51	0.55
time (sec)	N/A	0.172	0.040	0.051	1.549	0.391	0.000	0.193	5.237	4.560

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	58	57	64	73	0	210	58	84
N.S.	1	1.00	0.54	0.53	0.59	0.68	0.00	1.94	0.54	0.78
time (sec)	N/A	0.141	0.029	0.043	1.439	0.399	0.000	0.234	5.187	4.718

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	47	46	53	62	0	173	47	70
N.S.	1	1.00	0.59	0.58	0.66	0.78	0.00	2.16	0.59	0.88
time (sec)	N/A	0.133	0.027	0.045	1.547	0.397	0.000	0.239	5.175	4.661

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	36	35	41	50	0	136	36	60
N.S.	1	1.00	0.69	0.67	0.79	0.96	0.00	2.62	0.69	1.15
time (sec)	N/A	0.083	0.021	0.039	1.468	0.390	0.000	0.180	5.174	6.438

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	23	27	28	37	0	89	28	28
N.S.	1	1.00	0.92	1.08	1.12	1.48	0.00	3.56	1.12	1.12
time (sec)	N/A	0.041	0.014	0.041	1.388	0.401	0.000	0.162	5.618	8.588

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	68	61	0	130	0	85	-1	76
N.S.	1	1.00	0.92	0.82	0.00	1.76	0.00	1.15	-0.01	1.03
time (sec)	N/A	0.096	0.047	0.048	0.000	0.417	0.000	0.174	0.000	10.833

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	40	72	0	136	0	62	-1	75
N.S.	1	1.00	0.55	0.99	0.00	1.86	0.00	0.85	-0.01	1.03
time (sec)	N/A	0.093	0.015	0.055	0.000	0.432	0.000	0.207	0.000	13.190

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	72	74	0	154	0	70	-1	82
N.S.	1	1.00	0.89	0.91	0.00	1.90	0.00	0.86	-0.01	1.01
time (sec)	N/A	0.092	0.051	0.059	0.000	0.431	0.000	0.265	0.000	15.705

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	42	87	0	175	0	92	-1	97
N.S.	1	1.00	0.39	0.80	0.00	1.61	0.00	0.84	-0.01	0.89
time (sec)	N/A	0.134	0.022	0.056	0.000	0.433	0.000	0.229	0.000	15.701

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	42	101	0	197	0	109	-1	109
N.S.	1	1.00	0.31	0.74	0.00	1.44	0.00	0.80	-0.01	0.80
time (sec)	N/A	0.184	0.017	0.053	0.000	0.424	0.000	0.241	0.000	16.005

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	165	165	42	113	0	219	0	126	-1	121
N.S.	1	1.00	0.25	0.68	0.00	1.33	0.00	0.76	-0.01	0.73
time (sec)	N/A	0.236	0.029	0.054	0.000	0.414	0.000	0.267	0.000	16.399

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	53	55	53	51	0	0	51	55
N.S.	1	1.00	0.51	0.53	0.51	0.50	0.00	0.00	0.50	0.53
time (sec)	N/A	0.148	0.041	0.048	1.462	0.388	0.000	0.000	5.188	0.047

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	42	44	42	40	0	0	40	44
N.S.	1	1.00	0.56	0.59	0.56	0.53	0.00	0.00	0.53	0.59
time (sec)	N/A	0.100	0.027	0.046	1.459	0.391	0.000	0.000	5.201	0.041

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	30	33	30	28	0	0	31	32
N.S.	1	1.00	0.61	0.67	0.61	0.57	0.00	0.00	0.63	0.65
time (sec)	N/A	0.055	0.021	0.043	1.433	0.391	0.000	0.000	5.164	0.032

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	21	25	12	21	0	26	17	23
N.S.	1	1.00	0.91	1.09	0.52	0.91	0.00	1.13	0.74	1.00
time (sec)	N/A	0.010	0.008	0.043	1.410	0.404	0.000	0.197	5.142	0.026

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	46	39	0	74	0	45	-1	30
N.S.	1	1.00	1.53	1.30	0.00	2.47	0.00	1.50	-0.03	1.00
time (sec)	N/A	0.011	0.010	0.044	0.000	0.415	0.000	0.159	0.000	0.038

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	66	55	0	127	0	0	-1	54
N.S.	1	1.00	1.22	1.02	0.00	2.35	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.049	0.064	0.048	0.000	0.400	0.000	0.000	0.000	0.061

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	40	77	0	153	0	0	44	69
N.S.	1	1.00	0.46	0.89	0.00	1.76	0.00	0.00	0.51	0.79
time (sec)	N/A	0.091	0.011	0.046	0.000	0.410	0.000	0.000	5.408	0.074

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	40	95	0	175	0	0	-1	80
N.S.	1	1.00	0.35	0.83	0.00	1.52	0.00	0.00	-0.01	0.70
time (sec)	N/A	0.135	0.011	0.055	0.000	0.423	0.000	0.000	0.000	0.074

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	50	56	41	60	0	0	57	54
N.S.	1	1.00	0.51	0.57	0.42	0.61	0.00	0.00	0.58	0.55
time (sec)	N/A	0.151	0.035	0.046	1.481	0.397	0.000	0.000	5.282	3.874

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	39	46	30	49	0	0	47	42
N.S.	1	1.00	0.54	0.64	0.42	0.68	0.00	0.00	0.65	0.58
time (sec)	N/A	0.105	0.018	0.048	1.442	0.381	0.000	0.000	5.223	3.917

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	26	34	19	38	0	28	35	37
N.S.	1	1.00	0.55	0.72	0.40	0.81	0.00	0.60	0.74	0.79
time (sec)	N/A	0.057	0.014	0.047	1.536	0.389	0.000	0.280	5.175	2.409

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	19	27	12	29	0	37	28	30
N.S.	1	1.00	0.90	1.29	0.57	1.38	0.00	1.76	1.33	1.43
time (sec)	N/A	0.018	0.007	0.044	1.479	0.380	0.000	0.226	5.074	2.193

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	35	54	0	156	0	0	-1	63
N.S.	1	1.00	0.67	1.04	0.00	3.00	0.00	0.00	-0.02	1.21
time (sec)	N/A	0.058	0.029	0.053	0.000	0.409	0.000	0.000	0.000	2.410

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	36	62	0	189	0	0	-1	76
N.S.	1	1.00	0.48	0.83	0.00	2.52	0.00	0.00	-0.01	1.01
time (sec)	N/A	0.086	0.010	0.059	0.000	0.415	0.000	0.000	0.000	3.793

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	38	76	0	219	0	0	42	95
N.S.	1	1.00	0.35	0.69	0.00	1.99	0.00	0.00	0.38	0.86
time (sec)	N/A	0.105	0.019	0.057	0.000	0.409	0.000	0.000	5.432	3.723

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	38	86	0	241	0	0	-1	107
N.S.	1	1.00	0.28	0.62	0.00	1.75	0.00	0.00	-0.01	0.78
time (sec)	N/A	0.186	0.010	0.061	0.000	0.415	0.000	0.000	0.000	4.041

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	38	100	0	263	0	0	44	119
N.S.	1	1.00	0.23	0.60	0.00	1.58	0.00	0.00	0.27	0.72
time (sec)	N/A	0.232	0.012	0.064	0.000	0.429	0.000	0.000	5.683	6.307

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	125	104	103	0	180	0	64	-1	106
N.S.	1	1.00	0.83	0.82	0.00	1.44	0.00	0.51	-0.01	0.85
time (sec)	N/A	0.169	0.135	0.069	0.000	0.418	0.000	0.204	0.000	0.183

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	90	92	0	159	0	52	-1	95
N.S.	1	1.00	0.95	0.97	0.00	1.67	0.00	0.55	-0.01	1.00
time (sec)	N/A	0.125	0.055	0.048	0.000	0.407	0.000	0.185	0.000	0.163

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	73	78	0	131	0	38	-1	75
N.S.	1	1.00	1.22	1.30	0.00	2.18	0.00	0.63	-0.02	1.25
time (sec)	N/A	0.083	0.043	0.049	0.000	0.431	0.000	0.178	0.000	0.139

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	55	58	0	77	0	23	-1	50
N.S.	1	1.00	1.62	1.71	0.00	2.26	0.00	0.68	-0.03	1.47
time (sec)	N/A	0.042	0.018	0.047	0.000	0.423	0.000	0.175	0.000	0.090

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	23	27	0	21	0	30	-1	25
N.S.	1	1.00	0.92	1.08	0.00	0.84	0.00	1.20	-0.04	1.00
time (sec)	N/A	0.038	0.010	0.037	0.000	0.390	0.000	0.236	0.000	0.091

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	31	33	0	29	0	55	-1	35
N.S.	1	1.00	0.55	0.59	0.00	0.52	0.00	0.98	-0.02	0.62
time (sec)	N/A	0.077	0.016	0.063	0.000	0.403	0.000	0.214	0.000	0.118

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	44	46	0	40	0	77	-1	46
N.S.	1	1.00	0.51	0.53	0.00	0.47	0.00	0.90	-0.01	0.53
time (sec)	N/A	0.118	0.016	0.045	0.000	0.397	0.000	0.202	0.000	0.135

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	55	57	0	51	0	103	-1	57
N.S.	1	1.00	0.47	0.49	0.00	0.44	0.00	0.89	-0.01	0.49
time (sec)	N/A	0.163	0.020	0.045	0.000	0.403	0.000	0.211	0.000	0.145

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	30	36	0	38	0	0	54	0
N.S.	1	1.00	0.94	1.12	0.00	1.19	0.00	0.00	1.69	0.00
time (sec)	N/A	0.025	0.016	0.046	0.000	0.413	0.000	0.000	5.281	0.097

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	44	50	0	70	0	0	98	0
N.S.	1	1.00	0.63	0.71	0.00	1.00	0.00	0.00	1.40	0.00
time (sec)	N/A	0.054	0.023	0.048	0.000	0.421	0.000	0.000	5.275	0.068

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	72	84	0	111	0	0	157	0
N.S.	1	1.00	0.62	0.72	0.00	0.96	0.00	0.00	1.35	0.00
time (sec)	N/A	0.091	0.032	0.049	0.000	0.417	0.000	0.000	5.365	0.095

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	24	31	36	36	36	22	37	0
N.S.	1	1.00	1.26	1.63	1.89	1.89	1.89	1.16	1.95	0.00
time (sec)	N/A	0.010	0.008	0.056	1.286	0.376	0.405	0.176	5.110	0.001

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	46	48	46	42	0	0	42	46
N.S.	1	1.00	0.58	0.60	0.58	0.52	0.00	0.00	0.52	0.58
time (sec)	N/A	0.115	0.034	0.050	1.433	0.392	0.000	0.000	5.184	0.042

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	34	37	34	30	0	0	33	34
N.S.	1	1.00	0.65	0.71	0.65	0.58	0.00	0.00	0.63	0.65
time (sec)	N/A	0.065	0.019	0.056	1.443	0.403	0.000	0.000	5.327	0.040

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	27	14	21	0	0	21	25
N.S.	1	1.00	1.00	1.08	0.56	0.84	0.00	0.00	0.84	1.00
time (sec)	N/A	0.017	0.008	0.048	1.405	0.394	0.000	0.000	5.190	0.036

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	54	43	0	75	0	47	-1	32
N.S.	1	1.00	1.69	1.34	0.00	2.34	0.00	1.47	-0.03	1.00
time (sec)	N/A	0.011	0.011	0.048	0.000	0.417	0.000	0.191	0.000	0.047

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	71	66	0	127	0	57	-1	59
N.S.	1	1.00	1.20	1.12	0.00	2.15	0.00	0.97	-0.02	1.00
time (sec)	N/A	0.055	0.071	0.051	0.000	0.411	0.000	0.210	0.000	0.073

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	81	3347	0	148	0	44	-1	82
N.S.	1	1.00	1.25	51.49	0.00	2.28	0.00	0.68	-0.02	1.26
time (sec)	N/A	0.090	0.042	1.140	0.000	0.560	0.000	0.246	0.000	0.289

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	59	480	0	101	0	41	-1	53
N.S.	1	1.00	1.64	13.33	0.00	2.81	0.00	1.14	-0.03	1.47
time (sec)	N/A	0.048	0.023	1.237	0.000	0.556	0.000	0.201	0.000	0.220

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	29	26	21	0	23	-1	27
N.S.	1	1.00	1.00	1.07	0.96	0.78	0.00	0.85	-0.04	1.00
time (sec)	N/A	0.040	0.012	0.045	1.421	0.412	0.000	0.204	0.000	0.238

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	35	37	38	31	0	38	-1	37
N.S.	1	1.00	0.62	0.66	0.68	0.55	0.00	0.68	-0.02	0.66
time (sec)	N/A	0.083	0.019	0.043	1.421	0.397	0.000	0.232	0.000	0.291

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	29	28	26	19	30	25	0
N.S.	1	1.00	1.00	1.04	1.00	0.93	0.68	1.07	0.89	0.00
time (sec)	N/A	0.017	0.005	0.048	1.363	0.382	0.207	0.151	5.209	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	41	40	41	31	45	38	0
N.S.	1	1.00	1.00	0.98	0.95	0.98	0.74	1.07	0.90	0.00
time (sec)	N/A	0.019	0.005	0.048	1.358	0.390	0.216	0.144	5.724	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	94	120	0	171	0	0	-1	109
N.S.	1	1.00	0.84	1.07	0.00	1.53	0.00	0.00	-0.01	0.97
time (sec)	N/A	0.179	0.196	0.047	0.000	0.425	0.000	0.000	0.000	0.334

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	92	98	0	150	0	0	-1	98
N.S.	1	1.00	1.07	1.14	0.00	1.74	0.00	0.00	-0.01	1.14
time (sec)	N/A	0.127	0.073	0.053	0.000	0.401	0.000	0.000	0.000	0.295

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	75	78	0	122	0	48	-1	81
N.S.	1	1.00	1.34	1.39	0.00	2.18	0.00	0.86	-0.02	1.45
time (sec)	N/A	0.082	0.050	0.049	0.000	0.409	0.000	0.231	0.000	0.249

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	59	56	0	74	0	23	-1	49
N.S.	1	1.00	1.84	1.75	0.00	2.31	0.00	0.72	-0.03	1.53
time (sec)	N/A	0.034	0.018	0.051	0.000	0.412	0.000	0.281	0.000	0.176

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	21	25	0	21	0	27	21	23
N.S.	1	1.00	0.91	1.09	0.00	0.91	0.00	1.17	0.91	1.00
time (sec)	N/A	0.005	0.008	0.048	0.000	0.391	0.000	0.206	5.140	0.144

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	29	30	0	29	0	27	42	33
N.S.	1	1.00	0.56	0.58	0.00	0.56	0.00	0.52	0.81	0.63
time (sec)	N/A	0.045	0.015	0.054	0.000	0.392	0.000	0.231	5.062	0.171

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	42	46	0	40	0	43	40	44
N.S.	1	1.00	0.52	0.58	0.00	0.50	0.00	0.54	0.50	0.55
time (sec)	N/A	0.086	0.021	0.044	0.000	0.388	0.000	0.239	5.140	0.182

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	53	57	0	51	0	57	92	55
N.S.	1	1.00	0.49	0.53	0.00	0.47	0.00	0.53	0.85	0.51
time (sec)	N/A	0.134	0.024	0.046	0.000	0.401	0.000	0.268	5.137	0.184

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	64	68	0	62	0	71	116	66
N.S.	1	1.00	0.47	0.50	0.00	0.46	0.00	0.52	0.85	0.49
time (sec)	N/A	0.174	0.021	0.046	0.000	0.386	0.000	0.233	5.145	0.209

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	23	22	28	22	32	22	0
N.S.	1	1.00	1.00	0.88	0.85	1.08	0.85	1.23	0.85	0.00
time (sec)	N/A	0.018	0.014	0.045	1.337	0.398	0.221	0.147	0.050	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	23	22	28	22	32	22	0
N.S.	1	1.00	1.00	0.85	0.81	1.04	0.81	1.19	0.81	0.00
time (sec)	N/A	0.018	0.006	0.050	1.323	0.391	0.233	0.180	5.209	0.001

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	14	9	14	8	5	15	8	0
N.S.	1	1.00	1.75	1.12	1.75	1.00	0.62	1.88	1.00	0.00
time (sec)	N/A	0.003	0.005	0.043	1.333	0.389	0.072	0.148	5.274	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	11	16	18	15	16	10	0
N.S.	1	1.00	1.00	1.10	1.60	1.80	1.50	1.60	1.00	0.00
time (sec)	N/A	0.003	0.003	0.034	1.337	0.371	0.078	0.145	0.032	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	16	26	27	16	26	0
N.S.	1	1.00	1.00	0.92	1.33	2.17	2.25	1.33	2.17	0.00
time (sec)	N/A	0.003	0.004	0.046	1.310	0.371	0.088	0.147	0.037	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	11	10	10	7	10	10	0
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.00	1.00	0.00
time (sec)	N/A	0.002	0.001	0.043	1.357	0.375	0.077	0.148	0.027	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	19	21	22	32	0	19	0
N.S.	1	1.00	1.00	0.95	1.05	1.10	1.60	0.00	0.95	0.00
time (sec)	N/A	0.006	0.004	0.045	1.345	0.400	0.635	0.000	5.178	0.018

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	21	40	36	82	0	21	0
N.S.	1	1.00	1.00	1.05	2.00	1.80	4.10	0.00	1.05	0.00
time (sec)	N/A	0.008	0.004	0.043	1.361	0.412	1.027	0.000	5.135	0.016

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	21	53	52	119	0	21	0
N.S.	1	1.00	1.00	1.05	2.65	2.60	5.95	0.00	1.05	0.00
time (sec)	N/A	0.009	0.004	0.041	1.402	0.417	1.355	0.000	5.122	0.017

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	160	135	134	134	160	134	134	0
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38	0.00
time (sec)	N/A	0.004	0.005	0.048	1.314	0.349	0.118	0.145	5.157	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	160	135	134	134	160	134	134	0
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38	0.00
time (sec)	N/A	0.011	0.020	0.039	1.369	0.344	0.130	0.151	5.156	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	160	135	134	134	160	134	134	0
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38	0.00
time (sec)	N/A	0.012	0.014	0.037	1.312	0.343	0.138	0.149	5.153	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	24	339	275	231	0	285	287	196
N.S.	1	1.00	0.89	12.56	10.19	8.56	0.00	10.56	10.63	7.26
time (sec)	N/A	0.015	0.031	0.108	1.428	0.444	0.000	0.515	5.875	0.074

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	160	135	134	134	160	134	134	0
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38	0.00
time (sec)	N/A	0.004	0.004	0.047	1.325	0.339	0.128	0.167	0.002	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	160	135	134	134	160	134	134	0
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38	0.00
time (sec)	N/A	0.005	0.006	0.043	1.340	0.342	0.127	0.148	5.188	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	160	135	134	134	160	134	134	0
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38	0.00
time (sec)	N/A	0.004	0.007	0.043	1.347	0.340	0.127	0.164	5.158	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	24	287	275	205	0	205	285	196
N.S.	1	1.00	0.89	10.63	10.19	7.59	0.00	7.59	10.56	7.26
time (sec)	N/A	0.008	0.005	0.085	1.481	0.420	0.000	0.271	5.950	0.064

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	126	121	93	0	93	124	91
N.S.	1	1.00	1.00	4.67	4.48	3.44	0.00	3.44	4.59	3.37
time (sec)	N/A	0.010	0.014	0.064	1.406	0.413	0.000	0.199	5.437	0.047

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	39	66	82	0	0	38	0
N.S.	1	1.00	1.00	1.44	2.44	3.04	0.00	0.00	1.41	0.00
time (sec)	N/A	0.014	0.022	0.068	1.401	0.418	0.000	0.000	5.211	0.062

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	13	10	14	13	0
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87	0.00
time (sec)	N/A	0.005	0.003	0.039	1.288	0.375	0.122	0.147	0.046	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	13	10	14	13	0
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87	0.00
time (sec)	N/A	0.006	0.004	0.041	1.273	0.386	0.148	0.145	5.124	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	13	10	14	13	0
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87	0.00
time (sec)	N/A	0.005	0.005	0.036	1.368	0.383	0.164	0.146	0.047	0.001

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	24	31	36	36	36	22	37	0
N.S.	1	1.00	1.26	1.63	1.89	1.89	1.89	1.16	1.95	0.00
time (sec)	N/A	0.005	0.010	0.049	1.334	0.371	0.282	0.191	0.042	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	24	31	36	36	36	22	37	0
N.S.	1	1.00	1.26	1.63	1.89	1.89	1.89	1.16	1.95	0.00
time (sec)	N/A	0.006	0.017	0.051	1.290	0.366	0.500	0.173	0.062	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	24	31	36	36	36	22	37	0
N.S.	1	1.00	1.26	1.63	1.89	1.89	1.89	1.16	1.95	0.00
time (sec)	N/A	0.006	0.013	0.055	1.306	0.384	0.745	0.150	5.184	0.001

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	23	22	25	19	22	22	0
N.S.	1	1.00	1.00	0.96	0.92	1.04	0.79	0.92	0.92	0.00
time (sec)	N/A	0.011	0.002	0.043	1.359	0.375	0.103	0.154	0.042	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	40	35	34	38	37	46	34	0
N.S.	1	1.00	1.00	0.88	0.85	0.95	0.92	1.15	0.85	0.00
time (sec)	N/A	0.023	0.008	0.044	1.318	0.373	0.151	0.169	0.037	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	50	45	44	48	49	45	47	0
N.S.	1	1.00	1.00	0.90	0.88	0.96	0.98	0.90	0.94	0.00
time (sec)	N/A	0.021	0.007	0.048	1.322	0.378	0.188	0.150	0.048	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	185	185	144	156	124	637	36	112	197	0
N.S.	1	1.00	0.78	0.84	0.67	3.44	0.19	0.61	1.06	0.00
time (sec)	N/A	0.346	0.143	0.108	2.902	1.284	1.542	0.181	5.914	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	363	325	297	0	269	363	232
N.S.	1	1.00	1.00	12.52	11.21	10.24	0.00	9.28	12.52	8.00
time (sec)	N/A	0.017	0.017	0.211	1.479	0.433	0.000	2.472	6.777	0.095

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	160	135	14	134	160	14	14	0
N.S.	1	1.00	10.00	8.44	0.88	8.38	10.00	0.88	0.88	0.00
time (sec)	N/A	0.003	0.006	0.043	1.329	0.349	0.115	0.150	5.325	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	160	135	134	134	160	134	134	0
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38	0.00
time (sec)	N/A	0.009	0.006	0.044	1.343	0.348	0.127	0.155	0.002	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	160	135	134	134	160	134	134	0
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38	0.00
time (sec)	N/A	0.012	0.008	0.039	1.355	0.346	0.140	0.153	5.222	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	160	135	14	134	160	14	14	0
N.S.	1	1.00	10.00	8.44	0.88	8.38	10.00	0.88	0.88	0.00
time (sec)	N/A	0.003	0.005	0.039	1.352	0.348	0.111	0.147	5.202	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	160	135	134	134	160	134	134	0
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38	0.00
time (sec)	N/A	0.008	0.007	0.042	1.311	0.355	0.131	0.214	0.002	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	160	135	14	134	160	14	14	0
N.S.	1	1.00	10.00	8.44	0.88	8.38	10.00	0.88	0.88	0.00
time (sec)	N/A	0.003	0.006	0.043	1.298	0.358	0.117	0.153	5.177	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	36	37	27	53	0	26	0
N.S.	1	1.00	1.00	1.57	1.61	1.17	2.30	0.00	1.13	0.00
time (sec)	N/A	0.011	0.009	0.051	1.424	0.407	0.675	0.000	5.260	0.011

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	22	39	27	28	41	0	31	34
N.S.	1	1.00	0.96	1.70	1.17	1.22	1.78	0.00	1.35	1.48
time (sec)	N/A	0.013	0.006	0.053	1.335	0.423	1.810	0.000	5.227	0.034

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	41	19	28	39	0	34	18
N.S.	1	1.00	1.00	2.73	1.27	1.87	2.60	0.00	2.27	1.20
time (sec)	N/A	0.008	0.005	0.056	1.369	0.422	2.142	0.000	5.225	0.029

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	32	16	26	20	0	26	30
N.S.	1	1.00	1.00	1.45	0.73	1.18	0.91	0.00	1.18	1.36
time (sec)	N/A	0.010	0.006	0.051	1.321	0.402	1.508	0.000	5.231	0.028

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	34	11	26	22	0	28	18
N.S.	1	1.00	1.00	2.27	0.73	1.73	1.47	0.00	1.87	1.20
time (sec)	N/A	0.006	0.004	0.050	1.321	0.405	1.702	0.000	5.203	0.024

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	12	8	8	8	9	8	10
N.S.	1	1.00	1.00	1.00	0.67	0.67	0.67	0.75	0.67	0.83
time (sec)	N/A	0.004	0.003	0.046	1.330	0.389	0.178	0.153	0.096	0.009

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	116	17	8	22	18	8	25
N.S.	1	1.00	1.00	8.29	1.21	0.57	1.57	1.29	0.57	1.79
time (sec)	N/A	0.005	0.003	0.436	1.300	0.397	0.374	0.198	5.296	0.013

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	9	8	8	10	32	8	12
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.83	2.67	0.67	1.00
time (sec)	N/A	0.004	0.002	0.044	2.888	0.385	0.259	0.152	0.094	0.012

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	39	31	24	32	0	26	0
N.S.	1	1.00	1.00	1.62	1.29	1.00	1.33	0.00	1.08	0.00
time (sec)	N/A	0.016	0.024	0.162	2.929	0.422	0.445	0.000	5.249	0.013

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	104	0	0	0	0	0	-1	0
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.114	0.226	0.866	0.000	0.000	0.000	0.000	0.000	0.052

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	109	0	0	0	0	0	-1	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.157	0.067	0.950	0.000	0.000	0.000	0.000	0.000	0.121

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	103	0	0	0	0	0	-1	0
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.155	0.270	0.761	0.000	0.000	0.000	0.000	0.000	3.301

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	99	0	0	0	0	0	-1	0
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.076	0.160	0.723	0.000	0.000	0.000	0.000	0.000	0.057

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	100	0	0	0	0	0	-1	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.138	0.286	0.971	0.000	0.000	0.000	0.000	0.000	1.031

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	46	39	58	97	78	0	-1	51
N.S.	1	1.00	0.90	0.76	1.14	1.90	1.53	0.00	-0.02	1.00
time (sec)	N/A	0.029	0.014	0.044	2.956	0.431	1.776	0.000	0.000	0.051

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	85	0	0	0	0	0	-1	104
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01	1.24
time (sec)	N/A	0.171	0.068	0.715	0.000	0.000	0.000	0.000	0.000	1.271

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	78	0	0	0	0	0	97	83
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	1.59	1.36
time (sec)	N/A	0.083	0.056	0.716	0.000	0.000	0.000	0.000	5.169	0.100

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	85	0	0	0	0	0	-1	104
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.22
time (sec)	N/A	0.199	0.074	0.739	0.000	0.000	0.000	0.000	0.000	1.371

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	131	0	0	0	0	0	-1	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.225	0.263	0.868	0.000	0.000	0.000	0.000	0.000	0.135

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	126	0	0	0	0	0	-1	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.209	0.326	0.710	0.000	0.000	0.000	0.000	0.000	6.403

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	117	0	0	0	0	0	-1	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.133	0.173	0.709	0.000	0.000	0.000	0.000	0.000	0.243

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	120	0	0	0	0	0	-1	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.188	0.276	0.687	0.000	0.000	0.000	0.000	0.000	2.396

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	58	51	73	120	88	0	-1	68
N.S.	1	1.00	0.79	0.70	1.00	1.64	1.21	0.00	-0.01	0.93
time (sec)	N/A	0.041	0.028	0.049	3.001	0.422	3.076	0.000	0.000	0.062

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	97	0	0	0	0	0	-1	123
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01	1.05
time (sec)	N/A	0.230	0.078	0.668	0.000	0.000	0.000	0.000	0.000	1.504

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	94	0	0	0	0	0	-1	108
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01	1.10
time (sec)	N/A	0.172	0.072	0.696	0.000	0.000	0.000	0.000	0.000	0.143

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	100	0	0	0	0	0	-1	123
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01	1.01
time (sec)	N/A	0.276	0.090	0.708	0.000	0.000	0.000	0.000	0.000	1.687

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	96	0	0	0	0	0	-1	110
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01	1.10
time (sec)	N/A	0.212	0.079	0.727	0.000	0.000	0.000	0.000	0.000	0.150

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	58	47	0	93	0	67	67	59
N.S.	1	1.00	1.14	0.92	0.00	1.82	0.00	1.31	1.31	1.16
time (sec)	N/A	0.077	0.029	0.076	0.000	0.415	0.000	0.162	5.184	3.649

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	62	61	53	108	0	69	55	61
N.S.	1	1.00	1.48	1.45	1.26	2.57	0.00	1.64	1.31	1.45
time (sec)	N/A	0.023	0.032	0.076	2.935	0.426	0.000	0.169	5.574	3.844

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	66	55	0	104	0	71	63	69
N.S.	1	1.00	1.29	1.08	0.00	2.04	0.00	1.39	1.24	1.35
time (sec)	N/A	0.065	0.038	0.080	0.000	0.417	0.000	0.191	5.337	4.027

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	70	74	0	112	0	0	-1	73
N.S.	1	1.00	1.15	1.21	0.00	1.84	0.00	0.00	-0.02	1.20
time (sec)	N/A	0.080	0.022	1.870	0.000	0.419	0.000	0.000	0.000	0.045

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	66	55	0	98	0	61	67	67
N.S.	1	1.00	1.25	1.04	0.00	1.85	0.00	1.15	1.26	1.26
time (sec)	N/A	0.079	0.031	0.085	0.000	0.401	0.000	0.159	5.182	3.813

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	68	81	34	118	0	63	54	68
N.S.	1	1.00	1.58	1.88	0.79	2.74	0.00	1.47	1.26	1.58
time (sec)	N/A	0.023	0.034	0.087	2.995	0.419	0.000	0.240	5.384	3.954

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	73	73	0	109	0	65	63	76
N.S.	1	1.00	1.38	1.38	0.00	2.06	0.00	1.23	1.19	1.43
time (sec)	N/A	0.068	0.042	0.110	0.000	0.401	0.000	0.189	5.354	4.128

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	78	105	0	118	0	0	-1	81
N.S.	1	1.00	1.24	1.67	0.00	1.87	0.00	0.00	-0.02	1.29
time (sec)	N/A	0.080	0.032	1.987	0.000	0.415	0.000	0.000	0.000	0.096

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	98	0	0	0	0	0	-1	0
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.108	0.161	0.792	0.000	0.000	0.000	0.000	0.000	0.135

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	89	0	0	0	0	0	-1	0
N.S.	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.099	0.169	0.729	0.000	0.000	0.000	0.000	0.000	2.054

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F	F	F(-2)	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	78	0	0	0	0	0	67	0
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	1.81	0.00
time (sec)	N/A	0.015	0.074	0.709	0.000	0.000	0.000	0.000	5.387	0.030

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	87	0	0	0	0	0	-1	0
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.085	0.135	0.715	0.000	0.000	0.000	0.000	0.000	0.820

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	26	42	76	27	0	-1	31
N.S.	1	1.00	1.00	0.84	1.35	2.45	0.87	0.00	-0.03	1.00
time (sec)	N/A	0.020	0.007	0.050	2.957	0.418	1.976	0.000	0.000	0.052

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	68	0	0	0	0	0	-1	70
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.02	1.30
time (sec)	N/A	0.111	0.050	0.724	0.000	0.000	0.000	0.000	0.000	1.124

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	66	0	0	0	0	0	-1	64
N.S.	1	1.00	1.65	0.00	0.00	0.00	0.00	0.00	-0.02	1.60
time (sec)	N/A	0.074	0.048	0.704	0.000	0.000	0.000	0.000	0.000	0.109

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	68	0	0	0	0	0	-1	70
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.02	1.30
time (sec)	N/A	0.136	0.057	0.723	0.000	0.000	0.000	0.000	0.000	1.196

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	117	0	0	0	0	0	-1	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.187	0.199	0.832	0.000	0.000	0.000	0.000	0.000	0.147

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	109	0	0	0	0	0	-1	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.160	0.209	0.731	0.000	0.000	0.000	0.000	0.000	7.599

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	91	0	0	0	0	0	-1	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.088	0.152	0.702	0.000	0.000	0.000	0.000	0.000	0.067

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	104	0	0	0	0	0	-1	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.138	0.201	0.738	0.000	0.000	0.000	0.000	0.000	1.434

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	40	42	61	148	185	0	-1	54
N.S.	1	1.00	0.74	0.78	1.13	2.74	3.43	0.00	-0.02	1.00
time (sec)	N/A	0.033	0.014	0.051	3.065	0.415	3.721	0.000	0.000	0.067

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-2)	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	55	0	0	0	0	0	-1	107
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	-0.01	1.19
time (sec)	N/A	0.194	0.049	0.681	0.000	0.000	0.000	0.000	0.000	1.537

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-2)	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	51	0	0	0	0	0	-1	92
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01	1.28
time (sec)	N/A	0.156	0.043	0.720	0.000	0.000	0.000	0.000	0.000	0.133

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-2)	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	55	0	0	0	0	0	-1	107
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	-0.01	1.19
time (sec)	N/A	0.217	0.055	0.694	0.000	0.000	0.000	0.000	0.000	1.639

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-2)	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	51	0	0	0	0	0	-1	94
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01	1.31
time (sec)	N/A	0.152	0.065	0.730	0.000	0.000	0.000	0.000	0.000	0.145

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	63	477	0	102	0	0	-1	34
N.S.	1	1.00	1.97	14.91	0.00	3.19	0.00	0.00	-0.03	1.06
time (sec)	N/A	0.014	0.025	0.570	0.000	0.568	0.000	0.000	0.000	0.359

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	59	49	0	80	0	40	-1	58
N.S.	1	1.00	1.84	1.53	0.00	2.50	0.00	1.25	-0.03	1.81
time (sec)	N/A	0.016	0.020	0.194	0.000	0.419	0.000	0.214	0.000	3.536

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	63	0	0	102	0	0	-1	64
N.S.	1	1.00	1.97	0.00	0.00	3.19	0.00	0.00	-0.03	2.00
time (sec)	N/A	0.016	0.029	0.115	0.000	0.934	0.000	0.000	0.000	75.148

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	76	0	0	102	0	0	-1	0
N.S.	1	1.00	2.05	0.00	0.00	2.76	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.024	0.050	0.732	0.000	0.429	0.000	0.000	0.000	0.059

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	66	471	0	111	0	0	-1	35
N.S.	1	1.00	2.00	14.27	0.00	3.36	0.00	0.00	-0.03	1.06
time (sec)	N/A	0.014	0.033	6.219	0.000	0.539	0.000	0.000	0.000	0.383

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	62	53	0	88	0	47	-1	66
N.S.	1	1.00	1.88	1.61	0.00	2.67	0.00	1.42	-0.03	2.00
time (sec)	N/A	0.015	0.023	0.190	0.000	0.417	0.000	0.187	0.000	3.664

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	66	0	0	111	0	0	-1	72
N.S.	1	1.00	2.00	0.00	0.00	3.36	0.00	0.00	-0.03	2.18
time (sec)	N/A	0.016	0.028	0.157	0.000	0.929	0.000	0.000	0.000	74.851

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	78	0	0	106	0	0	-1	0
N.S.	1	1.00	2.05	0.00	0.00	2.79	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.023	0.059	1.377	0.000	0.426	0.000	0.000	0.000	0.094

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F	F	F(-2)	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	78	0	0	0	0	0	67	0
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	1.81	0.00
time (sec)	N/A	0.023	0.103	0.760	0.000	0.000	0.000	0.000	5.209	0.083

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F	F	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	78	0	0	109	0	0	67	0
N.S.	1	1.00	2.11	0.00	0.00	2.95	0.00	0.00	1.81	0.00
time (sec)	N/A	0.019	0.040	2.102	0.000	0.435	0.000	0.000	5.323	0.064

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F	F	F(-2)	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	78	0	0	0	0	0	67	0
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	1.81	0.00
time (sec)	N/A	0.018	0.029	0.720	0.000	0.000	0.000	0.000	5.127	0.062

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F	F	F(-2)	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	80	0	0	0	0	0	66	0
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	1.74	0.00
time (sec)	N/A	0.021	0.106	0.696	0.000	0.000	0.000	0.000	5.166	0.049

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F	F	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	80	0	0	109	0	0	66	0
N.S.	1	1.00	2.11	0.00	0.00	2.87	0.00	0.00	1.74	0.00
time (sec)	N/A	0.020	0.025	2.152	0.000	0.436	0.000	0.000	5.102	0.065

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F	F	F(-2)	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	80	0	0	0	0	0	66	0
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	1.74	0.00
time (sec)	N/A	0.019	0.028	0.694	0.000	0.000	0.000	0.000	5.117	0.064

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	19	16	0	16	0	50	15	18
N.S.	1	1.00	1.06	0.89	0.00	0.89	0.00	2.78	0.83	1.00
time (sec)	N/A	0.007	0.006	0.047	0.000	0.387	0.000	0.185	5.262	0.025

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	18	0	19	0	11	27	20
N.S.	1	1.00	1.00	0.90	0.00	0.95	0.00	0.55	1.35	1.00
time (sec)	N/A	0.003	0.010	0.054	0.000	0.428	0.000	0.172	5.322	0.040

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	8	7	6	6	7	6	6	8
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75	1.00
time (sec)	N/A	0.004	0.002	0.044	3.144	0.391	0.213	0.155	5.244	0.012

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	29	14	28	0	27	22	25
N.S.	1	1.00	1.00	1.16	0.56	1.12	0.00	1.08	0.88	1.00
time (sec)	N/A	0.010	0.013	0.050	1.463	0.377	0.000	0.169	5.231	0.021

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	29	14	28	0	27	29	25
N.S.	1	1.00	1.00	1.16	0.56	1.12	0.00	1.08	1.16	1.00
time (sec)	N/A	0.011	0.003	0.044	1.427	0.383	0.000	0.163	5.258	0.024

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	29	14	28	0	14	22	25
N.S.	1	1.00	1.00	1.16	0.56	1.12	0.00	0.56	0.88	1.00
time (sec)	N/A	0.008	0.012	0.053	1.465	0.392	0.000	0.186	5.211	0.021

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	43	0	0	64	0	0	76	0
N.S.	1	1.00	0.98	0.00	0.00	1.45	0.00	0.00	1.73	0.00
time (sec)	N/A	0.015	0.039	0.856	0.000	0.646	0.000	0.000	5.303	0.083

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	43	0	0	61	0	0	-1	0
N.S.	1	1.00	0.98	0.00	0.00	1.39	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.015	0.003	0.848	0.000	0.410	0.000	0.000	0.000	0.022

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	45	0	0	76	0	0	-1	0
N.S.	1	1.00	0.98	0.00	0.00	1.65	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.075	0.046	0.908	0.000	0.426	0.000	0.000	0.000	0.108

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	45	0	0	79	0	0	-1	0
N.S.	1	1.00	0.98	0.00	0.00	1.72	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.045	0.005	0.882	0.000	0.432	0.000	0.000	0.000	0.105

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	36	40	17	44	0	0	-1	43
N.S.	1	1.00	0.82	0.91	0.39	1.00	0.00	0.00	-0.02	0.98
time (sec)	N/A	0.018	0.033	0.188	1.551	0.412	0.000	0.000	0.000	0.065

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	36	40	17	44	0	0	-1	43
N.S.	1	1.00	0.82	0.91	0.39	1.00	0.00	0.00	-0.02	0.98
time (sec)	N/A	0.017	0.033	0.079	1.495	0.405	0.000	0.000	0.000	0.092

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	36	40	17	44	0	0	-1	43
N.S.	1	1.00	0.82	0.91	0.39	1.00	0.00	0.00	-0.02	0.98
time (sec)	N/A	0.018	0.033	0.069	1.580	0.415	0.000	0.000	0.000	0.083

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	47	0	0	47	0	0	-1	0
N.S.	1	1.00	0.82	0.00	0.00	0.82	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.028	0.023	0.720	0.000	0.424	0.000	0.000	0.000	0.096

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	45	0	0	54	0	0	-1	0
N.S.	1	1.00	0.74	0.00	0.00	0.89	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.025	0.027	0.726	0.000	0.434	0.000	0.000	0.000	0.135

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	40	0	0	76	0	0	-1	0
N.S.	1	1.00	1.03	0.00	0.00	1.95	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.050	0.026	1.269	0.000	0.445	0.000	0.000	0.000	0.136

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	38	0	0	64	0	0	-1	0
N.S.	1	1.00	0.95	0.00	0.00	1.60	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.074	0.045	0.869	0.000	0.412	0.000	0.000	0.000	0.135

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [250] had the largest ratio of [.7778]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	13	0.077
2	A	2	1	1.00	11	0.091
3	A	1	0	1.00	9	0.000
4	A	2	1	1.00	13	0.077
5	A	2	1	1.00	13	0.077
6	A	3	2	1.00	15	0.133
7	A	4	3	1.00	13	0.231
8	A	3	2	1.00	11	0.182
9	A	2	2	1.00	15	0.133
10	A	3	2	1.00	15	0.133
11	A	3	2	1.00	11	0.182
12	A	4	3	1.00	15	0.200
13	A	3	3	1.00	15	0.200
14	A	2	2	1.00	15	0.133
15	A	2	2	1.00	13	0.154
16	A	5	5	1.00	11	0.454
17	A	3	3	1.00	15	0.200
18	A	4	3	1.00	15	0.200
19	A	4	3	1.00	15	0.200
20	A	4	3	1.00	15	0.200
21	A	3	3	1.00	15	0.200
22	A	4	3	1.00	13	0.231
23	A	4	4	1.00	11	0.364
24	A	4	3	1.00	15	0.200
25	A	5	4	1.00	15	0.267
26	A	4	3	1.00	13	0.231
27	A	4	3	1.00	13	0.231
28	A	3	3	1.00	13	0.231
29	A	2	2	1.00	13	0.154
30	A	2	2	1.00	11	0.182
31	A	5	5	1.00	9	0.556
32	A	3	3	1.00	13	0.231
33	A	4	3	1.00	13	0.231
34	A	4	3	1.00	13	0.231
35	A	4	3	1.00	13	0.231
36	A	5	5	1.00	9	0.556
37	A	5	5	1.00	11	0.454
38	A	7	4	1.00	19	0.210
39	A	5	2	1.00	19	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
40	A	6	3	1.00	19	0.158
41	A	4	2	1.00	19	0.105
42	A	1	1	1.00	19	0.053
43	A	3	2	1.00	19	0.105
44	A	2	2	1.00	19	0.105
45	A	2	2	1.00	19	0.105
46	A	3	2	1.00	19	0.105
47	A	1	1	1.00	19	0.053
48	A	4	2	1.00	19	0.105
49	A	6	3	1.00	19	0.158
50	A	5	2	1.00	19	0.105
51	A	7	4	1.00	19	0.210
52	A	6	3	1.00	19	0.158
53	A	8	4	1.00	19	0.210
54	A	7	3	1.00	19	0.158
55	A	3	3	1.00	17	0.176
56	A	2	2	1.00	15	0.133
57	A	1	1	1.00	17	0.059
58	A	2	2	1.00	17	0.118
59	A	3	2	1.00	17	0.118
60	A	8	5	1.00	19	0.263
61	A	6	5	1.00	17	0.294
62	A	4	4	1.00	15	0.267
63	A	1	1	1.00	19	0.053
64	A	3	2	1.00	19	0.105
65	A	5	2	1.00	19	0.105
66	A	7	2	1.00	19	0.105
67	A	9	6	1.00	19	0.316
68	A	7	6	1.00	19	0.316
69	A	5	5	1.00	17	0.294
70	A	1	1	1.00	15	0.067
71	A	3	3	1.00	19	0.158
72	A	5	3	1.00	19	0.158
73	A	7	3	1.00	19	0.158
74	A	9	5	1.00	21	0.238
75	A	7	5	1.00	21	0.238
76	A	5	5	1.00	21	0.238
77	A	3	3	1.00	21	0.143
78	A	2	2	1.00	21	0.095
79	A	4	2	1.00	21	0.095
80	A	6	2	1.00	21	0.095
81	A	8	6	1.00	21	0.286
82	A	6	6	1.00	21	0.286
83	A	4	4	1.00	21	0.190
84	A	2	2	1.00	21	0.095
85	A	4	3	1.00	21	0.143
86	A	6	3	1.00	21	0.143
87	A	8	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
88	A	13	3	1.00	19	0.158
89	A	10	3	1.00	19	0.158
90	A	7	3	1.00	17	0.176
91	A	4	3	1.00	15	0.200
92	A	1	1	1.00	19	0.053
93	A	4	4	1.00	19	0.210
94	A	7	4	1.00	19	0.210
95	A	10	4	1.00	19	0.210
96	A	13	4	1.00	19	0.210
97	A	12	3	1.00	19	0.158
98	A	9	3	1.00	17	0.176
99	A	6	3	1.00	15	0.200
100	A	3	2	1.00	19	0.105
101	A	4	3	1.00	19	0.158
102	A	5	4	1.00	19	0.210
103	A	8	4	1.00	19	0.210
104	A	11	4	1.00	19	0.210
105	A	14	4	1.00	19	0.210
106	A	14	3	1.00	19	0.158
107	A	11	3	1.00	19	0.158
108	A	8	3	1.00	19	0.158
109	A	5	3	1.00	17	0.176
110	A	2	2	1.00	15	0.133
111	A	3	3	1.00	19	0.158
112	A	6	3	1.00	19	0.158
113	A	9	3	1.00	19	0.158
114	A	12	3	1.00	19	0.158
115	A	12	4	1.00	19	0.210
116	A	9	4	1.00	19	0.210
117	A	6	4	1.00	19	0.210
118	A	3	3	1.00	17	0.176
119	A	3	3	1.00	15	0.200
120	A	6	4	1.00	19	0.210
121	A	9	4	1.00	19	0.210
122	A	12	4	1.00	19	0.210
123	A	15	4	1.00	19	0.210
124	A	2	1	1.00	15	0.067
125	A	2	1	1.00	13	0.077
126	A	1	0	1.00	11	0.000
127	A	2	1	1.00	15	0.067
128	A	2	1	1.00	15	0.067
129	A	3	2	1.00	17	0.118
130	A	3	2	1.00	15	0.133
131	A	3	2	1.00	13	0.154
132	A	3	2	1.00	17	0.118
133	A	3	2	1.00	17	0.118
134	A	3	2	1.00	17	0.118
135	A	3	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
136	A	3	2	1.00	17	0.118
137	A	3	2	1.00	17	0.118
138	A	2	2	1.00	17	0.118
139	A	4	4	1.00	15	0.267
140	A	3	2	1.00	13	0.154
141	A	3	2	1.00	17	0.118
142	A	3	2	1.00	17	0.118
143	A	3	2	1.00	17	0.118
144	A	3	2	1.00	17	0.118
145	A	3	2	1.00	17	0.118
146	A	3	2	1.00	17	0.118
147	A	2	2	1.00	17	0.118
148	A	3	2	1.00	17	0.118
149	A	3	2	1.00	17	0.118
150	A	3	2	1.00	15	0.133
151	A	3	2	1.00	13	0.154
152	A	3	2	1.00	17	0.118
153	A	4	3	1.00	19	0.158
154	A	3	3	1.00	17	0.176
155	A	2	2	1.00	15	0.133
156	A	1	1	1.00	19	0.053
157	A	3	3	1.00	19	0.158
158	A	3	3	1.00	19	0.158
159	A	4	4	1.00	19	0.210
160	A	5	4	1.00	19	0.210
161	A	6	3	1.00	19	0.158
162	A	5	3	1.00	17	0.176
163	A	4	3	1.00	15	0.200
164	A	3	2	1.00	19	0.105
165	A	2	2	1.00	19	0.105
166	A	1	1	1.00	19	0.053
167	A	4	3	1.00	19	0.158
168	A	4	4	1.00	19	0.210
169	A	4	3	1.00	19	0.158
170	A	5	4	1.00	19	0.210
171	A	6	4	1.00	19	0.210
172	A	7	4	1.00	19	0.210
173	A	4	2	1.00	19	0.105
174	A	3	2	1.00	19	0.105
175	A	2	2	1.00	19	0.105
176	A	1	1	1.00	17	0.059
177	A	2	2	1.00	15	0.133
178	A	3	3	1.00	19	0.158
179	A	4	3	1.00	19	0.158
180	A	5	3	1.00	19	0.158
181	A	4	3	1.00	19	0.158
182	A	3	3	1.00	19	0.158
183	A	2	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	A	1	1	1.00	19	0.053
185	A	3	3	1.00	19	0.158
186	A	4	4	1.00	17	0.235
187	A	5	4	1.00	15	0.267
188	A	6	4	1.00	19	0.210
189	A	7	4	1.00	19	0.210
190	A	5	3	1.00	21	0.143
191	A	4	3	1.00	21	0.143
192	A	3	3	1.00	21	0.143
193	A	2	2	1.00	21	0.095
194	A	1	1	1.00	21	0.048
195	A	2	2	1.00	21	0.095
196	A	3	2	1.00	21	0.095
197	A	4	2	1.00	21	0.095
198	A	1	1	1.00	21	0.048
199	A	2	2	1.00	21	0.095
200	A	3	2	1.00	21	0.095
201	A	2	2	1.00	17	0.118
202	A	3	2	1.00	19	0.105
203	A	2	2	1.00	19	0.105
204	A	1	1	1.00	19	0.053
205	A	2	2	1.00	15	0.133
206	A	3	3	1.00	19	0.158
207	A	3	3	1.00	21	0.143
208	A	2	2	1.00	21	0.095
209	A	1	1	1.00	21	0.048
210	A	2	2	1.00	21	0.095
211	A	3	2	1.00	15	0.133
212	A	3	2	1.00	13	0.154
213	A	5	3	1.00	19	0.158
214	A	4	3	1.00	19	0.158
215	A	3	3	1.00	19	0.158
216	A	2	2	1.00	17	0.118
217	A	1	1	1.00	15	0.067
218	A	2	2	1.00	19	0.105
219	A	3	2	1.00	19	0.105
220	A	4	2	1.00	19	0.105
221	A	5	2	1.00	19	0.105
222	A	4	3	1.00	11	0.273
223	A	4	3	1.00	13	0.231
224	A	3	3	1.00	9	0.333
225	A	3	3	1.00	9	0.333
226	A	3	3	1.00	9	0.333
227	A	3	3	1.00	13	0.231
228	A	3	3	1.00	13	0.231
229	A	3	3	1.00	13	0.231
230	A	3	3	1.00	13	0.231
231	A	2	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
232	A	2	2	1.00	15	0.133
233	A	2	2	1.00	15	0.133
234	A	2	2	1.00	23	0.087
235	A	2	2	1.00	11	0.182
236	A	2	2	1.00	13	0.154
237	A	2	2	1.00	13	0.154
238	A	2	2	1.00	17	0.118
239	A	2	2	1.00	17	0.118
240	A	2	2	1.00	17	0.118
241	A	2	2	1.00	11	0.182
242	A	2	2	1.00	11	0.182
243	A	2	2	1.00	11	0.182
244	A	2	2	1.00	11	0.182
245	A	2	2	1.00	13	0.154
246	A	2	2	1.00	13	0.154
247	A	3	2	1.00	11	0.182
248	A	4	3	1.00	11	0.273
249	A	3	2	1.00	11	0.182
250	A	7	7	1.00	9	0.778
251	A	2	2	1.00	22	0.091
252	A	1	1	1.00	13	0.077
253	A	2	2	1.00	15	0.133
254	A	2	2	1.00	17	0.118
255	A	1	1	1.00	13	0.077
256	A	2	2	1.00	15	0.133
257	A	1	1	1.00	13	0.077
258	A	2	2	1.00	11	0.182
259	A	5	5	1.00	13	0.385
260	A	2	2	1.00	15	0.133
261	A	5	5	1.00	13	0.385
262	A	2	2	1.00	15	0.133
263	A	2	2	1.00	11	0.182
264	A	2	2	1.00	11	0.182
265	A	2	2	1.00	9	0.222
266	A	5	5	1.00	11	0.454
267	A	3	3	1.00	25	0.120
268	A	4	4	1.00	27	0.148
269	A	4	4	1.00	23	0.174
270	A	4	4	1.00	22	0.182
271	A	4	4	1.00	21	0.190
272	A	5	5	1.00	18	0.278
273	A	4	4	1.00	23	0.174
274	A	3	3	1.00	15	0.200
275	A	4	4	1.00	23	0.174
276	A	5	4	1.00	27	0.148
277	A	5	4	1.00	23	0.174
278	A	5	4	1.00	22	0.182
279	A	5	4	1.00	21	0.190

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
280	A	6	5	1.00	18	0.278
281	A	5	4	1.00	23	0.174
282	A	5	5	1.00	22	0.227
283	A	5	4	1.00	23	0.174
284	A	5	4	1.00	22	0.182
285	A	5	5	1.00	13	0.385
286	A	5	5	1.00	15	0.333
287	A	4	4	1.00	15	0.267
288	A	4	4	1.00	15	0.267
289	A	5	5	1.00	15	0.333
290	A	5	5	1.00	17	0.294
291	A	4	4	1.00	17	0.235
292	A	4	4	1.00	17	0.235
293	A	3	3	1.00	27	0.111
294	A	3	3	1.00	23	0.130
295	A	2	2	1.00	15	0.133
296	A	3	3	1.00	21	0.143
297	A	4	4	1.00	18	0.222
298	A	3	3	1.00	23	0.130
299	A	3	3	1.00	22	0.136
300	A	3	3	1.00	23	0.130
301	A	4	4	1.00	27	0.148
302	A	4	4	1.00	23	0.174
303	A	4	4	1.00	22	0.182
304	A	4	4	1.00	21	0.190
305	A	5	5	1.00	18	0.278
306	A	4	4	1.00	23	0.174
307	A	4	4	1.00	22	0.182
308	A	4	4	1.00	23	0.174
309	A	4	4	1.00	22	0.182
310	A	3	3	1.00	15	0.200
311	A	3	3	1.00	15	0.200
312	A	3	3	1.00	15	0.200
313	A	3	3	1.00	19	0.158
314	A	3	3	1.00	16	0.188
315	A	3	3	1.00	16	0.188
316	A	3	3	1.00	16	0.188
317	A	3	3	1.00	20	0.150
318	A	3	3	1.00	19	0.158
319	A	3	3	1.00	17	0.176
320	A	3	3	1.00	17	0.176
321	A	3	3	1.00	20	0.150
322	A	3	3	1.00	19	0.158
323	A	3	3	1.00	18	0.167
324	A	2	2	1.00	11	0.182
325	A	1	1	1.00	11	0.091
326	A	3	3	1.00	13	0.231
327	A	1	1	1.00	17	0.059

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
328	A	1	1	1.00	17	0.059
329	A	2	2	1.00	15	0.133
330	A	1	1	1.00	18	0.056
331	A	2	2	1.00	17	0.118
332	A	2	2	1.00	22	0.091
333	A	1	1	1.00	23	0.043
334	A	2	2	1.00	19	0.105
335	A	2	2	1.00	19	0.105
336	A	2	2	1.00	19	0.105
337	A	2	2	1.00	19	0.105
338	A	2	2	1.00	19	0.105
339	A	1	1	1.00	25	0.040
340	A	2	2	1.00	28	0.071

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^2 \left(ax + bx^3\right) dx$

Optimal. Leaf size=17

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x + b*x^3),x]
```

```
[Out] (a*x^4)/4 + (b*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int x^2 \left(ax + bx^3\right) dx = \int \left(ax^3 + bx^5\right) dx$$
$$= \frac{ax^4}{4} + \frac{bx^6}{6}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x + b*x^3),x]
```

```
[Out] (a*x^4)/4 + (b*x^6)/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(ax + bx^3\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a*x + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[x^2*(a*x + b*x^3), x]
```

fricas [A] time = 0.35, size = 13, normalized size = 0.76

$$\frac{1}{6}x^6b + \frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x),x, algorithm="fricas")
[Out] 1/6*x^6*b + 1/4*x^4*a
```

giac [A]   time = 0.16, size = 13, normalized size = 0.76

$$\frac{1}{6}\,bx^6 + \frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x),x, algorithm="giac")
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

maple [A]   time = 0.04, size = 14, normalized size = 0.82

$$\frac{1}{6}b\,x^6 + \frac{1}{4}a\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x),x)
[Out] 1/4*a*x^4+1/6*b*x^6
```

maxima [A]   time = 1.30, size = 13, normalized size = 0.76

$$\frac{1}{6}\,bx^6 + \frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x),x, algorithm="maxima")
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

mupad [B]   time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\,x^6}{6} + \frac{a\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x + b*x^3),x)
[Out] (a*x^4)/4 + (b*x^6)/6
```

sympy [A]   time = 0.10, size = 12, normalized size = 0.71

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x),x)
[Out] a*x**4/4 + b*x**6/6
```

3.2 $\int x\left(ax+bx^3\right)\,dx$

Optimal. Leaf size=17

$$\frac{ax^3}{3}+\frac{bx^5}{5}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$\frac{ax^3}{3}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x + b*x^3),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(ax+bx^3\right)\,dx&=\int\left(ax^2+bx^4\right)\,dx\\&=\frac{ax^3}{3}+\frac{bx^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^3}{3}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x + b*x^3),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(ax+bx^3\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a*x + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[x*(a*x + b*x^3), x]
```

fricas [A] time = 0.36, size = 13, normalized size = 0.76

$$\frac{1}{5}x^5b+\frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x),x, algorithm="fricas")
[Out] 1/5*x^5*b + 1/3*x^3*a
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^5 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x),x, algorithm="giac")
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

maple [A] time = 0.04, size = 14, normalized size = 0.82

$$\frac{1}{5}b\,x^5 + \frac{1}{3}a\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x),x)
[Out] 1/3*a*x^3+1/5*b*x^5
```

maxima [A] time = 1.30, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^5 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x),x, algorithm="maxima")
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\,x^5}{5} + \frac{a\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x + b*x^3),x)
[Out] (a*x^3)/3 + (b*x^5)/5
```

sympy [A] time = 0.08, size = 12, normalized size = 0.71

$$\frac{ax^3}{3} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x),x)
[Out] a*x**3/3 + b*x**5/5
```

3.3 $\int (ax + bx^3) \, dx$

Optimal. Leaf size=17

$$\frac{ax^2}{2} + \frac{bx^4}{4}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{ax^2}{2} + \frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[a*x + b*x^3,x]
[Out] (a*x^2)/2 + (b*x^4)/4
```

Rubi steps

$$\int (ax + bx^3) \, dx = \frac{ax^2}{2} + \frac{bx^4}{4}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^2}{2} + \frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*x + b*x^3,x]
[Out] (a*x^2)/2 + (b*x^4)/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ax + bx^3) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[a*x + b*x^3,x]
[Out] IntegrateAlgebraic[a*x + b*x^3, x]
```

fricas [A] time = 0.35, size = 13, normalized size = 0.76

$$\frac{1}{4}x^4b + \frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x,x, algorithm="fricas")
[Out] 1/4*x^4*b + 1/2*x^2*a
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$\frac{1}{4}bx^4 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x,x, algorithm="giac")
```

```
[Out] 1/4*b*x^4 + 1/2*a*x^2
```

maple [A] time = 0.04, size = 14, normalized size = 0.82

$$\frac{1}{4}b\,x^{4}+\frac{1}{2}a\,x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x^3+a*x,x)
```

```
[Out] 1/2*a*x^2+1/4*b*x^4
```

maxima [A] time = 1.35, size = 13, normalized size = 0.76

$$\frac{1}{4}\,bx^{4}+\frac{1}{2}\,ax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x,x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4 + 1/2*a*x^2
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\,x^4}{4}+\frac{a\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*x + b*x^3,x)
```

```
[Out] (a*x^2)/2 + (b*x^4)/4
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^{2}}{2}+\frac{bx^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x**3+a*x,x)
```

```
[Out] a*x**2/2 + b*x**4/4
```

3.4 $\int \frac{ax+bx^3}{x}\,dx$

Optimal. Leaf size=12

$$ax+\frac{bx^3}{3}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$ax+\frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)/x,x]
```

```
[Out] a*x + (b*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{ax+bx^3}{x}\,dx &= \int \left(a+bx^2\right)dx\\ &= ax+\frac{bx^3}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$ax+\frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)/x,x]
```

```
[Out] a*x + (b*x^3)/3
```

IntegrateAlgebraic [A] time = 0.01, size = 12, normalized size = 1.00

$$ax+\frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x + b*x^3)/x,x]
```

```
[Out] a*x + (b*x^3)/3
```

fricas [A] time = 0.37, size = 10, normalized size = 0.83

$$\frac{1}{3}\,bx^3+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)/x,x, algorithm="fricas")
[Out] 1/3*b*x^3 + a*x
```

giac [A] time = 0.15, size = 10, normalized size = 0.83

$$\frac{1}{3}\,bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)/x,x, algorithm="giac")
[Out] 1/3*b*x^3 + a*x
```

maple [A] time = 0.04, size = 11, normalized size = 0.92

$$\frac{1}{3}b\,x^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)/x,x)
[Out] a*x+1/3*b*x^3
```

maxima [A] time = 1.34, size = 10, normalized size = 0.83

$$\frac{1}{3}\,bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)/x,x, algorithm="maxima")
[Out] 1/3*b*x^3 + a*x
```

mupad [B] time = 0.02, size = 10, normalized size = 0.83

$$\frac{b\,x^3}{3} + a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)/x,x)
[Out] a*x + (b*x^3)/3
```

sympy [A] time = 0.06, size = 8, normalized size = 0.67

$$ax + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)/x,x)
[Out] a*x + b*x**3/3
```

3.5 $\int \frac{ax+bx^3}{x^2}\,dx$

Optimal. Leaf size=13

$$a\log(x)+\frac{bx^2}{2}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$a\log(x)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)/x^2,x]
```

```
[Out] (b*x^2)/2 + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{ax+bx^3}{x^2}\,dx &= \int \left(\frac{a}{x}+bx\right)dx\\ &= \frac{bx^2}{2}+a\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$a\log(x)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)/x^2,x]
```

```
[Out] (b*x^2)/2 + a*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+bx^3}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x + b*x^3)/x^2,x]
```

```
[Out] IntegrateAlgebraic[(a*x + b*x^3)/x^2, x]
```

fricas [A] time = 0.39, size = 11, normalized size = 0.85

$$\frac{1}{2}\,bx^2+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*log(x)
```

giac [A] time = 0.15, size = 14, normalized size = 1.08

$$\frac{1}{2}\,bx^2 + \frac{1}{2}\,a\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + 1/2*a*log(x^2)
```

maple [A] time = 0.05, size = 12, normalized size = 0.92

$$\frac{b\,x^2}{2} + a\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)/x^2,x)
```

```
[Out] 1/2*b*x^2+a*ln(x)
```

maxima [A] time = 1.30, size = 11, normalized size = 0.85

$$\frac{1}{2}\,bx^2 + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*log(x)
```

mupad [B] time = 0.02, size = 11, normalized size = 0.85

$$\frac{b\,x^2}{2} + a\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)/x^2,x)
```

```
[Out] (b*x^2)/2 + a*log(x)
```

sympy [A] time = 0.11, size = 10, normalized size = 0.77

$$a\log\left(x\right) + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)/x**2,x)
```

```
[Out] a*log(x) + b*x**2/2
```

3.6 $\int x^2 \left(ax + bx^3\right)^2 dx$

Optimal. Leaf size=30

$$\frac{a^2x^5}{5} + \frac{2}{7}abx^7 + \frac{b^2x^9}{9}$$

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1584, 270}

$$\frac{a^2x^5}{5} + \frac{2}{7}abx^7 + \frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(ax + bx^3\right)^2 dx &= \int x^4 \left(a + bx^2\right)^2 dx\\ &= \int \left(a^2x^4 + 2abx^6 + b^2x^8\right) dx\\ &= \frac{a^2x^5}{5} + \frac{2}{7}abx^7 + \frac{b^2x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^5}{5} + \frac{2}{7}abx^7 + \frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(ax + bx^3\right)^2 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a*x + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[x^2*(a*x + b*x^3)^2, x]
```

fricas [A] time = 0.36, size = 24, normalized size = 0.80

$$\frac{1}{9}x^9b^2 + \frac{2}{7}x^7ba + \frac{1}{5}x^5a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^2,x, algorithm="fricas")
[Out] 1/9*x^9*b^2 + 2/7*x^7*b*a + 1/5*x^5*a^2
```

giac [A] time = 0.15, size = 24, normalized size = 0.80

$$\frac{1}{9}b^2x^9 + \frac{2}{7}abx^7 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^2,x, algorithm="giac")
[Out] 1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

maple [A] time = 0.04, size = 25, normalized size = 0.83

$$\frac{1}{9}b^2x^9 + \frac{2}{7}ab\,x^7 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x)^2,x)
[Out] 1/5*a^2*x^5+2/7*a*b*x^7+1/9*b^2*x^9
```

maxima [A] time = 1.31, size = 24, normalized size = 0.80

$$\frac{1}{9}b^2x^9 + \frac{2}{7}abx^7 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^2,x, algorithm="maxima")
[Out] 1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

mupad [B] time = 0.04, size = 24, normalized size = 0.80

$$\frac{a^2\,x^5}{5} + \frac{2\,a\,b\,x^7}{7} + \frac{b^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x + b*x^3)^2,x)
[Out] (a^2*x^5)/5 + (b^2*x^9)/9 + (2*a*b*x^7)/7
```

sympy [A] time = 0.07, size = 26, normalized size = 0.87

$$\frac{a^2x^5}{5} + \frac{2abx^7}{7} + \frac{b^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x)**2,x)
[Out] a**2*x**5/5 + 2*a*b*x**7/7 + b**2*x**9/9
```

3.7 $\int x \left(ax + bx^3\right)^2 \, dx$

Optimal. Leaf size=30

$$\frac{a^2x^4}{4} + \frac{1}{3}abx^6 + \frac{b^2x^8}{8}$$

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1584, 266, 43}

$$\frac{a^2x^4}{4} + \frac{1}{3}abx^6 + \frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int x \left(ax + bx^3\right)^2 \, dx &= \int x^3 \left(a + bx^2\right)^2 \, dx \\
&= \frac{1}{2}\text{Subst}\left(\int x(a + bx)^2 \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^2x + 2abx^2 + b^2x^3\right) \, dx, x, x^2\right) \\
&= \frac{a^2x^4}{4} + \frac{1}{3}abx^6 + \frac{b^2x^8}{8}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^4}{4} + \frac{1}{3}abx^6 + \frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(ax+bx^3\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a*x + b*x^3)^2,x]
[Out] IntegrateAlgebraic[x*(a*x + b*x^3)^2, x]
```

fricas [A] time = 0.34, size = 24, normalized size = 0.80

$$\frac{1}{8}x^8b^2+\frac{1}{3}x^6ba+\frac{1}{4}x^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^2,x, algorithm="fricas")
[Out] 1/8*x^8*b^2 + 1/3*x^6*b*a + 1/4*x^4*a^2
```

giac [A] time = 0.16, size = 24, normalized size = 0.80

$$\frac{1}{8}\,b^2x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^2,x, algorithm="giac")
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

maple [A] time = 0.04, size = 25, normalized size = 0.83

$$\frac{1}{8}b^2x^8+\frac{1}{3}ab\,x^6+\frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x)^2,x)
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

maxima [A] time = 1.34, size = 24, normalized size = 0.80

$$\frac{1}{8}\,b^2x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^2,x, algorithm="maxima")
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^4}{4}+\frac{a\,b\,x^6}{3}+\frac{b^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x + b*x^3)^2,x)
```

```
[Out] (a^2*x^4)/4 + (b^2*x^8)/8 + (a*b*x^6)/3
```

sympy [A] time = 0.07, size = 24, normalized size = 0.80

$$\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8
```

3.8 $\int \left(ax+bx^3\right)^2 \, dx$

Optimal. Leaf size=30

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 270}

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(ax+bx^3\right)^2 \, dx &= \int x^2 \left(a+bx^2\right)^2 \, dx\\ &= \int \left(a^2x^2+2abx^4+b^2x^6\right) \, dx\\ &= \frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax+bx^3\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(a*x + b*x^3)^2, x]
```

fricas [A] time = 0.35, size = 24, normalized size = 0.80

$$\frac{1}{7}x^7b^2 + \frac{2}{5}x^5ba + \frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2,x, algorithm="fricas")
[Out] 1/7*x^7*b^2 + 2/5*x^5*b*a + 1/3*x^3*a^2
```

giac [A] time = 0.15, size = 24, normalized size = 0.80

$$\frac{1}{7}\,b^2x^7 + \frac{2}{5}\,abx^5 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2,x, algorithm="giac")
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

maple [A] time = 0.04, size = 25, normalized size = 0.83

$$\frac{1}{7}b^2x^7 + \frac{2}{5}ab\,x^5 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^2,x)
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

maxima [A] time = 1.32, size = 24, normalized size = 0.80

$$\frac{1}{7}\,b^2x^7 + \frac{2}{5}\,abx^5 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2,x, algorithm="maxima")
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^3}{3} + \frac{2\,a\,b\,x^5}{5} + \frac{b^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^2,x)
[Out] (a^2*x^3)/3 + (b^2*x^7)/7 + (2*a*b*x^5)/5
```

sympy [A] time = 0.07, size = 26, normalized size = 0.87

$$\frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**2,x)
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7
```

3.9 $\int \frac{(ax+bx^3)^2}{x} dx$

Optimal. Leaf size=16

$$\frac{(a+bx^2)^3}{6b}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 261}

$$\frac{(a+bx^2)^3}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^2/x,x]
```

```
[Out] (a + b*x^2)^3/(6*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{(ax+bx^3)^2}{x} dx &= \int x(a+bx^2)^2 dx \\ &= \frac{(a+bx^2)^3}{6b} \end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{(a+bx^2)^3}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^2/x,x]
```

```
[Out] (a + b*x^2)^3/(6*b)
```

IntegrateAlgebraic [A] time = 0.01, size = 27, normalized size = 1.69

$$\frac{1}{6}x^2(3a^2+3abx^2+b^2x^4)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x + b*x^3)^2/x,x]
```

```
[Out] (x^2*(3*a^2 + 3*a*b*x^2 + b^2*x^4))/6
```

fricas [A] time = 0.37, size = 24, normalized size = 1.50

$$\frac{1}{6}\,b^2x^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2/x,x, algorithm="fricas")
```

```
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

giac [A] time = 0.20, size = 24, normalized size = 1.50

$$\frac{1}{6}\,b^2x^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2/x,x, algorithm="giac")
```

```
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

maple [A] time = 0.04, size = 25, normalized size = 1.56

$$\frac{1}{6}b^2x^6 + \frac{1}{2}ab\,x^4 + \frac{1}{2}a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^2/x,x)
```

```
[Out] 1/6*b^2*x^6+1/2*a*b*x^4+1/2*a^2*x^2
```

maxima [A] time = 1.29, size = 24, normalized size = 1.50

$$\frac{1}{6}\,b^2x^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2/x,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

mupad [B] time = 0.03, size = 24, normalized size = 1.50

$$\frac{a^2\,x^2}{2} + \frac{a\,b\,x^4}{2} + \frac{b^2\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^2/x,x)
```

```
[Out] (a^2*x^2)/2 + (b^2*x^6)/6 + (a*b*x^4)/2
```

sympy [B] time = 0.07, size = 24, normalized size = 1.50

$$\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**2/x,x)
```

```
[Out] a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6
```

3.10 $\int \frac{(ax+bx^3)^2}{x^2}\,dx$

Optimal. Leaf size=25

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 194}

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^2/x^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(ax+bx^3)^2}{x^2}\,dx &= \int \left(a+bx^2\right)^2\,dx\\ &= \int \left(a^2+2abx^2+b^2x^4\right)\,dx\\ &= a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^2/x^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

IntegrateAlgebraic [A] time = 0.02, size = 25, normalized size = 1.00

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x + b*x^3)^2/x^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

fricas [A] time = 0.39, size = 21, normalized size = 0.84

$$\frac{1}{5}b^2x^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

giac [A] time = 0.15, size = 21, normalized size = 0.84

$$\frac{1}{5}b^2x^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

maple [A] time = 0.05, size = 22, normalized size = 0.88

$$\frac{1}{5}b^2x^5 + \frac{2}{3}ab\,x^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^2/x^2,x)
```

```
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

maxima [A] time = 1.30, size = 21, normalized size = 0.84

$$\frac{1}{5}b^2x^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

mupad [B] time = 0.03, size = 21, normalized size = 0.84

$$a^2\,x + \frac{2\,a\,b\,x^3}{3} + \frac{b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^2/x^2,x)
```

```
[Out] a^2*x + (b^2*x^5)/5 + (2*a*b*x^3)/3
```

sympy [A] time = 0.08, size = 22, normalized size = 0.88

$$a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**2/x**2,x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

3.11 $\int \left(-4x+3x^3\right)^6\,dx$

Optimal. Leaf size=46

$$\frac{729x^{19}}{19}-\frac{5832x^{17}}{17}+1296x^{15}-\frac{34560x^{13}}{13}+\frac{34560x^{11}}{11}-2048x^9+\frac{4096x^7}{7}$$

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 270}

$$\frac{729x^{19}}{19}-\frac{5832x^{17}}{17}+1296x^{15}-\frac{34560x^{13}}{13}+\frac{34560x^{11}}{11}-2048x^9+\frac{4096x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(-4*x + 3*x^3)^6,x]
```

```
[Out] (4096*x^7)/7 - 2048*x^9 + (34560*x^11)/11 - (34560*x^13)/13 + 1296*x^15 - (
5832*x^17)/17 + (729*x^19)/19
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(-4x+3x^3\right)^6\,dx &= \int x^6\left(-4+3x^2\right)^6\,dx\\ &= \int \left(4096x^6-18432x^8+34560x^{10}-34560x^{12}+19440x^{14}-5832x^{16}+729x^{18}\right)dx\\ &= \frac{4096x^7}{7}-2048x^9+\frac{34560x^{11}}{11}-\frac{34560x^{13}}{13}+1296x^{15}-\frac{5832x^{17}}{17}+\frac{729x^{19}}{19}\end{aligned}$$

Mathematica [A] time = 0.00, size = 46, normalized size = 1.00

$$\frac{729x^{19}}{19}-\frac{5832x^{17}}{17}+1296x^{15}-\frac{34560x^{13}}{13}+\frac{34560x^{11}}{11}-2048x^9+\frac{4096x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4*x + 3*x^3)^6,x]
```

```
[Out] (4096*x^7)/7 - 2048*x^9 + (34560*x^11)/11 - (34560*x^13)/13 + 1296*x^15 - (
5832*x^17)/17 + (729*x^19)/19
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-4x+3x^3\right)^6\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-4*x + 3*x^3)^6,x]
```

```
[Out] IntegrateAlgebraic[(-4*x + 3*x^3)^6, x]
```

fricas [A] time = 0.35, size = 36, normalized size = 0.78

$$\frac{729}{19}x^{19} - \frac{5832}{17}x^{17} + 1296x^{15} - \frac{34560}{13}x^{13} + \frac{34560}{11}x^{11} - 2048x^9 + \frac{4096}{7}x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-4*x)^6,x, algorithm="fricas")
```

```
[Out] 729/19*x^19 - 5832/17*x^17 + 1296*x^15 - 34560/13*x^13 + 34560/11*x^11 - 20
48*x^9 + 4096/7*x^7
```

giac [A] time = 0.15, size = 36, normalized size = 0.78

$$\frac{729}{19}\,x^{19} - \frac{5832}{17}\,x^{17} + 1296\,x^{15} - \frac{34560}{13}\,x^{13} + \frac{34560}{11}\,x^{11} - 2048\,x^9 + \frac{4096}{7}\,x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-4*x)^6,x, algorithm="giac")
```

```
[Out] 729/19*x^19 - 5832/17*x^17 + 1296*x^15 - 34560/13*x^13 + 34560/11*x^11 - 20
48*x^9 + 4096/7*x^7
```

maple [A] time = 0.04, size = 37, normalized size = 0.80

$$\frac{729}{19}x^{19} - \frac{5832}{17}x^{17} + 1296x^{15} - \frac{34560}{13}x^{13} + \frac{34560}{11}x^{11} - 2048x^9 + \frac{4096}{7}x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^3-4*x)^6,x)
```

```
[Out] 4096/7*x^7-2048*x^9+34560/11*x^11-34560/13*x^13+1296*x^15-5832/17*x^17+729/
19*x^19
```

maxima [A] time = 1.33, size = 36, normalized size = 0.78

$$\frac{729}{19}\,x^{19} - \frac{5832}{17}\,x^{17} + 1296\,x^{15} - \frac{34560}{13}\,x^{13} + \frac{34560}{11}\,x^{11} - 2048\,x^9 + \frac{4096}{7}\,x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-4*x)^6,x, algorithm="maxima")
```

```
[Out] 729/19*x^19 - 5832/17*x^17 + 1296*x^15 - 34560/13*x^13 + 34560/11*x^11 - 20
48*x^9 + 4096/7*x^7
```

mupad [B] time = 0.04, size = 36, normalized size = 0.78

$$\frac{729\,x^{19}}{19} - \frac{5832\,x^{17}}{17} + 1296\,x^{15} - \frac{34560\,x^{13}}{13} + \frac{34560\,x^{11}}{11} - 2048\,x^9 + \frac{4096\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x - 3*x^3)^6,x)
```

```
[Out] (4096*x^7)/7 - 2048*x^9 + (34560*x^11)/11 - (34560*x^13)/13 + 1296*x^15 - (
5832*x^17)/17 + (729*x^19)/19
```

sympy [**A**] time = 0.07, size = 42, normalized size = 0.91

$$\frac{729x^{19}}{19}-\frac{5832x^{17}}{17}+1296x^{15}-\frac{34560x^{13}}{13}+\frac{34560x^{11}}{11}-2048x^{9}+\frac{4096x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**3-4*x)**6,x)
```

```
[Out] 729*x**19/19 - 5832*x**17/17 + 1296*x**15 - 34560*x**13/13 + 34560*x**11/11
 - 2048*x**9 + 4096*x**7/7
```

3.12 $\int \frac{x^4}{ax+bx^3}\,dx$

Optimal. Leaf size=27

$$\frac{x^2}{2b}-\frac{a\log\left(a+bx^2\right)}{2b^2}$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1584, 266, 43}

$$\frac{x^2}{2b}-\frac{a\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x + b*x^3),x]
```

```
[Out] x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{ax+bx^3}\,dx &= \int \frac{x^3}{a+bx^2}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{a+bx}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)\,dx,x,x^2\right)\\
&= \frac{x^2}{2b}-\frac{a\log\left(a+bx^2\right)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$\frac{x^2}{2b}-\frac{a\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x + b*x^3),x]
[Out] x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{ax + bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a*x + b*x^3),x]
[Out] IntegrateAlgebraic[x^4/(a*x + b*x^3), x]
```

fricas [A] time = 0.39, size = 22, normalized size = 0.81

$$\frac{bx^2 - a\log\left(bx^2 + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x),x, algorithm="fricas")
[Out] 1/2*(b*x^2 - a*log(b*x^2 + a))/b^2
```

giac [A] time = 0.15, size = 24, normalized size = 0.89

$$\frac{x^2}{2\,b} - \frac{a\log\left(\left|bx^2 + a\right|\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x),x, algorithm="giac")
[Out] 1/2*x^2/b - 1/2*a*log(abs(b*x^2 + a))/b^2
```

maple [A] time = 0.04, size = 24, normalized size = 0.89

$$\frac{x^2}{2b} - \frac{a\ln\left(b\,x^2 + a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x),x)
[Out] 1/2/b*x^2-1/2*a*ln(b*x^2+a)/b^2
```

maxima [A] time = 1.32, size = 23, normalized size = 0.85

$$\frac{x^2}{2\,b} - \frac{a\log\left(bx^2 + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x),x, algorithm="maxima")
[Out] 1/2*x^2/b - 1/2*a*log(b*x^2 + a)/b^2
```

mupad [B] time = 0.04, size = 22, normalized size = 0.81

$$-\frac{a\,\ln\left(b\,x^2 + a\right) - b\,x^2}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x + b*x^3),x)
```

```
[Out] -(a*log(a + b*x^2) - b*x^2)/(2*b^2)
```

sympy [**A**]   time = 0.15, size = 20, normalized size = 0.74

$$-\frac{a\log\left(a+bx^2\right)}{2b^2}+\frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x),x)
```

```
[Out] -a*log(a + b*x**2)/(2*b**2) + x**2/(2*b)
```

3.13 $\int \frac{x^3}{ax+bx^3}\,dx$

Optimal. Leaf size=31

$$\frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1584, 321, 205}

$$\frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x + b*x^3),x]
```

```
[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{ax+bx^3}\,dx &= \int \frac{x^2}{a+bx^2}\,dx\\ &= \frac{x}{b}-\frac{a\int \frac{1}{a+bx^2}\,dx}{b}\\ &= \frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x + b*x^3),x]
[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{ax + bx^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a*x + b*x^3),x]
[Out] IntegrateAlgebraic[x^3/(a*x + b*x^3), x]
```

fricas [A] time = 0.42, size = 82, normalized size = 2.65

$$\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2\,x}{2\,b}, -\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-x}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x),x, algorithm="fricas")
[Out] [1/2*(sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*x)/b,
-(sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - x)/b]
```

giac [A] time = 0.15, size = 26, normalized size = 0.84

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x),x, algorithm="giac")
[Out] -a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

maple [A] time = 0.04, size = 27, normalized size = 0.87

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a*x),x)
[Out] 1/b*x-a/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.94, size = 26, normalized size = 0.84

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

mupad [B] time = 4.91, size = 23, normalized size = 0.74

$$\frac{x}{b}-\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^3),x)
```

```
[Out] x/b - (a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/b^(3/2)
```

sympy [B] time = 0.17, size = 56, normalized size = 1.81

$$\frac{\sqrt{-\frac{a}{b^3}}\log\left(-b\sqrt{-\frac{a}{b^3}}+x\right)}{2}-\frac{\sqrt{-\frac{a}{b^3}}\log\left(b\sqrt{-\frac{a}{b^3}}+x\right)}{2}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x),x)
```

```
[Out] sqrt(-a/b**3)*log(-b*sqrt(-a/b**3) + x)/2 - sqrt(-a/b**3)*log(b*sqrt(-a/b**
3) + x)/2 + x/b
```

3.14 $\int \frac{x^2}{ax+bx^3} \, dx$

Optimal. Leaf size=15

$$\frac{\log\left(a+bx^2\right)}{2b}$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 260}

$$\frac{\log\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x + b*x^3),x]
```

```
[Out] Log[a + b*x^2]/(2*b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{ax+bx^3} \, dx &= \int \frac{x}{a+bx^2} \, dx\\ &= \frac{\log\left(a+bx^2\right)}{2b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x + b*x^3),x]
```

```
[Out] Log[a + b*x^2]/(2*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{ax+bx^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a*x + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[x^2/(a*x + b*x^3), x]
```

fricas [A] time = 0.40, size = 13, normalized size = 0.87

$$\frac{\log\left(bx^2 + a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x),x, algorithm="fricas")
[Out] 1/2*log(b*x^2 + a)/b
```

giac [A] time = 0.19, size = 14, normalized size = 0.93

$$\frac{\log\left(|bx^2 + a|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x),x, algorithm="giac")
[Out] 1/2*log(abs(b*x^2 + a))/b
```

maple [A] time = 0.04, size = 14, normalized size = 0.93

$$\frac{\ln\left(b\,x^2 + a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x),x)
[Out] 1/2*ln(b*x^2+a)/b
```

maxima [A] time = 1.42, size = 13, normalized size = 0.87

$$\frac{\log\left(bx^2 + a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x),x, algorithm="maxima")
[Out] 1/2*log(b*x^2 + a)/b
```

mupad [B] time = 4.92, size = 13, normalized size = 0.87

$$\frac{\ln\left(b\,x^2 + a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^3),x)
[Out] log(a + b*x^2)/(2*b)
```

sympy [A] time = 0.13, size = 10, normalized size = 0.67

$$\frac{\log\left(a + bx^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x),x)
[Out] log(a + b*x**2)/(2*b)
```

3.15 $\int \frac{x}{ax+bx^3}\,dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1584, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x + b*x^3),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{ax+bx^3}\,dx &= \int \frac{1}{a+bx^2}\,dx\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x + b*x^3),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{ax+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a*x + b*x^3),x]
[Out] IntegrateAlgebraic[x/(a*x + b*x^3), x]
```

fricas [A] time = 0.41, size = 67, normalized size = 2.79

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x),x, algorithm="fricas")
[Out] [-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt(
a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

giac [A] time = 0.15, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x),x, algorithm="giac")
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

maple [A] time = 0.04, size = 16, normalized size = 0.67

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x),x)
[Out] 1/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.95, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x),x, algorithm="maxima")
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

mupad [B] time = 0.04, size = 16, normalized size = 0.67

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^3),x)
[Out] atan((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

sympy [B] time = 0.15, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-1
/(a*b)) + x)/2
```

3.16 $\int \frac{1}{ax+bx^3} \, dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^2\right)}{2a}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1593, 266, 36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(-1),x]
```

```
[Out] Log[x]/a - Log[a + b*x^2]/(2*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{ax+bx^3} \, dx &= \int \frac{1}{x\left(a+bx^2\right)} \, dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)} \, dx, x, x^2\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{x} \, dx, x, x^2\right)}{2a}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx} \, dx, x, x^2\right)}{2a} \\
&= \frac{\log(x)}{a}-\frac{\log\left(a+bx^2\right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(-1),x]
```

```
[Out] Log[x]/a - Log[a + b*x^2]/(2*a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ax+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x + b*x^3)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a*x + b*x^3)^(-1), x]
```

fricas [A] time = 0.40, size = 18, normalized size = 0.82

$$-\frac{\log\left(bx^2+a\right)-2\,\log(x)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] -1/2*(log(b*x^2 + a) - 2*log(x))/a
```

giac [A] time = 0.15, size = 24, normalized size = 1.09

$$\frac{\log\left(x^2\right)}{2\,a}-\frac{\log\left(\left|bx^2+a\right|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a - 1/2*log(abs(b*x^2 + a))/a
```

maple [A] time = 0.05, size = 21, normalized size = 0.95

$$\frac{\ln(x)}{a}-\frac{\ln\left(b\,x^2+a\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x),x)
```

```
[Out] 1/a*ln(x)-1/2*ln(b*x^2+a)/a
```

maxima [A] time = 1.34, size = 20, normalized size = 0.91

$$-\frac{\log\left(bx^2+a\right)}{2\,a}+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] -1/2*log(b*x^2 + a)/a + log(x)/a
```

mupad [B] time = 0.06, size = 18, normalized size = 0.82

$$-\frac{\ln\left(b\,x^2+a\right)-2\,\ln(x)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^3),x)
[Out] -(log(a + b*x^2) - 2*log(x))/(2*a)
```

sympy [A] time = 0.21, size = 15, normalized size = 0.68

$$\frac{\log\left(x\right)}{a}-\frac{\log\left(\frac{a}{b}+x^{2}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x),x)
[Out] log(x)/a - log(a/b + x**2)/(2*a)
```

3.17 $\int \frac{1}{x(ax+bx^3)}\,dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1584, 325, 205}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x + b*x^3)),x]
```

```
[Out] -(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(ax+bx^3\right)}\,dx &= \int \frac{1}{x^2\left(a+bx^2\right)}\,dx\\ &= -\frac{1}{ax}-\frac{b\int \frac{1}{a+bx^2}\,dx}{a}\\ &= -\frac{1}{ax}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x + b*x^3)),x]
[Out] -(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(ax+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a*x + b*x^3)),x]
[Out] IntegrateAlgebraic[1/(x*(a*x + b*x^3)), x]
```

fricas [A] time = 0.42, size = 82, normalized size = 2.41

$$\left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2}{2\,ax}, -\frac{x\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+1}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x),x, algorithm="fricas")
[Out] [1/2*(x*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2)/(a*
x), -(x*sqrt(b/a)*arctan(x*sqrt(b/a)) + 1)/(a*x)]
```

giac [A] time = 0.15, size = 29, normalized size = 0.85

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x),x, algorithm="giac")
[Out] -b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

maple [A] time = 0.05, size = 30, normalized size = 0.88

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x),x)
[Out] -1/a*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)-1/a/x
```

maxima [A] time = 2.96, size = 29, normalized size = 0.85

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] -b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

mupad [B] time = 4.96, size = 26, normalized size = 0.76

$$-\frac{1}{a\,x}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^3)),x)
```

```
[Out] - 1/(a*x) - (b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2)
```

sympy [B] time = 0.22, size = 65, normalized size = 1.91

$$\frac{\sqrt{-\frac{b}{a^3}}\log\left(-\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^3}}\log\left(\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x),x)
```

```
[Out] sqrt(-b/a**3)*log(-a**2*sqrt(-b/a**3)/b + x)/2 - sqrt(-b/a**3)*log(a**2*sqr
t(-b/a**3)/b + x)/2 - 1/(a*x)
```

3.18 $\int \frac{1}{x^2(ax+bx^3)}\,dx$

Optimal. Leaf size=35

$$\frac{b\log\left(a+bx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1584, 266, 44}

$$\frac{b\log\left(a+bx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x + b*x^3)),x]
```

```
[Out] -1/(2*a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(ax+bx^3\right)}\,dx&=\int \frac{1}{x^3\left(a+bx^2\right)}\,dx\\&=\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)\,dx,x,x^2\right)\\&=-\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2\right)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$\frac{b\log\left(a+bx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x + b*x^3)),x]
```

```
[Out] -1/2*1/(a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(ax+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a*x + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a*x + b*x^3)), x]
```

fricas [A] time = 0.40, size = 33, normalized size = 0.94

$$\frac{bx^2\log\left(bx^2+a\right)-2\,bx^2\log(x)-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*log(b*x^2 + a) - 2*b*x^2*log(x) - a)/(a^2*x^2)
```

giac [A] time = 0.17, size = 43, normalized size = 1.23

$$-\frac{b\log\left(x^2\right)}{2\,a^2}+\frac{b\log\left(\left|bx^2+a\right|\right)}{2\,a^2}+\frac{bx^2-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -1/2*b*log(x^2)/a^2 + 1/2*b*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 - a)/(a^2*
x^2)
```

maple [A] time = 0.05, size = 32, normalized size = 0.91

$$-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(b\,x^2+a\right)}{2a^2}-\frac{1}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x),x)
```

```
[Out] -1/2/a/x^2-1/a^2*b*ln(x)+1/2*b*ln(b*x^2+a)/a^2
```

maxima [A] time = 1.27, size = 31, normalized size = 0.89

$$\frac{b\log\left(bx^2+a\right)}{2\,a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/2*b*log(b*x^2 + a)/a^2 - b*log(x)/a^2 - 1/2/(a*x^2)
```

mupad [B] time = 0.06, size = 31, normalized size = 0.89

$$\frac{b\,\ln\left(b\,x^2+a\right)}{2\,a^2}-\frac{1}{2\,a\,x^2}-\frac{b\,\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^3)),x)
[Out] (b*log(a + b*x^2))/(2*a^2) - 1/(2*a*x^2) - (b*log(x))/a^2
```

sympy [A] time = 0.29, size = 31, normalized size = 0.89

$$-\frac{1}{2ax^2} - \frac{b\log(x)}{a^2} + \frac{b\log\left(\frac{a}{b} + x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x),x)
[Out] -1/(2*a*x**2) - b*log(x)/a**2 + b*log(a/b + x**2)/(2*a**2)
```

3.19 $\int \frac{1}{x^3(ax+bx^3)}\,dx$

Optimal. Leaf size=43

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1584, 325, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a*x + b*x^3)),x]
```

```
[Out] -1/(3*a*x^3) + b/(a^2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(ax+bx^3\right)}\,dx &= \int \frac{1}{x^4\left(a+bx^2\right)}\,dx\\
&= -\frac{1}{3ax^3}-\frac{b\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{a}\\
&= -\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{b^2\int \frac{1}{a+bx^2}\,dx}{a^2}\\
&= -\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a*x + b*x^3)),x]
[Out] -1/3*1/(a*x^3) + b/(a^2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(ax+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a*x + b*x^3)),x]
[Out] IntegrateAlgebraic[1/(x^3*(a*x + b*x^3)), x]
```

fricas [A] time = 0.42, size = 106, normalized size = 2.47

$$\left[\frac{3\,bx^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+6\,bx^2-2\,a}{6\,a^2x^3},\ \frac{3\,bx^3\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+3\,bx^2-a}{3\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x),x, algorithm="fricas")
[Out] [1/6*(3*b*x^3*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) +
6*b*x^2 - 2*a)/(a^2*x^3), 1/3*(3*b*x^3*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*b*
x^2 - a)/(a^2*x^3)]
```

giac [A] time = 0.15, size = 40, normalized size = 0.93

$$\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{3\,bx^2-a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x),x, algorithm="giac")
[Out] b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

maple [A] time = 0.05, size = 39, normalized size = 0.91

$$\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^2}+\frac{b}{a^2x}-\frac{1}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a*x),x)
[Out] 1/a^2*b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)-1/3/a/x^3+b/a^2/x
```

maxima [A] time = 2.97, size = 40, normalized size = 0.93

$$\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{3\,bx^2-a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

mupad [B] time = 4.94, size = 37, normalized size = 0.86

$$\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{1}{3\,a}-\frac{b\,x^2}{a^2}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x + b*x^3)),x)
```

```
[Out] (b^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(5/2) - (1/(3*a) - (b*x^2)/a^2)/x^3
```

sympy [B] time = 0.25, size = 87, normalized size = 2.02

$$-\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{-a+3bx^2}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a*x),x)
```

```
[Out] -sqrt(-b**3/a**5)*log(-a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + sqrt(-b**3/a**5)
*log(a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + (-a + 3*b*x**2)/(3*a**2*x**3)
```

3.20 $\int \frac{1}{x^4(ax+bx^3)}\,dx$

Optimal. Leaf size=49

$$-\frac{b^2\log\left(a+bx^2\right)}{2a^3}+\frac{b^2\log(x)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1584, 266, 44}

$$-\frac{b^2\log\left(a+bx^2\right)}{2a^3}+\frac{b^2\log(x)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a*x + b*x^3)),x]
```

```
[Out] -1/(4*a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(2*a
^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(ax+bx^3\right)}\,dx &= \int \frac{1}{x^5\left(a+bx^2\right)}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(a+bx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log\left(a+bx^2\right)}{2a^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$-\frac{b^2\log\left(a+bx^2\right)}{2a^3}+\frac{b^2\log(x)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a*x + b*x^3)),x]
```

```
[Out] -1/4*1/(a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(2
*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(ax + bx^3\right)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a*x + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a*x + b*x^3)), x]
```

fricas [A] time = 0.41, size = 45, normalized size = 0.92

$$-\frac{2\,b^2x^4\log\left(bx^2+a\right)-4\,b^2x^4\log(x)-2\,abx^2+a^2}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*x^4*log(b*x^2 + a) - 4*b^2*x^4*log(x) - 2*a*b*x^2 + a^2)/(a^3*x
^4)
```

giac [A] time = 0.16, size = 57, normalized size = 1.16

$$\frac{b^2\log\left(x^2\right)}{2\,a^3}-\frac{b^2\log\left(\left|bx^2+a\right|\right)}{2\,a^3}-\frac{3\,b^2x^4-2\,abx^2+a^2}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/2*b^2*log(x^2)/a^3 - 1/2*b^2*log(abs(b*x^2 + a))/a^3 - 1/4*(3*b^2*x^4 - 2
*a*b*x^2 + a^2)/(a^3*x^4)
```

maple [A] time = 0.05, size = 44, normalized size = 0.90

$$\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln\left(b\,x^2+a\right)}{2a^3}+\frac{b}{2a^2x^2}-\frac{1}{4a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a*x),x)
```

```
[Out] -1/4/a/x^4+1/2*b/a^2/x^2+1/a^3*b^2*ln(x)-1/2*b^2*ln(b*x^2+a)/a^3
```

maxima [A] time = 1.39, size = 44, normalized size = 0.90

$$-\frac{b^2\log\left(bx^2+a\right)}{2\,a^3}+\frac{b^2\log(x)}{a^3}+\frac{2\,bx^2-a}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] -1/2*b^2*log(b*x^2 + a)/a^3 + b^2*log(x)/a^3 + 1/4*(2*b*x^2 - a)/(a^2*x^4)
```

mupad [B] time = 0.06, size = 46, normalized size = 0.94

$$\frac{b^2\,\ln(x)}{a^3}-\frac{b^2\,\ln\left(b\,x^2+a\right)}{2\,a^3}-\frac{\frac{1}{4\,a}-\frac{b\,x^2}{2\,a^2}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a*x + b*x^3)),x)
```

```
[Out] (b^2*log(x))/a^3 - (b^2*log(a + b*x^2))/(2*a^3) - (1/(4*a) - (b*x^2)/(2*a^2
))/x^4
```

sympy [A] time = 0.34, size = 42, normalized size = 0.86

$$\frac{-a+2bx^2}{4a^2x^4}+\frac{b^2\log{(x)}}{a^3}-\frac{b^2\log{\left(\frac{a}{b}+x^2\right)}}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a*x),x)
```

```
[Out] (-a + 2*b*x**2)/(4*a**2*x**4) + b**2*log(x)/a**3 - b**2*log(a/b + x**2)/(2*
a**3)
```

3.21 $\int \frac{x^2}{\left(ax+bx^3\right)^2}\,dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1584, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x + b*x^3)^2,x]
```

```
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(ax+bx^3\right)^2}\,dx &= \int \frac{1}{\left(a+bx^2\right)^2}\,dx\\ &= \frac{x}{2a\left(a+bx^2\right)}+\frac{\int \frac{1}{a+bx^2}\,dx}{2a}\\ &= \frac{x}{2a\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x + b*x^3)^2,x]
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a*x + b*x^3)^2,x]
[Out] IntegrateAlgebraic[x^2/(a*x + b*x^3)^2, x]
```

fricas [A] time = 0.43, size = 120, normalized size = 2.67

$$\left[\frac{2\,abx-\left(bx^2+a\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{4\left(a^2b^2x^2+a^3b\right)},\ \frac{abx+\left(bx^2+a\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\left(a^2b^2x^2+a^3b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^2,x, algorithm="fricas")
[Out] [1/4*(2*a*b*x - (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)))/(a^2*b^2*x^2 + a^3*b), 1/2*(a*b*x + (b*x^2 + a)*sqrt(a*b)*arctan
(sqrt(a*b)*x/a))/(a^2*b^2*x^2 + a^3*b)]
```

giac [A] time = 0.17, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a}+\frac{x}{2\left(bx^2+a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^2,x, algorithm="giac")
[Out] 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*x/((b*x^2 + a)*a)
```

maple [A] time = 0.05, size = 36, normalized size = 0.80

$$\frac{x}{2\left(b\,x^2+a\right)a}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x)^2,x)
[Out] 1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.98, size = 35, normalized size = 0.78

$$\frac{x}{2\left(abx^2+a^2\right)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x/(a*b*x^2 + a^2) + 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a)
```

mupad [B] time = 4.96, size = 33, normalized size = 0.73

$$\frac{x}{2\,a\,\left(b\,x^2+a\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^3)^2,x)
```

```
[Out] x/(2*a*(a + b*x^2)) + atan((b^(1/2)*x)/a^(1/2))/(2*a^(3/2)*b^(1/2))
```

sympy [B] time = 0.24, size = 78, normalized size = 1.73

$$\frac{x}{2a^2+2abx^2}-\frac{\sqrt{-\frac{1}{a^3b}}\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b}}\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x)**2,x)
```

```
[Out] x/(2*a**2 + 2*a*b*x**2) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b)) + x
)/4 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x)/4
```

3.22 $\int \frac{x}{\left(ax+bx^3\right)^2} \, dx$

Optimal. Leaf size=38

$$-\frac{\log \left(a+bx^2\right)}{2a^2}+\frac{\log (x)}{a^2}+\frac{1}{2a \left(a+bx^2\right)}$$

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 266, 44}

$$-\frac{\log \left(a+bx^2\right)}{2a^2}+\frac{\log (x)}{a^2}+\frac{1}{2a \left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x + b*x^3)^2,x]
```

```
[Out] 1/(2*a*(a + b*x^2)) + Log[x]/a^2 - Log[a + b*x^2]/(2*a^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(ax+bx^3\right)^2} \, dx &= \int \frac{1}{x \left(a+bx^2\right)^2} \, dx\\
&= \frac{1}{2} \text{Subst}\left(\int \frac{1}{x(a+bx)^2} \, dx,x,x^2\right)\\
&= \frac{1}{2} \text{Subst}\left(\int \left(\frac{1}{a^2 x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right) \, dx,x,x^2\right)\\
&= \frac{1}{2a \left(a+bx^2\right)}+\frac{\log (x)}{a^2}-\frac{\log \left(a+bx^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.87

$$\frac{\frac{a}{a+bx^2}-\log \left(a+bx^2\right)+2 \log (x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x + b*x^3)^2,x]
```

```
[Out] (a/(a + b*x^2) + 2*Log[x] - Log[a + b*x^2])/(2*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(ax+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a*x + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[x/(a*x + b*x^3)^2, x]
```

fricas [A] time = 0.41, size = 47, normalized size = 1.24

$$-\frac{\left(bx^2+a\right)\log\left(bx^2+a\right)-2\left(bx^2+a\right)\log(x)-a}{2\left(a^2bx^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((b*x^2 + a)*log(b*x^2 + a) - 2*(b*x^2 + a)*log(x) - a)/(a^2*b*x^2 + a
^3)
```

giac [A] time = 0.16, size = 47, normalized size = 1.24

$$\frac{\log\left(x^2\right)}{2\,a^2}-\frac{\log\left(\left|bx^2+a\right|\right)}{2\,a^2}+\frac{bx^2+2\,a}{2\left(bx^2+a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^2 - 1/2*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 + 2*a)/((b*x^2
+ a)*a^2)
```

maple [A] time = 0.05, size = 35, normalized size = 0.92

$$\frac{1}{2\left(b\,x^2+a\right)a}+\frac{\ln(x)}{a^2}-\frac{\ln\left(b\,x^2+a\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x)^2,x)
```

```
[Out] 1/2/a/(b*x^2+a)+1/a^2*ln(x)-1/2*ln(b*x^2+a)/a^2
```

maxima [A] time = 1.40, size = 34, normalized size = 0.89

$$\frac{1}{2\left(abx^2+a^2\right)}-\frac{\log\left(bx^2+a\right)}{2\,a^2}+\frac{\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2/(a*b*x^2 + a^2) - 1/2*log(b*x^2 + a)/a^2 + log(x)/a^2
```

mupad [B] time = 0.05, size = 34, normalized size = 0.89

$$\frac{\ln(x)}{a^2}+\frac{1}{2\,a\,\left(b\,x^2+a\right)}-\frac{\ln\left(b\,x^2+a\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^3)^2,x)
[Out] log(x)/a^2 + 1/(2*a*(a + b*x^2)) - log(a + b*x^2)/(2*a^2)
```

sympy [A] time = 0.33, size = 34, normalized size = 0.89

$$\frac{1}{2a^2+2abx^2}+\frac{\log{(x)}}{a^2}-\frac{\log{\left(\frac{a}{b}+x^2\right)}}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x)**2,x)
[Out] 1/(2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(a/b + x**2)/(2*a**2)
```

3.23 $\int \frac{1}{(ax+bx^3)^2}\,dx$

Optimal. Leaf size=57

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}$$

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1593, 290, 325, 205}

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(-2),x]
```

```
[Out] -3/(2*a^2*x) + 1/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(2*a^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\left(ax+bx^3\right)^2}\,dx = \int \frac{1}{x^2\left(a+bx^2\right)^2}\,dx$$
$$= \frac{1}{2ax\left(a+bx^2\right)} + \frac{3\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{2a}$$
$$= -\frac{3}{2a^2x} + \frac{1}{2ax\left(a+bx^2\right)} - \frac{(3b)\int \frac{1}{a+bx^2}\,dx}{2a^2}$$
$$= -\frac{3}{2a^2x} + \frac{1}{2ax\left(a+bx^2\right)} - \frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.95

$$-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}} - \frac{bx}{2a^2\left(a+bx^2\right)} - \frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(-2),x]
[Out] -(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(2*a^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x + b*x^3)^(-2),x]
[Out] IntegrateAlgebraic[(a*x + b*x^3)^(-2), x]
```

fricas [A] time = 0.42, size = 136, normalized size = 2.39

$$\left[-\frac{6\,bx^2 - 3\left(bx^3+ax\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right) + 4\,a}{4\left(a^2bx^3+a^3x\right)},\ -\frac{3\,bx^2 + 3\left(bx^3+ax\right)\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right) + 2\,a}{2\left(a^2bx^3+a^3x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^2,x, algorithm="fricas")
[Out] [-1/4*(6*b*x^2 - 3*(b*x^3 + a*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
 a)/(b*x^2 + a)) + 4*a)/(a^2*b*x^3 + a^3*x), -1/2*(3*b*x^2 + 3*(b*x^3 + a*x
)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 2*a)/(a^2*b*x^3 + a^3*x)]
```

giac [A] time = 0.15, size = 47, normalized size = 0.82

$$-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2} - \frac{3\,bx^2+2\,a}{2\left(bx^3+ax\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*x^2 + 2*a)/((b*x^3
+ a*x)*a^2)
```

maple [A] time = 0.06, size = 46, normalized size = 0.81

$$-\frac{bx}{2\left(b\,x^{2}+a\right)a^{2}}-\frac{3b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^{2}}-\frac{1}{a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x)^2,x)
```

```
[Out] -1/2/a^2*b*x/(b*x^2+a)-3/2/a^2*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)-1/a^
2/x
```

maxima [A] time = 2.96, size = 49, normalized size = 0.86

$$-\frac{3\,bx^{2}+2\,a}{2\left(a^{2}bx^{3}+a^{3}x\right)}-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*b*x^2 + 2*a)/(a^2*b*x^3 + a^3*x) - 3/2*b*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*a^2)
```

mupad [B] time = 4.98, size = 44, normalized size = 0.77

$$-\frac{\frac{1}{a}+\frac{3\,b\,x^{2}}{2\,a^{2}}}{b\,x^{3}+a\,x}-\frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^3)^2,x)
```

```
[Out] - (1/a + (3*b*x^2)/(2*a^2))/(a*x + b*x^3) - (3*b^(1/2)*atan((b^(1/2)*x)/a^(
1/2)))/(2*a^(5/2))
```

sympy [A] time = 0.33, size = 92, normalized size = 1.61

$$\frac{3\sqrt{-\frac{b}{a^{5}}}\log\left(-\frac{a^{3}\sqrt{-\frac{b}{a^{5}}}}{b}+x\right)}{4}-\frac{3\sqrt{-\frac{b}{a^{5}}}\log\left(\frac{a^{3}\sqrt{-\frac{b}{a^{5}}}}{b}+x\right)}{4}+\frac{-2a-3bx^{2}}{2a^{3}x+2a^{2}bx^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x)**2,x)
```

```
[Out] 3*sqrt(-b/a**5)*log(-a**3*sqrt(-b/a**5)/b + x)/4 - 3*sqrt(-b/a**5)*log(a**3
*sqrt(-b/a**5)/b + x)/4 + (-2*a - 3*b*x**2)/(2*a**3*x + 2*a**2*b*x**3)
```

3.24 $\int \frac{1}{x\left(ax+bx^3\right)^2} dx$

Optimal. Leaf size=49

$$\frac{b\log\left(a+bx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{b}{2a^2\left(a+bx^2\right)}-\frac{1}{2a^2x^2}$$

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1584, 266, 44}

$$-\frac{b}{2a^2\left(a+bx^2\right)}+\frac{b\log\left(a+bx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x + b*x^3)^2),x]
```

```
[Out] -1/(2*a^2*x^2) - b/(2*a^2*(a + b*x^2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x^
2])/a^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(ax+bx^3\right)^2} dx &= \int \frac{1}{x^3\left(a+bx^2\right)^2} dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right) dx, x, x^2\right) \\
&= -\frac{1}{2a^2x^2}-\frac{b}{2a^2\left(a+bx^2\right)}-\frac{2b\log(x)}{a^3}+\frac{b\log\left(a+bx^2\right)}{a^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 41, normalized size = 0.84

$$-\frac{a\left(\frac{b}{a+bx^2}+\frac{1}{x^2}\right)-2b\log\left(a+bx^2\right)+4b\log(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x + b*x^3)^2),x]
```

```
[Out] -1/2*(a*(x^(-2) + b/(a + b*x^2)) + 4*b*Log[x] - 2*b*Log[a + b*x^2])/a^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(ax+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a*x + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a*x + b*x^3)^2), x]
```

fricas [A] time = 0.40, size = 73, normalized size = 1.49

$$-\frac{2\,abx^2+a^2-2\left(b^2x^4+abx^2\right)\log\left(bx^2+a\right)+4\left(b^2x^4+abx^2\right)\log(x)}{2\left(a^3bx^4+a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*x^2 + a^2 - 2*(b^2*x^4 + a*b*x^2)*log(b*x^2 + a) + 4*(b^2*x^4 +
 a*b*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

giac [A] time = 0.21, size = 51, normalized size = 1.04

$$-\frac{b\log\left(x^2\right)}{a^3}+\frac{b\log\left(\left|bx^2+a\right|\right)}{a^3}-\frac{2\,bx^2+a}{2\left(bx^4+ax^2\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -b*log(x^2)/a^3 + b*log(abs(b*x^2 + a))/a^3 - 1/2*(2*b*x^2 + a)/((b*x^4 + a
*x^2)*a^2)
```

maple [A] time = 0.05, size = 46, normalized size = 0.94

$$-\frac{b}{2\left(b\,x^2+a\right)a^2}-\frac{2b\ln(x)}{a^3}+\frac{b\ln\left(b\,x^2+a\right)}{a^3}-\frac{1}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x)^2,x)
```

```
[Out] -1/2/a^2/x^2-1/2*b/a^2/(b*x^2+a)-2/a^3*b*ln(x)+b*ln(b*x^2+a)/a^3
```

maxima [A] time = 1.36, size = 50, normalized size = 1.02

$$-\frac{2\,bx^2+a}{2\left(a^2bx^4+a^3x^2\right)}+\frac{b\log\left(bx^2+a\right)}{a^3}-\frac{2\,b\log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x^2 + a)/(a^2*b*x^4 + a^3*x^2) + b*log(b*x^2 + a)/a^3 - 2*b*log(x
)/a^3
```

mupad [B] time = 0.05, size = 51, normalized size = 1.04

$$\frac{b\ \ln\left(b\,x^2+a\right)}{a^3}-\frac{\frac{1}{2\,a}+\frac{b\,x^2}{a^2}}{b\,x^4+a\,x^2}-\frac{2\,b\ \ln(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^3)^2),x)
```

```
[Out] (b*log(a + b*x^2))/a^3 - (1/(2*a) + (b*x^2)/a^2)/(a*x^2 + b*x^4) - (2*b*log
(x))/a^3
```

sympy [A] time = 0.41, size = 51, normalized size = 1.04

$$\frac{-a-2bx^2}{2a^3x^2+2a^2bx^4}-\frac{2b\log\left(x\right)}{a^3}+\frac{b\log\left(\frac{a}{b}+x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x)**2,x)
```

```
[Out] (-a - 2*b*x**2)/(2*a**3*x**2 + 2*a**2*b*x**4) - 2*b*log(x)/a**3 + b*log(a/b
 + x**2)/a**3
```

3.25 $\int \frac{1}{x^2\left(ax+bx^3\right)^2}\,dx$

Optimal. Leaf size=68

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{5b}{2a^3x}-\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}$$

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1584, 290, 325, 205}

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{5b}{2a^3x}-\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x + b*x^3)^2),x]
```

```
[Out] -5/(6*a^2*x^3) + (5*b)/(2*a^3*x) + 1/(2*a*x^3*(a + b*x^2)) + (5*b^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x^2\left(ax+bx^3\right)^2}\,dx = \int \frac{1}{x^4\left(a+bx^2\right)^2}\,dx$$
$$= \frac{1}{2ax^3\left(a+bx^2\right)} + \frac{5\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{2a}$$
$$= -\frac{5}{6a^2x^3} + \frac{1}{2ax^3\left(a+bx^2\right)} - \frac{(5b)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{2a^2}$$
$$= -\frac{5}{6a^2x^3} + \frac{5b}{2a^3x} + \frac{1}{2ax^3\left(a+bx^2\right)} + \frac{\left(5b^2\right)\int \frac{1}{a+bx^2}\,dx}{2a^3}$$
$$= -\frac{5}{6a^2x^3} + \frac{5b}{2a^3x} + \frac{1}{2ax^3\left(a+bx^2\right)} + \frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}$$

Mathematica [A] time = 0.04, size = 67, normalized size = 0.99

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}} + \frac{b^2x}{2a^3\left(a+bx^2\right)} + \frac{2b}{a^3x} - \frac{1}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x + b*x^3)^2),x]
```

```
[Out] -1/3*1/(a^2*x^3) + (2*b)/(a^3*x) + (b^2*x)/(2*a^3*(a + b*x^2)) + (5*b^(3/2)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(ax+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a*x + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a*x + b*x^3)^2), x]
```

fricas [A] time = 0.41, size = 172, normalized size = 2.53

$$\left[\frac{30\,b^2x^4 + 20\,abx^2 + 15\left(b^2x^5 + abx^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right) - 4\,a^2}{12\left(a^3bx^5 + a^4x^3\right)},\ \frac{15\,b^2x^4 + 10\,abx^2 + 15\left(b^2x^5 + abx^3\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right) - 2\,a^2}{6\left(a^3bx^5 + a^4x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(30*b^2*x^4 + 20*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(-b/a)*log((b*x
^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 4*a^2)/(a^3*b*x^5 + a^4*x^3), 1/6
*(15*b^2*x^4 + 10*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(b/a)*arctan(x*sqrt(
b/a)) - 2*a^2)/(a^3*b*x^5 + a^4*x^3)]
```

giac [A] time = 0.17, size = 59, normalized size = 0.87

$$\frac{5\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3}+\frac{b^2x}{2\left(bx^2+a\right)a^3}+\frac{6\,bx^2-a}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 5/2*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/2*b^2*x/((b*x^2 + a)*a^3)
 + 1/3*(6*b*x^2 - a)/(a^3*x^3)
```

maple [A] time = 0.06, size = 59, normalized size = 0.87

$$\frac{b^2x}{2\left(b\,x^2+a\right)a^3}+\frac{5b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^3}+\frac{2b}{a^3x}-\frac{1}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x)^2,x)
```

```
[Out] 1/2/a^3*b^2*x/(b*x^2+a)+5/2/a^3*b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)-1
/3/a^2/x^3+2*b/a^3/x
```

maxima [A] time = 2.92, size = 64, normalized size = 0.94

$$\frac{15\,b^2x^4+10\,abx^2-2\,a^2}{6\left(a^3bx^5+a^4x^3\right)}+\frac{5\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(15*b^2*x^4 + 10*a*b*x^2 - 2*a^2)/(a^3*b*x^5 + a^4*x^3) + 5/2*b^2*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

mupad [B] time = 5.03, size = 58, normalized size = 0.85

$$\frac{\frac{5\,b\,x^2}{3\,a^2}-\frac{1}{3\,a}+\frac{5\,b^2\,x^4}{2\,a^3}}{b\,x^5+a\,x^3}+\frac{5\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^3)^2),x)
```

```
[Out] ((5*b*x^2)/(3*a^2) - 1/(3*a) + (5*b^2*x^4)/(2*a^3))/(a*x^3 + b*x^5) + (5*b^
(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(7/2))
```

sympy [A] time = 0.39, size = 114, normalized size = 1.68

$$-\frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(-\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2}+x\right)}{4}+\frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2}+x\right)}{4}+\frac{-2a^2+10abx^2+15b^2x^4}{6a^4x^3+6a^3bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x)**2,x)
```

```
[Out] -5*sqrt(-b**3/a**7)*log(-a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + 5*sqrt(-b**3/a
**7)*log(a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + (-2*a**2 + 10*a*b*x**2 + 15*b*
*2*x**4)/(6*a**4*x**3 + 6*a**3*b*x**5)
```

3.26 $\int \frac{x^5}{x-x^3}\,dx$

Optimal. Leaf size=13

$$-\frac{x^3}{3}-x+\tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 302, 206}

$$-\frac{x^3}{3}-x+\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(x - x^3),x]
```

```
[Out] -x - x^3/3 + ArcTanh[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{x-x^3}\,dx &= \int \frac{x^4}{1-x^2}\,dx\\
&= \int \left(-1-x^2+\frac{1}{1-x^2}\right)dx\\
&= -x-\frac{x^3}{3}+\int \frac{1}{1-x^2}\,dx\\
&= -x-\frac{x^3}{3}+\tanh^{-1}(x)
\end{aligned}$$

Mathematica [B] time = 0.00, size = 29, normalized size = 2.23

$$-\frac{x^3}{3}-x-\frac{1}{2}\log(1-x)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(x - x^3),x]
```

```
[Out] -x - x^3/3 - Log[1 - x]/2 + Log[1 + x]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{x - x^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(x - x^3),x]
[Out] IntegrateAlgebraic[x^5/(x - x^3), x]
```

fricas [A] time = 0.40, size = 21, normalized size = 1.62

$$-\frac{1}{3} x^3 - x + \frac{1}{2} \log(x+1) - \frac{1}{2} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+x),x, algorithm="fricas")
[Out] -1/3*x^3 - x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

giac [B] time = 0.17, size = 23, normalized size = 1.77

$$-\frac{1}{3} x^3 - x + \frac{1}{2} \log(|x+1|) - \frac{1}{2} \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+x),x, algorithm="giac")
[Out] -1/3*x^3 - x + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

maple [A] time = 0.05, size = 22, normalized size = 1.69

$$-\frac{x^3}{3} - x - \frac{\ln(x-1)}{2} + \frac{\ln(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^3+x),x)
[Out] -1/3*x^3-x-1/2*ln(x-1)+1/2*ln(x+1)
```

maxima [A] time = 1.38, size = 21, normalized size = 1.62

$$-\frac{1}{3} x^3 - x + \frac{1}{2} \log(x+1) - \frac{1}{2} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+x),x, algorithm="maxima")
[Out] -1/3*x^3 - x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

mupad [B] time = 4.98, size = 11, normalized size = 0.85

$$\operatorname{atanh}(x) - x - \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x - x^3),x)
```

```
[Out] atanh(x) - x - x^3/3
```

sympy [B] time = 0.11, size = 19, normalized size = 1.46

$$-\frac{x^3}{3} - x - \frac{\log(x-1)}{2} + \frac{\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**3+x),x)
```

```
[Out] -x**3/3 - x - log(x - 1)/2 + log(x + 1)/2
```

3.27 $\int \frac{x^4}{x-x^3} \, dx$

Optimal. Leaf size=20

$$-\frac{x^2}{2}-\frac{1}{2}\log\left(1-x^2\right)$$

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 266, 43}

$$-\frac{x^2}{2}-\frac{1}{2}\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(x - x^3),x]
```

```
[Out] -x^2/2 - Log[1 - x^2]/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{x-x^3} \, dx &= \int \frac{x^3}{1-x^2} \, dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{1-x} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(-1+\frac{1}{1-x}\right) dx, x, x^2\right) \\
&= -\frac{x^2}{2}-\frac{1}{2}\log\left(1-x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.90

$$-\frac{x^2}{2}-\frac{1}{2}\log\left(x^2-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(x - x^3),x]
```

```
[Out] -1/2*x^2 - Log[-1 + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{x-x^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(x - x^3),x]
[Out] IntegrateAlgebraic[x^4/(x - x^3), x]
```

fricas [A] time = 0.39, size = 14, normalized size = 0.70

$$-\frac{1}{2}\,x^2 - \frac{1}{2}\,\log\left(x^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+x),x, algorithm="fricas")
[Out] -1/2*x^2 - 1/2*log(x^2 - 1)
```

giac [A] time = 0.15, size = 15, normalized size = 0.75

$$-\frac{1}{2}\,x^2 - \frac{1}{2}\,\log\left(\left|x^2 - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+x),x, algorithm="giac")
[Out] -1/2*x^2 - 1/2*log(abs(x^2 - 1))
```

maple [A] time = 0.04, size = 19, normalized size = 0.95

$$-\frac{x^2}{2} - \frac{\ln(x-1)}{2} - \frac{\ln(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^3+x),x)
[Out] -1/2*x^2-1/2*ln(x-1)-1/2*ln(x+1)
```

maxima [A] time = 1.32, size = 18, normalized size = 0.90

$$-\frac{1}{2}\,x^2 - \frac{1}{2}\,\log(x+1) - \frac{1}{2}\,\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+x),x, algorithm="maxima")
[Out] -1/2*x^2 - 1/2*log(x + 1) - 1/2*log(x - 1)
```

mupad [B] time = 0.04, size = 14, normalized size = 0.70

$$-\frac{\ln\left(x^2-1\right)}{2} - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x - x^3),x)
```

```
[Out] - log(x^2 - 1)/2 - x^2/2
```

sympy [A] time = 0.08, size = 14, normalized size = 0.70

$$-\frac{x^2}{2} - \frac{\log\left(x^2 - 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**3+x),x)
[Out] -x**2/2 - log(x**2 - 1)/2
```

3.28 $\int \frac{x^3}{x-x^3} \, dx$

Optimal. Leaf size=6

$$\tanh^{-1}(x) - x$$

Rubi [A] time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 321, 206}

$$\tanh^{-1}(x) - x$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(x - x^3),x]
```

```
[Out] -x + ArcTanh[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{x-x^3} \, dx &= \int \frac{x^2}{1-x^2} \, dx \\
&= -x + \int \frac{1}{1-x^2} \, dx \\
&= -x + \tanh^{-1}(x)
\end{aligned}$$

Mathematica [B] time = 0.00, size = 22, normalized size = 3.67

$$-x - \frac{1}{2}\log(1-x) + \frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(x - x^3),x]
```

```
[Out] -x - Log[1 - x]/2 + Log[1 + x]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{x - x^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(x - x^3),x]
[Out] IntegrateAlgebraic[x^3/(x - x^3), x]
```

fricas [B] time = 0.39, size = 16, normalized size = 2.67

$$-x + \frac{1}{2} \log\left(x + 1\right) - \frac{1}{2} \log\left(x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+x),x, algorithm="fricas")
[Out] -x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

giac [B] time = 0.17, size = 18, normalized size = 3.00

$$-x + \frac{1}{2} \log\left({\left| x + 1 \right|}\right) - \frac{1}{2} \log\left({\left| x - 1 \right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+x),x, algorithm="giac")
[Out] -x + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

maple [B] time = 0.05, size = 17, normalized size = 2.83

$$-x - \frac{\ln\left(x - 1\right)}{2} + \frac{\ln\left(x + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^3+x),x)
[Out] -x-1/2*ln(x-1)+1/2*ln(x+1)
```

maxima [B] time = 1.29, size = 16, normalized size = 2.67

$$-x + \frac{1}{2} \log\left(x + 1\right) - \frac{1}{2} \log\left(x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+x),x, algorithm="maxima")
[Out] -x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

mupad [B] time = 0.06, size = 6, normalized size = 1.00

$$\mathrm{atanh}(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x - x^3),x)
[Out] atanh(x) - x
```

sympy [B] time = 0.11, size = 14, normalized size = 2.33

$$-x - \frac{\log(x-1)}{2} + \frac{\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**3+x),x)
[Out] -x - log(x - 1)/2 + log(x + 1)/2
```

3.29 $\int \frac{x^2}{x-x^3} \, dx$

Optimal. Leaf size=12

$$-\frac{1}{2} \log \left(1-x^2\right)$$

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1584, 260}

$$-\frac{1}{2} \log \left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(x - x^3),x]
```

```
[Out] -Log[1 - x^2]/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{x-x^3} \, dx &= \int \frac{x}{1-x^2} \, dx \\ &= -\frac{1}{2} \log \left(1-x^2\right) \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{2} \log \left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(x - x^3),x]
```

```
[Out] -1/2*Log[1 - x^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x-x^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(x - x^3),x]
```

```
[Out] IntegrateAlgebraic[x^2/(x - x^3), x]
```

fricas [A] time = 0.39, size = 8, normalized size = 0.67

$$-\frac{1}{2}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+x),x, algorithm="fricas")
[Out] -1/2*log(x^2 - 1)
```

giac [A] time = 0.15, size = 15, normalized size = 1.25

$$-\frac{1}{2}\log\left(|x+1|\right)-\frac{1}{2}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+x),x, algorithm="giac")
[Out] -1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

maple [A] time = 0.05, size = 14, normalized size = 1.17

$$-\frac{\ln\left(x-1\right)}{2}-\frac{\ln\left(x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+x),x)
[Out] -1/2*ln(x-1)-1/2*ln(x+1)
```

maxima [A] time = 1.29, size = 13, normalized size = 1.08

$$-\frac{1}{2}\log\left(x+1\right)-\frac{1}{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+x),x, algorithm="maxima")
[Out] -1/2*log(x + 1) - 1/2*log(x - 1)
```

mupad [B] time = 0.03, size = 8, normalized size = 0.67

$$-\frac{\ln\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x - x^3),x)
[Out] -log(x^2 - 1)/2
```

sympy [A] time = 0.10, size = 8, normalized size = 0.67

$$-\frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+x),x)
[Out] -log(x**2 - 1)/2
```

3.30 $\int \frac{x}{x-x^3} dx$

Optimal. Leaf size=2

$$\tanh^{-1}(x)$$

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1584, 206}

$$\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/(x - x^3),x]
```

```
[Out] ArcTanh[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{x-x^3} dx &= \int \frac{1}{1-x^2} dx \\ &= \tanh^{-1}(x) \end{aligned}$$

Mathematica [B] time = 0.00, size = 19, normalized size = 9.50

$$\frac{1}{2}\log(x+1) - \frac{1}{2}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x - x^3),x]
```

```
[Out] -1/2*Log[1 - x] + Log[1 + x]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x-x^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(x - x^3),x]
```

```
[Out] IntegrateAlgebraic[x/(x - x^3), x]
```

fricas [B] time = 0.41, size = 13, normalized size = 6.50

$$\frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+x),x, algorithm="fricas")
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

giac [B] time = 0.17, size = 15, normalized size = 7.50

$$\frac{1}{2}\log\left(|x+1|\right)-\frac{1}{2}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+x),x, algorithm="giac")
[Out] 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

maple [A] time = 0.04, size = 3, normalized size = 1.50

$$\operatorname{arctanh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+x),x)
[Out] arctanh(x)
```

maxima [B] time = 1.28, size = 13, normalized size = 6.50

$$\frac{1}{2}\log\left(x+1\right)-\frac{1}{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+x),x, algorithm="maxima")
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

mupad [B] time = 0.03, size = 2, normalized size = 1.00

$$\operatorname{atanh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x - x^3),x)
[Out] atanh(x)
```

sympy [B] time = 0.12, size = 12, normalized size = 6.00

$$-\frac{\log\left(x-1\right)}{2}+\frac{\log\left(x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+x),x)
[Out] -log(x - 1)/2 + log(x + 1)/2
```

3.31 $\int \frac{1}{x-x^3} dx$

Optimal. Leaf size=15

$$\log(x) - \frac{1}{2}\log\left(1-x^2\right)$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {1593, 266, 36, 31, 29}

$$\log(x) - \frac{1}{2}\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x - x^3)^(-1),x]
```

```
[Out] Log[x] - Log[1 - x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x-x^3}\,dx &= \int \frac{1}{x\left(1-x^2\right)}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)x}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, x^2\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)\\
&= \log(x) - \frac{1}{2}\log\left(1-x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\log(x) - \frac{1}{2}\log\left(1 - x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - x^3)^(-1),x]
[Out] Log[x] - Log[1 - x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x - x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x - x^3)^(-1),x]
[Out] IntegrateAlgebraic[(x - x^3)^(-1), x]
```

fricas [A] time = 0.39, size = 11, normalized size = 0.73

$$-\frac{1}{2}\log\left(x^2 - 1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+x),x, algorithm="fricas")
[Out] -1/2*log(x^2 - 1) + log(x)
```

giac [A] time = 0.16, size = 16, normalized size = 1.07

$$\frac{1}{2}\log\left(x^2\right) - \frac{1}{2}\log\left(\left|x^2 - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+x),x, algorithm="giac")
[Out] 1/2*log(x^2) - 1/2*log(abs(x^2 - 1))
```

maple [A] time = 0.06, size = 16, normalized size = 1.07

$$\ln(x) - \frac{\ln(x-1)}{2} - \frac{\ln(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+x),x)
[Out] -1/2*ln(x-1)-1/2*ln(x+1)+ln(x)
```

maxima [A] time = 1.32, size = 15, normalized size = 1.00

$$-\frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+x),x, algorithm="maxima")
[Out] -1/2*log(x + 1) - 1/2*log(x - 1) + log(x)
```

mupad [B] time = 4.96, size = 11, normalized size = 0.73

$$\ln(x) - \frac{\ln\left(x^2 - 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - x^3),x)
[Out] log(x) - log(x^2 - 1)/2
```

sympy [A] time = 0.11, size = 10, normalized size = 0.67

$$\log\left(x\right) - \frac{\log\left(x^2 - 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+x),x)
[Out] log(x) - log(x**2 - 1)/2
```

3.32 $\int \frac{1}{x(x-x^3)}\,dx$

Optimal. Leaf size=8

$$\tanh^{-1}(x) - \frac{1}{x}$$

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 325, 206}

$$\tanh^{-1}(x) - \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(x - x^3)),x]
```

```
[Out] -x^(-1) + ArcTanh[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(x-x^3\right)}\,dx &= \int \frac{1}{x^2\left(1-x^2\right)}\,dx\\ &= -\frac{1}{x}+\int \frac{1}{1-x^2}\,dx\\ &= -\frac{1}{x}+\tanh^{-1}(x)\end{aligned}$$

Mathematica [B] time = 0.00, size = 24, normalized size = 3.00

$$-\frac{1}{x}-\frac{1}{2}\log(1-x)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(x - x^3)),x]
```

```
[Out] -x^(-1) - Log[1 - x]/2 + Log[1 + x]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(x - x^3\right)}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(x - x^3)),x]
[Out] IntegrateAlgebraic[1/(x*(x - x^3)), x]
```

fricas [B] time = 0.41, size = 20, normalized size = 2.50

$$\frac{x \log\left(x + 1\right) - x \log\left(x - 1\right) - 2}{2\, x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+x),x, algorithm="fricas")
[Out] 1/2*(x*log(x + 1) - x*log(x - 1) - 2)/x
```

giac [B] time = 0.15, size = 20, normalized size = 2.50

$$-\frac{1}{x} + \frac{1}{2}\, \log\left(|x + 1|\right) - \frac{1}{2}\, \log\left(|x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+x),x, algorithm="giac")
[Out] -1/x + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

maple [B] time = 0.05, size = 19, normalized size = 2.38

$$-\frac{\ln\left(x - 1\right)}{2} + \frac{\ln\left(x + 1\right)}{2} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^3+x),x)
[Out] -1/2*ln(x-1)-1/x+1/2*ln(x+1)
```

maxima [B] time = 1.33, size = 18, normalized size = 2.25

$$-\frac{1}{x} + \frac{1}{2}\, \log\left(x + 1\right) - \frac{1}{2}\, \log\left(x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+x),x, algorithm="maxima")
[Out] -1/x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

mupad [B] time = 0.03, size = 8, normalized size = 1.00

$$\mathrm{atanh}(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x - x^3)),x)
[Out] atanh(x) - 1/x
```

sympy [B] time = 0.13, size = 15, normalized size = 1.88

$$-\frac{\log(x-1)}{2}+\frac{\log(x+1)}{2}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**3+x),x)
```

```
[Out] -log(x - 1)/2 + log(x + 1)/2 - 1/x
```

3.33 $\int \frac{1}{x^2(x-x^3)}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{2x^2}-\frac{1}{2}\log\left(1-x^2\right)+\log(x)$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 266, 44}

$$-\frac{1}{2x^2}-\frac{1}{2}\log\left(1-x^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(x - x^3)),x]
```

```
[Out] -1/(2*x^2) + Log[x] - Log[1 - x^2]/2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(x-x^3\right)}\,dx &= \int \frac{1}{x^3\left(1-x^2\right)}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)x^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{1-x}+\frac{1}{x^2}+\frac{1}{x}\right)\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2}+\log(x)-\frac{1}{2}\log\left(1-x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$-\frac{1}{2x^2}-\frac{1}{2}\log\left(1-x^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(x - x^3)),x]
```

```
[Out] -1/2*1/x^2 + Log[x] - Log[1 - x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(x - x^3\right)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(x - x^3)),x]
[Out] IntegrateAlgebraic[1/(x^2*(x - x^3)), x]
```

fricas [A] time = 0.40, size = 24, normalized size = 1.09

$$-\frac{x^2 \log\left(x^2 - 1\right) - 2\,x^2 \log(x) + 1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+x),x, algorithm="fricas")
[Out] -1/2*(x^2*log(x^2 - 1) - 2*x^2*log(x) + 1)/x^2
```

giac [A] time = 0.15, size = 26, normalized size = 1.18

$$-\frac{x^2 + 1}{2\,x^2} + \frac{1}{2} \log\left(x^2\right) - \frac{1}{2} \log\left(\left|x^2 - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+x),x, algorithm="giac")
[Out] -1/2*(x^2 + 1)/x^2 + 1/2*log(x^2) - 1/2*log(abs(x^2 - 1))
```

maple [A] time = 0.05, size = 21, normalized size = 0.95

$$\ln(x) - \frac{\ln\left(x - 1\right)}{2} - \frac{\ln\left(x + 1\right)}{2} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^3+x),x)
[Out] -1/2*ln(x-1)-1/2*ln(x+1)-1/2/x^2+ln(x)
```

maxima [A] time = 1.29, size = 20, normalized size = 0.91

$$-\frac{1}{2\,x^2} - \frac{1}{2} \log\left(x + 1\right) - \frac{1}{2} \log\left(x - 1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+x),x, algorithm="maxima")
[Out] -1/2/x^2 - 1/2*log(x + 1) - 1/2*log(x - 1) + log(x)
```

mupad [B] time = 0.03, size = 16, normalized size = 0.73

$$\ln(x) - \frac{\ln\left(x^2 - 1\right)}{2} - \frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x - x^3)),x)
```

```
[Out] log(x) - log(x^2 - 1)/2 - 1/(2*x^2)
```

sympy [**A**] time = 0.11, size = 17, normalized size = 0.77

$$\log(x) - \frac{\log\left(x^2 - 1\right)}{2} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**3+x),x)
```

```
[Out] log(x) - log(x**2 - 1)/2 - 1/(2*x**2)
```

3.34 $\int \frac{1}{x^3\left(x-x^3\right)}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{3x^3}-\frac{1}{x}+\tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 325, 206}

$$-\frac{1}{3x^3}-\frac{1}{x}+\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(x - x^3)),x]
```

```
[Out] -1/(3*x^3) - x^(-1) + ArcTanh[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(x-x^3\right)}\,dx &= \int \frac{1}{x^4\left(1-x^2\right)}\,dx\\
&= -\frac{1}{3x^3}+\int \frac{1}{x^2\left(1-x^2\right)}\,dx\\
&= -\frac{1}{3x^3}-\frac{1}{x}+\int \frac{1}{1-x^2}\,dx\\
&= -\frac{1}{3x^3}-\frac{1}{x}+\tanh^{-1}(x)
\end{aligned}$$

Mathematica [B] time = 0.00, size = 31, normalized size = 2.07

$$-\frac{1}{3x^3}-\frac{1}{x}-\frac{1}{2}\log(1-x)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(x - x^3)),x]
[Out] -1/3*1/x^3 - x^(-1) - Log[1 - x]/2 + Log[1 + x]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(x - x^3\right)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(x - x^3)),x]
[Out] IntegrateAlgebraic[1/(x^3*(x - x^3)), x]
```

fricas [B] time = 0.40, size = 30, normalized size = 2.00

$$\frac{3\,x^3 \log\left(x+1\right) - 3\,x^3 \log\left(x-1\right) - 6\,x^2 - 2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+x),x, algorithm="fricas")
[Out] 1/6*(3*x^3*log(x + 1) - 3*x^3*log(x - 1) - 6*x^2 - 2)/x^3
```

giac [B] time = 0.15, size = 27, normalized size = 1.80

$$-\frac{3\,x^2+1}{3\,x^3} + \frac{1}{2}\,\log\left(\left|x+1\right|\right) - \frac{1}{2}\,\log\left(\left|x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+x),x, algorithm="giac")
[Out] -1/3*(3*x^2 + 1)/x^3 + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

maple [A] time = 0.06, size = 24, normalized size = 1.60

$$-\frac{\ln\left(x-1\right)}{2} + \frac{\ln\left(x+1\right)}{2} - \frac{1}{x} - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^3+x),x)
[Out] -1/2*ln(x-1)+1/2*ln(x+1)-1/3/x^3-1/x
```

maxima [A] time = 1.26, size = 25, normalized size = 1.67

$$-\frac{3\,x^2+1}{3\,x^3} + \frac{1}{2}\,\log\left(x+1\right) - \frac{1}{2}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+x),x, algorithm="maxima")
[Out] -1/3*(3*x^2 + 1)/x^3 + 1/2*log(x + 1) - 1/2*log(x - 1)
```

mupad [B] time = 4.92, size = 13, normalized size = 0.87

$$\operatorname{atanh}(x) - \frac{x^2 + \frac{1}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x - x^3)),x)
```

```
[Out] atanh(x) - (x^2 + 1/3)/x^3
```

sympy [**A**] time = 0.14, size = 24, normalized size = 1.60

$$-\frac{\log(x-1)}{2}+\frac{\log(x+1)}{2}-\frac{3x^2+1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**3+x),x)
```

```
[Out] -log(x - 1)/2 + log(x + 1)/2 - (3*x**2 + 1)/(3*x**3)
```

3.35 $\int \frac{1}{x^4(x-x^3)}\,dx$

Optimal. Leaf size=29

$$-\frac{1}{4x^4}-\frac{1}{2x^2}-\frac{1}{2}\log\left(1-x^2\right)+\log(x)$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 266, 44}

$$-\frac{1}{2x^2}-\frac{1}{4x^4}-\frac{1}{2}\log\left(1-x^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(x - x^3)),x]
```

```
[Out] -1/(4*x^4) - 1/(2*x^2) + Log[x] - Log[1 - x^2]/2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(x-x^3\right)}\,dx &= \int \frac{1}{x^5\left(1-x^2\right)}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(1-x)x^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{1-x}+\frac{1}{x^3}+\frac{1}{x^2}+\frac{1}{x}\right)dx,x,x^2\right)\\
&= -\frac{1}{4x^4}-\frac{1}{2x^2}+\log(x)-\frac{1}{2}\log\left(1-x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 29, normalized size = 1.00

$$-\frac{1}{4x^4}-\frac{1}{2x^2}-\frac{1}{2}\log\left(1-x^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(x - x^3)),x]
```

```
[Out] -1/4*1/x^4 - 1/(2*x^2) + Log[x] - Log[1 - x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(x - x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(x - x^3)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(x - x^3)), x]
```

fricas [A] time = 0.38, size = 30, normalized size = 1.03

$$-\frac{2\,x^4\log\left(x^2-1\right)-4\,x^4\log(x)+2\,x^2+1}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+x),x, algorithm="fricas")
```

```
[Out] -1/4*(2*x^4*log(x^2 - 1) - 4*x^4*log(x) + 2*x^2 + 1)/x^4
```

giac [A] time = 0.16, size = 33, normalized size = 1.14

$$-\frac{3\,x^4+2\,x^2+1}{4\,x^4}+\frac{1}{2}\,\log\left(x^2\right)-\frac{1}{2}\,\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+x),x, algorithm="giac")
```

```
[Out] -1/4*(3*x^4 + 2*x^2 + 1)/x^4 + 1/2*log(x^2) - 1/2*log(abs(x^2 - 1))
```

maple [A] time = 0.05, size = 26, normalized size = 0.90

$$\ln(x)-\frac{\ln\left(x-1\right)}{2}-\frac{\ln\left(x+1\right)}{2}-\frac{1}{2x^2}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^3+x),x)
```

```
[Out] -1/2*ln(x-1)-1/2*ln(x+1)-1/4/x^4-1/2/x^2+ln(x)
```

maxima [A] time = 1.35, size = 27, normalized size = 0.93

$$-\frac{2\,x^2+1}{4\,x^4}-\frac{1}{2}\,\log\left(x+1\right)-\frac{1}{2}\,\log\left(x-1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+x),x, algorithm="maxima")
```

```
[Out] -1/4*(2*x^2 + 1)/x^4 - 1/2*log(x + 1) - 1/2*log(x - 1) + log(x)
```

mupad [B] time = 0.03, size = 23, normalized size = 0.79

$$\ln(x)-\frac{\ln\left(x^2-1\right)}{2}-\frac{\frac{x^2}{2}+\frac{1}{4}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x - x^3)),x)
[Out] log(x) - log(x^2 - 1)/2 - (x^2/2 + 1/4)/x^4
```

sympy [**A**] time = 0.13, size = 22, normalized size = 0.76

$$\log(x) - \frac{\log(x^2 - 1)}{2} - \frac{2x^2 + 1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**3+x),x)
[Out] log(x) - log(x**2 - 1)/2 - (2*x**2 + 1)/(4*x**4)
```

3.36 $\int \frac{1}{x+bx^3}\,dx$

Optimal. Leaf size=15

$$\log(x)-\frac{1}{2}\log\left(bx^2+1\right)$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {1593, 266, 36, 29, 31}

$$\log(x)-\frac{1}{2}\log\left(bx^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + b*x^3)^(-1),x]
```

```
[Out] Log[x] - Log[1 + b*x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x+bx^3}\,dx &= \int \frac{1}{x\left(1+bx^2\right)}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(1+bx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)-\frac{1}{2}b\operatorname{Subst}\left(\int \frac{1}{1+bx}\,dx,x,x^2\right)\\
&= \log(x)-\frac{1}{2}\log\left(1+bx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\log(x) - \frac{1}{2}\log\left(bx^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + b*x^3)^(-1),x]
[Out] Log[x] - Log[1 + b*x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x + b*x^3)^(-1),x]
[Out] IntegrateAlgebraic[(x + b*x^3)^(-1), x]
```

fricas [A] time = 0.40, size = 13, normalized size = 0.87

$$-\frac{1}{2}\log\left(bx^2+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+x),x, algorithm="fricas")
[Out] -1/2*log(b*x^2 + 1) + log(x)
```

giac [A] time = 0.15, size = 18, normalized size = 1.20

$$\frac{1}{2}\log\left(x^2\right)-\frac{1}{2}\log\left(\left|bx^2+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+x),x, algorithm="giac")
[Out] 1/2*log(x^2) - 1/2*log(abs(b*x^2 + 1))
```

maple [A] time = 0.04, size = 14, normalized size = 0.93

$$\ln(x)-\frac{\ln\left(b\,x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+x),x)
[Out] ln(x)-1/2*ln(b*x^2+1)
```

maxima [A] time = 1.29, size = 13, normalized size = 0.87

$$-\frac{1}{2}\log\left(bx^2+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+x),x, algorithm="maxima")
[Out] -1/2*log(b*x^2 + 1) + log(x)
```

mupad [B] time = 4.95, size = 14, normalized size = 0.93

$$\ln(x) - \frac{\ln\left(\frac{3\,b\,x^2}{2} + \frac{3}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + b*x^3),x)
```

```
[Out] log(x) - log((3*b*x^2)/2 + 3/2)/2
```

sympy [A] time = 0.15, size = 12, normalized size = 0.80

$$\log(x) - \frac{\log\left(x^2 + \frac{1}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+x),x)
```

```
[Out] log(x) - log(x**2 + 1/b)/2
```

3.37 $\int \frac{1}{-x+bx^3} \, dx$

Optimal. Leaf size=18

$$\frac{1}{2}\log\left(1-bx^2\right)-\log(x)$$

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1593, 266, 36, 29, 31}

$$\frac{1}{2}\log\left(1-bx^2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + b*x^3)^(-1),x]
```

```
[Out] -Log[x] + Log[1 - b*x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-x+bx^3} \, dx &= \int \frac{1}{x\left(-1+bx^2\right)} \, dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(-1+bx)} \, dx, x, x^2\right) \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x} \, dx, x, x^2\right)\right)+\frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{-1+bx} \, dx, x, x^2\right) \\
&= -\log(x)+\frac{1}{2}\log\left(1-bx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{1}{2}\log\left(1-bx^2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + b*x^3)^(-1),x]
```

```
[Out] -Log[x] + Log[1 - b*x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{-x+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-x + b*x^3)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(-x + b*x^3)^(-1), x]
```

fricas [A] time = 0.38, size = 15, normalized size = 0.83

$$\frac{1}{2}\log\left(bx^2-1\right)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3-x),x, algorithm="fricas")
```

```
[Out] 1/2*log(b*x^2 - 1) - log(x)
```

giac [A] time = 0.15, size = 18, normalized size = 1.00

$$-\frac{1}{2}\log\left(x^2\right)+\frac{1}{2}\log\left(\left|bx^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3-x),x, algorithm="giac")
```

```
[Out] -1/2*log(x^2) + 1/2*log(abs(b*x^2 - 1))
```

maple [A] time = 0.05, size = 16, normalized size = 0.89

$$-\ln(x)+\frac{\ln\left(b\,x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3-x),x)
```

```
[Out] 1/2*ln(b*x^2-1)-ln(x)
```

maxima [A] time = 1.26, size = 15, normalized size = 0.83

$$\frac{1}{2}\log\left(bx^2-1\right)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3-x),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x^2 - 1) - log(x)
```

mupad [**B**] time = 0.05, size = 16, normalized size = 0.89

$$\frac{\ln\left(\frac{3}{2}-\frac{3\,b\,x^2}{2}\right)}{2}-\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x - b*x^3),x)
```

```
[Out] log(3/2 - (3*b*x^2)/2)/2 - log(x)
```

sympy [**A**] time = 0.14, size = 12, normalized size = 0.67

$$-\log\left(x\right)+\frac{\log\left(x^2-\frac{1}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3-x),x)
```

```
[Out] -log(x) + log(x**2 - 1/b)/2
```

3.38 $\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}} \, dx$

Optimal. Leaf size=159

$$-\frac{9a\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax+bx^3}}\right)}{2b^{11/2}}+\frac{9\sqrt{x}\sqrt{ax+bx^3}}{2b^5}-\frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}}-\frac{3x^{13/2}}{5b^3(ax+bx^3)^{3/2}}-\frac{9x^{19/2}}{35b^2(ax+bx^3)^{5/2}}-\frac{x^{25/2}}{7b(ax+bx^3)}$$

Rubi [A] time = 0.25, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2022, 2024, 2029, 206}

$$-\frac{9x^{19/2}}{35b^2(ax+bx^3)^{5/2}}-\frac{3x^{13/2}}{5b^3(ax+bx^3)^{3/2}}-\frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}}+\frac{9\sqrt{x}\sqrt{ax+bx^3}}{2b^5}-\frac{9a\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax+bx^3}}\right)}{2b^{11/2}}-\frac{x^{25/2}}{7b(ax+bx^3)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(29/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -x^(25/2)/(7*b*(a*x + b*x^3)^(7/2)) - (9*x^(19/2))/(35*b^2*(a*x + b*x^3)^(5
/2)) - (3*x^(13/2))/(5*b^3*(a*x + b*x^3)^(3/2)) - (3*x^(7/2))/(b^4*Sqrt[a*x
 + b*x^3]) + (9*Sqrt[x]*Sqrt[a*x + b*x^3])/(2*b^5) - (9*a*ArcTanh[(Sqrt[b]*
x^(3/2))/Sqrt[a*x + b*x^3]])/(2*b^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{x^{25/2}}{7b(ax+bx^3)^{7/2}} + \frac{9\int \frac{x^{23/2}}{(ax+bx^3)^{7/2}}\,dx}{7b}$$

$$= -\frac{x^{25/2}}{7b(ax+bx^3)^{7/2}} - \frac{9x^{19/2}}{35b^2(ax+bx^3)^{5/2}} + \frac{9\int \frac{x^{17/2}}{(ax+bx^3)^{5/2}}\,dx}{5b^2}$$

$$= -\frac{x^{25/2}}{7b(ax+bx^3)^{7/2}} - \frac{9x^{19/2}}{35b^2(ax+bx^3)^{5/2}} - \frac{3x^{13/2}}{5b^3(ax+bx^3)^{3/2}} + \frac{3\int \frac{x^{11/2}}{(ax+bx^3)^{3/2}}\,dx}{b^3}$$

$$= -\frac{x^{25/2}}{7b(ax+bx^3)^{7/2}} - \frac{9x^{19/2}}{35b^2(ax+bx^3)^{5/2}} - \frac{3x^{13/2}}{5b^3(ax+bx^3)^{3/2}} - \frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}} + \frac{9\int \frac{x^{5/2}}{\sqrt{ax+bx^3}}}{b^4}$$

$$= -\frac{x^{25/2}}{7b(ax+bx^3)^{7/2}} - \frac{9x^{19/2}}{35b^2(ax+bx^3)^{5/2}} - \frac{3x^{13/2}}{5b^3(ax+bx^3)^{3/2}} - \frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}} + \frac{9\sqrt{x}\sqrt{ax+}}{2b^5}$$

$$= -\frac{x^{25/2}}{7b(ax+bx^3)^{7/2}} - \frac{9x^{19/2}}{35b^2(ax+bx^3)^{5/2}} - \frac{3x^{13/2}}{5b^3(ax+bx^3)^{3/2}} - \frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}} + \frac{9\sqrt{x}\sqrt{ax+}}{2b^5}$$

$$= -\frac{x^{25/2}}{7b(ax+bx^3)^{7/2}} - \frac{9x^{19/2}}{35b^2(ax+bx^3)^{5/2}} - \frac{3x^{13/2}}{5b^3(ax+bx^3)^{3/2}} - \frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}} + \frac{9\sqrt{x}\sqrt{ax+}}{2b^5}$$

Mathematica [A] time = 0.21, size = 130, normalized size = 0.82

$$\frac{\sqrt{x}\left(\sqrt{b}\,x\left(315a^4+1050a^3bx^2+1218a^2b^2x^4+528ab^3x^6+35b^4x^8\right)-\frac{315\sqrt{a}\left(a+bx^2\right)^4\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{70b^{11/2}\left(a+bx^2\right)^3\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(29/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (Sqrt[x]*(Sqrt[b]*x*(315*a^4 + 1050*a^3*b*x^2 + 1218*a^2*b^2*x^4 + 528*a*b^
3*x^6 + 35*b^4*x^8) - (315*Sqrt[a]*(a + b*x^2)^4*ArcSinh[(Sqrt[b]*x)/Sqrt[a
]])/Sqrt[1 + (b*x^2)/a]))/(70*b^(11/2)*(a + b*x^2)^3*Sqrt[x*(a + b*x^2)])
```

IntegrateAlgebraic [A] time = 0.13, size = 124, normalized size = 0.78

$$\frac{x^{9/2}\left(a+bx^2\right)^{9/2}\left(\frac{315a^4x+1050a^3bx^3+1218a^2b^2x^5+528ab^3x^7+35b^4x^9}{70b^5\left(a+bx^2\right)^{7/2}}+\frac{9a\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{2b^{11/2}}\right)}{\left(x\left(a+bx^2\right)\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(29/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(9/2)*(a + b*x^2)^(9/2)*((315*a^4*x + 1050*a^3*b*x^3 + 1218*a^2*b^2*x^5
+ 528*a*b^3*x^7 + 35*b^4*x^9)/(70*b^5*(a + b*x^2)^(7/2)) + (9*a*Log[-(Sqrt[
b]*x) + Sqrt[a + b*x^2]])/(2*b^(11/2))))/(x*(a + b*x^2))^(9/2)
```

fricas [A] time = 0.45, size = 376, normalized size = 2.36

$$\left[\frac{315\left(ab^4x^8+4a^2b^3x^6+6a^3b^2x^4+4a^4bx^2+a^5\right)\sqrt{b}\log\left(2bx^2-2\sqrt{bx^3+ax}\sqrt{b}\sqrt{x}+a\right)+2\left(35b^5x^8+528ab^4x^6+1218a^2b^3x^4+1050a^3b^2x^2+315a^4b\right)\sqrt{bx^3+ax}\sqrt{x}}{140\left(b^{10}x^8+4ab^9x^6+6a^2b^8x^4+4a^3b^7x^2+a^4b^6\right)}, \frac{315\left(ab^4x^8+4a^2b^3x^6+6a^3b^2x^4+4a^4bx^2+a^5\right)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^3+ax}\sqrt{-b}}{bx^{\frac{3}{2}}}\right)+\left(35b^5x^8+528ab^4x^6+1218a^2b^3x^4+1050a^3b^2x^2+315a^4b\right)\sqrt{bx^3+ax}\sqrt{x}}{70\left(b^{10}x^8+4ab^9x^6+6a^2b^8x^4+4a^3b^7x^2+a^4b^6\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(29/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/140*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*b*x^2 + a^5)
*sqrt(b)*log(2*b*x^2 - 2*sqrt(b*x^3 + a*x)*sqrt(b)*sqrt(x) + a) + 2*(35*b^5
*x^8 + 528*a*b^4*x^6 + 1218*a^2*b^3*x^4 + 1050*a^3*b^2*x^2 + 315*a^4*b)*sqr
t(b*x^3 + a*x)*sqrt(x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7
*x^2 + a^4*b^6), 1/70*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a
^4*b*x^2 + a^5)*sqrt(-b)*arctan(sqrt(b*x^3 + a*x)*sqrt(-b)/(b*x^(3/2))) + (
35*b^5*x^8 + 528*a*b^4*x^6 + 1218*a^2*b^3*x^4 + 1050*a^3*b^2*x^2 + 315*a^4*
b)*sqrt(b*x^3 + a*x)*sqrt(x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a
^3*b^7*x^2 + a^4*b^6)]
```

giac [A] time = 0.33, size = 100, normalized size = 0.63

$$\frac{\left(\left(\left(x^2\left(\frac{35x^2}{b}+\frac{528a}{b^2}\right)+\frac{1218a^2}{b^3}\right)x^2+\frac{1050a^3}{b^4}\right)x^2+\frac{315a^4}{b^5}\right)x}{70\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{9a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2b^{\frac{11}{2}}}-\frac{9a\log\left(|a|\right)}{4b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(29/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/70*(((x^2*(35*x^2/b + 528*a/b^2) + 1218*a^2/b^3)*x^2 + 1050*a^3/b^4)*x^2
+ 315*a^4/b^5)*x/(b*x^2 + a)^(7/2) + 9/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2
+ a)))/b^(11/2) - 9/4*a*log(abs(a))/b^(11/2)
```

maple [A] time = 0.09, size = 212, normalized size = 1.33

$$-\frac{\sqrt{\left(bx^2+a\right)x}\left(-35b^{\frac{9}{2}}x^9-528ab^{\frac{7}{2}}x^7+315\sqrt{bx^2+a}\,ab^3x^6\ln\left(\sqrt{b}x+\sqrt{bx^2+a}\right)-1218a^2b^{\frac{5}{2}}x^5+945\sqrt{bx^2+a}\,a^2b^2x^4\ln\left(\sqrt{b}x+\sqrt{bx^2+a}\right)-1050a^3b^{\frac{3}{2}}x^3+945\sqrt{bx^2+a}\,a^3bx^2\ln\left(\sqrt{b}x+\sqrt{bx^2+a}\right)-315a^4\sqrt{b}x+315\sqrt{bx^2+a}\,a^4\ln\left(\sqrt{b}x+\sqrt{bx^2+a}\right)\right)}{70\left(bx^2+a\right)^4b^{\frac{11}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(29/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/70*((b*x^2+a)*x)^(1/2)/b^(11/2)*(-35*x^9*b^(9/2)+315*ln(b^(1/2)*x+(b*x^2
+a)^(1/2))*x^6*a*b^3*(b*x^2+a)^(1/2)-528*b^(7/2)*x^7*a+945*ln(b^(1/2)*x+(b*
x^2+a)^(1/2))*x^4*a^2*b^2*(b*x^2+a)^(1/2)-1218*b^(5/2)*x^5*a^2+945*ln(b^(1/
2)*x+(b*x^2+a)^(1/2))*x^2*a^3*b*(b*x^2+a)^(1/2)-1050*b^(3/2)*x^3*a^3+315*ln
(b^(1/2)*x+(b*x^2+a)^(1/2))*a^4*(b*x^2+a)^(1/2)-315*b^(1/2)*x*a^4)/x^(1/2)/
(b*x^2+a)^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{29}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(29/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(29/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{29/2}}{\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(29/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(29/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(29/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

3.39 $\int \frac{x^{27/2}}{\left(ax+bx^3\right)^{9/2}} dx$

Optimal. Leaf size=126

$$\frac{128\sqrt{ax+bx^3}}{35b^5\sqrt{x}}-\frac{64x^{5/2}}{35b^4\sqrt{ax+bx^3}}-\frac{16x^{11/2}}{35b^3\left(ax+bx^3\right)^{3/2}}-\frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Rubi [A] time = 0.20, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 2014}

$$-\frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{16x^{11/2}}{35b^3\left(ax+bx^3\right)^{3/2}}-\frac{64x^{5/2}}{35b^4\sqrt{ax+bx^3}}+\frac{128\sqrt{ax+bx^3}}{35b^5\sqrt{x}}-\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(27/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -x^(23/2)/(7*b*(a*x + b*x^3)^(7/2)) - (8*x^(17/2))/(35*b^2*(a*x + b*x^3)^(5
/2)) - (16*x^(11/2))/(35*b^3*(a*x + b*x^3)^(3/2)) - (64*x^(5/2))/(35*b^4*Sq
rt[a*x + b*x^3]) + (128*Sqrt[a*x + b*x^3])/(35*b^5*Sqrt[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{27/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = -\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}} + \frac{8\int \frac{x^{21/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7b}$$
$$= -\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}} + \frac{48\int \frac{x^{15/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{35b^2}$$
$$= -\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{16x^{11/2}}{35b^3\left(ax+bx^3\right)^{3/2}} + \frac{64\int \frac{x^{9/2}}{\left(ax+bx^3\right)^{3/2}}\,dx}{35b^3}$$
$$= -\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{16x^{11/2}}{35b^3\left(ax+bx^3\right)^{3/2}} - \frac{64x^{5/2}}{35b^4\sqrt{ax+bx^3}} + \frac{128\int \frac{}{\sqrt{a}}}{35}$$
$$= -\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{16x^{11/2}}{35b^3\left(ax+bx^3\right)^{3/2}} - \frac{64x^{5/2}}{35b^4\sqrt{ax+bx^3}} + \frac{128\sqrt{ax}}{35b^5}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 0.61

$$\frac{\sqrt{x}\left(128a^4+448a^3bx^2+560a^2b^2x^4+280ab^3x^6+35b^4x^8\right)}{35b^5\left(a+bx^2\right)^3\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(27/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (Sqrt[x]*(128*a^4 + 448*a^3*b*x^2 + 560*a^2*b^2*x^4 + 280*a*b^3*x^6 + 35*b^
4*x^8))/(35*b^5*(a + b*x^2)^3*Sqrt[x*(a + b*x^2)])
```

IntegrateAlgebraic [A] time = 0.11, size = 75, normalized size = 0.60

$$\frac{x^{9/2}\left(a+bx^2\right)\left(128a^4+448a^3bx^2+560a^2b^2x^4+280ab^3x^6+35b^4x^8\right)}{35b^5\left(x\left(a+bx^2\right)\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(27/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(9/2)*(a + b*x^2)*(128*a^4 + 448*a^3*b*x^2 + 560*a^2*b^2*x^4 + 280*a*b^3
*x^6 + 35*b^4*x^8))/(35*b^5*(x*(a + b*x^2))^(9/2))
```

fricas [A] time = 0.41, size = 108, normalized size = 0.86

$$\frac{\left(35\,b^4x^8+280\,ab^3x^6+560\,a^2b^2x^4+448\,a^3bx^2+128\,a^4\right)\sqrt{bx^3+ax}\,\sqrt{x}}{35\left(b^9x^9+4\,ab^8x^7+6\,a^2b^7x^5+4\,a^3b^6x^3+a^4b^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(27/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(35*b^4*x^8 + 280*a*b^3*x^6 + 560*a^2*b^2*x^4 + 448*a^3*b*x^2 + 128*a^
4)*sqrt(b*x^3 + a*x)*sqrt(x)/(b^9*x^9 + 4*a*b^8*x^7 + 6*a^2*b^7*x^5 + 4*a^3
*b^6*x^3 + a^4*b^5*x)
```

giac [A] time = 0.21, size = 80, normalized size = 0.63

$$\frac{\sqrt{bx^2+a}}{b^5}-\frac{128\sqrt{a}}{35\,b^5}+\frac{140\left(bx^2+a\right)^3a-70\left(bx^2+a\right)^2a^2+28\left(bx^2+a\right)a^3-5\,a^4}{35\left(bx^2+a\right)^{\frac{7}{2}}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(27/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] sqrt(b*x^2 + a)/b^5 - 128/35*sqrt(a)/b^5 + 1/35*(140*(b*x^2 + a)^3*a - 70*(
b*x^2 + a)^2*a^2 + 28*(b*x^2 + a)*a^3 - 5*a^4)/((b*x^2 + a)^(7/2)*b^5)
```

maple [A] time = 0.04, size = 70, normalized size = 0.56

$$\frac{\left(b\,x^2+a\right)\left(35x^8b^4+280a\,x^6b^3+560a^2x^4b^2+448a^3x^2b+128a^4\right)x^{\frac{9}{2}}}{35\left(b\,x^3+ax\right)^{\frac{9}{2}}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(27/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] 1/35*(b*x^2+a)*(35*b^4*x^8+280*a*b^3*x^6+560*a^2*b^2*x^4+448*a^3*b*x^2+128*
a^4)*x^(9/2)/b^5/(b*x^3+a*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{27}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(27/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(27/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^{27/2}}{\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(27/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(27/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(27/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

3.40 $\int \frac{x^{25/2}}{\left(ax+bx^3\right)^{9/2}} \, dx$

Optimal. Leaf size=130

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax+bx^3}}\right)}{b^{9/2}}-\frac{x^{3/2}}{b^4\sqrt{ax+bx^3}}-\frac{x^{9/2}}{3b^3\left(ax+bx^3\right)^{3/2}}-\frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Rubi [A] time = 0.21, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2022, 2029, 206}

$$-\frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{9/2}}{3b^3\left(ax+bx^3\right)^{3/2}}-\frac{x^{3/2}}{b^4\sqrt{ax+bx^3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax+bx^3}}\right)}{b^{9/2}}-\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(25/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -x^(21/2)/(7*b*(a*x + b*x^3)^(7/2)) - x^(15/2)/(5*b^2*(a*x + b*x^3)^(5/2))
- x^(9/2)/(3*b^3*(a*x + b*x^3)^(3/2)) - x^(3/2)/(b^4*Sqrt[a*x + b*x^3]) + A
rcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x + b*x^3]]/b^(9/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{x^{25/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = -\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}} + \frac{\int \frac{x^{19/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{b}$$

$$= -\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}} + \frac{\int \frac{x^{13/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{b^2}$$

$$= -\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}} - \frac{x^{9/2}}{3b^3\left(ax+bx^3\right)^{3/2}} + \frac{\int \frac{x^{7/2}}{\left(ax+bx^3\right)^{3/2}}\,dx}{b^3}$$

$$= -\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}} - \frac{x^{9/2}}{3b^3\left(ax+bx^3\right)^{3/2}} - \frac{x^{3/2}}{b^4\sqrt{ax+bx^3}} + \frac{\int \frac{\sqrt{x}}{\sqrt{ax+bx^3}}}{b^4}$$

$$= -\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}} - \frac{x^{9/2}}{3b^3\left(ax+bx^3\right)^{3/2}} - \frac{x^{3/2}}{b^4\sqrt{ax+bx^3}} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}} - \frac{x^{9/2}}{3b^3\left(ax+bx^3\right)^{3/2}} - \frac{x^{3/2}}{b^4\sqrt{ax+bx^3}} + \frac{\tanh^{-1}\left(\right.}{b^{9/2}}$$

Mathematica [A] time = 0.29, size = 120, normalized size = 0.92

$$\frac{\sqrt{x}\left(105\sqrt{a}\left(a+bx^2\right)^3\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\sqrt{b}\,x\left(105a^3+350a^2bx^2+406ab^2x^4+176b^3x^6\right)\right)}{105b^{9/2}\left(a+bx^2\right)^3\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(25/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (Sqrt[x]*(-(Sqrt[b]*x*(105*a^3 + 350*a^2*b*x^2 + 406*a*b^2*x^4 + 176*b^3*x^
6)) + 105*Sqrt[a]*(a + b*x^2)^3*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqr
t[a]]))/(105*b^(9/2)*(a + b*x^2)^3*Sqrt[x*(a + b*x^2)])
```

IntegrateAlgebraic [A] time = 0.11, size = 110, normalized size = 0.85

$$\frac{x^{9/2}\left(a+bx^2\right)^{9/2}\left(\frac{-105a^3x-350a^2bx^3-406ab^2x^5-176b^3x^7}{105b^4\left(a+bx^2\right)^{7/2}}-\frac{\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{b^{9/2}}\right)}{\left(x\left(a+bx^2\right)\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(25/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(9/2)*(a + b*x^2)^(9/2)*((-105*a^3*x - 350*a^2*b*x^3 - 406*a*b^2*x^5 - 1
76*b^3*x^7)/(105*b^4*(a + b*x^2)^(7/2)) - Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2
]]/b^(9/2)))/(x*(a + b*x^2))^(9/2)
```

fricas [A] time = 0.43, size = 348, normalized size = 2.68

$$\left[\frac{105\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{b}\,\log\left(2\,bx^2+2\,\sqrt{bx^3+ax}\,\sqrt{b}\,\sqrt{x}+a\right)-2\left(176\,b^4x^6+406\,ab^3x^4+350\,a^2b^2x^2+105\,a^3b\right)\sqrt{bx^3+ax}\,\sqrt{x}}{210\left(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5\right)}, -\frac{105\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^3+ax}\,\sqrt{-b}}{bx^{\frac{3}{2}}}\right)+\left(176\,b^4x^6+406\,ab^3x^4+350\,a^2b^2x^2+105\,a^3b\right)\sqrt{bx^3+ax}\,\sqrt{x}}{105\left(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(25/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqr
t(b)*log(2*b*x^2 + 2*sqrt(b*x^3 + a*x)*sqrt(b)*sqrt(x) + a) - 2*(176*b^4*x^
6 + 406*a*b^3*x^4 + 350*a^2*b^2*x^2 + 105*a^3*b)*sqrt(b*x^3 + a*x)*sqrt(x))
/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5), -1/105*
(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-b)*a
rctan(sqrt(b*x^3 + a*x)*sqrt(-b)/(b*x^(3/2))) + (176*b^4*x^6 + 406*a*b^3*x^
4 + 350*a^2*b^2*x^2 + 105*a^3*b)*sqrt(b*x^3 + a*x)*sqrt(x))/(b^9*x^8 + 4*a*
b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)]
```

giac [A] time = 0.30, size = 86, normalized size = 0.66

$$-\frac{\left(2\left(x^2\left(\frac{88\,x^2}{b}+\frac{203\,a}{b^2}\right)+\frac{175\,a^2}{b^3}\right)x^2+\frac{105\,a^3}{b^4}\right)x}{105\left(bx^2+a\right)^{\frac{7}{2}}}-\frac{\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}+\frac{\log\left(|a|\right)}{2\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(25/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(2*(x^2*(88*x^2/b + 203*a/b^2) + 175*a^2/b^3)*x^2 + 105*a^3/b^4)*x/(
b*x^2 + a)^(7/2) - log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2) + 1/2*log
(abs(a))/b^(9/2)
```

maple [A] time = 0.08, size = 198, normalized size = 1.52

$$\frac{\sqrt{\left(b\,x^2+a\right)x}\,\left(-176b^{\frac{7}{2}}x^7+105\sqrt{b\,x^2+a}\,b^3x^6\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)-406a\,b^{\frac{5}{2}}x^5+315\sqrt{b\,x^2+a}\,a\,b^2x^4\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)-350a^2b^{\frac{3}{2}}x^3+315\sqrt{b\,x^2+a}\,a^2b\,x^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)-105a^3\sqrt{b}\,x+105\sqrt{b\,x^2+a}\,a^3\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)\right)}{105\left(b\,x^2+a\right)^4b^{\frac{9}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(25/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] 1/105*((b*x^2+a)*x)^(1/2)/b^(9/2)*(105*ln(b^(1/2)*x+(b*x^2+a)^(1/2))*x^6*b^
3*(b*x^2+a)^(1/2)-176*x^7*b^(7/2)+315*ln(b^(1/2)*x+(b*x^2+a)^(1/2))*x^4*a*b
^2*(b*x^2+a)^(1/2)-406*b^(5/2)*x^5*a+315*ln(b^(1/2)*x+(b*x^2+a)^(1/2))*x^2*
a^2*b*(b*x^2+a)^(1/2)-350*b^(3/2)*x^3*a^2+105*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
*a^3*(b*x^2+a)^(1/2)-105*b^(1/2)*x*a^3)/x^(1/2)/(b*x^2+a)^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{25}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(25/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(25/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^{25/2}}{{\left(b\,x^3+a\,x\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(25/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(25/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(25/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

3.41 $\int \frac{x^{23/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=101

$$-\frac{16\sqrt{x}}{35b^4\sqrt{ax+bx^3}}-\frac{8x^{7/2}}{35b^3\left(ax+bx^3\right)^{3/2}}-\frac{6x^{13/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Rubi [A] time = 0.16, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 2014}

$$-\frac{6x^{13/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{8x^{7/2}}{35b^3\left(ax+bx^3\right)^{3/2}}-\frac{16\sqrt{x}}{35b^4\sqrt{ax+bx^3}}-\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(23/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -x^(19/2)/(7*b*(a*x + b*x^3)^(7/2)) - (6*x^(13/2))/(35*b^2*(a*x + b*x^3)^(5
/2)) - (8*x^(7/2))/(35*b^3*(a*x + b*x^3)^(3/2)) - (16*Sqrt[x])/(35*b^4*Sqrt
[a*x + b*x^3])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{23/2}}{\left(ax+bx^3\right)^{9/2}}\,dx &= -\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}}+\frac{6\int \frac{x^{17/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7b}\\
&= -\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{6x^{13/2}}{35b^2\left(ax+bx^3\right)^{5/2}}+\frac{24\int \frac{x^{11/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{35b^2}\\
&= -\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{6x^{13/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{8x^{7/2}}{35b^3\left(ax+bx^3\right)^{3/2}}+\frac{16\int \frac{x^{5/2}}{\left(ax+bx^3\right)^{3/2}}\,dx}{35b^3}\\
&= -\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{6x^{13/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{8x^{7/2}}{35b^3\left(ax+bx^3\right)^{3/2}}-\frac{16\sqrt{x}}{35b^4\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.65

$$-\frac{\sqrt{x}\left(16a^3+56a^2bx^2+70ab^2x^4+35b^3x^6\right)}{35b^4\left(a+bx^2\right)^3\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(23/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/35*(Sqrt[x]*(16*a^3 + 56*a^2*b*x^2 + 70*a*b^2*x^4 + 35*b^3*x^6))/(b^4*(a
 + b*x^2)^3*Sqrt[x*(a + b*x^2)])
```

IntegrateAlgebraic [A] time = 0.11, size = 64, normalized size = 0.63

$$\frac{x^{9/2}\left(a+bx^2\right)\left(-16a^3-56a^2bx^2-70ab^2x^4-35b^3x^6\right)}{35b^4\left(x\left(a+bx^2\right)\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(23/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(9/2)*(a + b*x^2)*(-16*a^3 - 56*a^2*b*x^2 - 70*a*b^2*x^4 - 35*b^3*x^6))/
(35*b^4*(x*(a + b*x^2))^(9/2))
```

fricas [A] time = 0.41, size = 97, normalized size = 0.96

$$-\frac{\left(35\,b^3x^6+70\,ab^2x^4+56\,a^2bx^2+16\,a^3\right)\sqrt{bx^3+ax}\,\sqrt{x}}{35\left(b^8x^9+4\,ab^7x^7+6\,a^2b^6x^5+4\,a^3b^5x^3+a^4b^4x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(35*b^3*x^6 + 70*a*b^2*x^4 + 56*a^2*b*x^2 + 16*a^3)*sqrt(b*x^3 + a*x)
*sqrt(x)/(b^8*x^9 + 4*a*b^7*x^7 + 6*a^2*b^6*x^5 + 4*a^3*b^5*x^3 + a^4*b^4*x
)
```

giac [A] time = 0.21, size = 64, normalized size = 0.63

$$\frac{16}{35\,\sqrt{a}\,b^4}-\frac{35\left(bx^2+a\right)^3-35\left(bx^2+a\right)^2a+21\left(bx^2+a\right)a^2-5\,a^3}{35\left(bx^2+a\right)^{\frac{7}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 16/35/(sqrt(a)*b^4) - 1/35*(35*(b*x^2 + a)^3 - 35*(b*x^2 + a)^2*a + 21*(b*x
^2 + a)*a^2 - 5*a^3)/((b*x^2 + a)^(7/2)*b^4)
```

maple [A] time = 0.05, size = 59, normalized size = 0.58

$$-\frac{\left(b\,x^2+a\right)\left(35b^3x^6+70a\,b^2x^4+56a^2b\,x^2+16a^3\right)x^{\frac{9}{2}}}{35\left(b\,x^3+ax\right)^{\frac{9}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(23/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/35*(b*x^2+a)*(35*b^3*x^6+70*a*b^2*x^4+56*a^2*b*x^2+16*a^3)*x^(9/2)/b^4/(
b*x^3+a*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{23}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(23/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{23/2}}{\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(23/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(23/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(23/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

3.42 $\int \frac{x^{21/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=25

$$\frac{x^{21/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2014}

$$\frac{x^{21/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(21/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(21/2)/(7*a*(a*x + b*x^3)^(7/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{21/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = \frac{x^{21/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{x^{21/2}}{7a\left(x\left(a+bx^2\right)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(21/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(21/2)/(7*a*(x*(a + b*x^2))^(7/2))
```

IntegrateAlgebraic [A] time = 0.11, size = 32, normalized size = 1.28

$$\frac{x^{23/2}\left(a+bx^2\right)}{7a\left(x\left(a+bx^2\right)\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(21/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(23/2)*(a + b*x^2))/(7*a*(x*(a + b*x^2))^(9/2))
```

fricas [B] time = 0.39, size = 61, normalized size = 2.44

$$\frac{\sqrt{bx^3 + ax}\, x^{\frac{13}{2}}}{7 \left(ab^4x^8 + 4\, a^2b^3x^6 + 6\, a^3b^2x^4 + 4\, a^4bx^2 + a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/7*sqrt(b*x^3 + a*x)*x^(13/2)/(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 +
 4*a^4*b*x^2 + a^5)
```

giac [A] time = 0.29, size = 17, normalized size = 0.68

$$\frac{x^7}{7 \left(bx^2 + a\right)^{\frac{7}{2}} a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/7*x^7/((b*x^2 + a)^(7/2)*a)
```

maple [A] time = 0.05, size = 27, normalized size = 1.08

$$\frac{\left(b\, x^2 + a\right) x^{\frac{23}{2}}}{7 \left(b\, x^3 + ax\right)^{\frac{9}{2}} a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(21/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] 1/7*(b*x^2+a)/a*x^(23/2)/(b*x^3+a*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{21}{2}}}{\left(bx^3 + ax\right)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(21/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^{21/2}}{\left(b\, x^3 + a\, x\right)^{9/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(21/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(21/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(21/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

3.43 $\int \frac{x^{19/2}}{\left(ax+bx^3\right)^{9/2}} dx$

Optimal. Leaf size=76

$$-\frac{8x^{3/2}}{105b^3\left(ax+bx^3\right)^{3/2}}-\frac{4x^{9/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{15/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Rubi [A] time = 0.12, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 2014}

$$-\frac{4x^{9/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{8x^{3/2}}{105b^3\left(ax+bx^3\right)^{3/2}}-\frac{x^{15/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(19/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -x^(15/2)/(7*b*(a*x + b*x^3)^(7/2)) - (4*x^(9/2))/(35*b^2*(a*x + b*x^3)^(5/
2)) - (8*x^(3/2))/(105*b^3*(a*x + b*x^3)^(3/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{19/2}}{\left(ax+bx^3\right)^{9/2}} dx &= -\frac{x^{15/2}}{7b\left(ax+bx^3\right)^{7/2}}+\frac{4\int \frac{x^{13/2}}{\left(ax+bx^3\right)^{7/2}} dx}{7b}\\
&= -\frac{x^{15/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{4x^{9/2}}{35b^2\left(ax+bx^3\right)^{5/2}}+\frac{8\int \frac{x^{7/2}}{\left(ax+bx^3\right)^{5/2}} dx}{35b^2}\\
&= -\frac{x^{15/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{4x^{9/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{8x^{3/2}}{105b^3\left(ax+bx^3\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.72

$$-\frac{\sqrt{x}\left(8a^2+28abx^2+35b^2x^4\right)}{105b^3\left(a+bx^2\right)^3\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(19/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/105*(Sqrt[x]*(8*a^2 + 28*a*b*x^2 + 35*b^2*x^4))/(b^3*(a + b*x^2)^3*Sqrt[
x*(a + b*x^2)])
```

IntegrateAlgebraic [A] time = 0.11, size = 53, normalized size = 0.70

$$\frac{x^{9/2}\left(a+bx^2\right)\left(-8a^2-28abx^2-35b^2x^4\right)}{105b^3\left(x\left(a+bx^2\right)\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(19/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(9/2)*(a + b*x^2)*(-8*a^2 - 28*a*b*x^2 - 35*b^2*x^4))/(105*b^3*(x*(a + b
*x^2))^(9/2))
```

fricas [A] time = 0.41, size = 86, normalized size = 1.13

$$-\frac{\left(35\,b^2x^4+28\,abx^2+8\,a^2\right)\sqrt{bx^3+ax}\,\sqrt{x}}{105\left(b^7x^9+4\,ab^6x^7+6\,a^2b^5x^5+4\,a^3b^4x^3+a^4b^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/105*(35*b^2*x^4 + 28*a*b*x^2 + 8*a^2)*sqrt(b*x^3 + a*x)*sqrt(x)/(b^7*x^9
 + 4*a*b^6*x^7 + 6*a^2*b^5*x^5 + 4*a^3*b^4*x^3 + a^4*b^3*x)
```

giac [A] time = 0.20, size = 50, normalized size = 0.66

$$\frac{8}{105\,a^{\frac{3}{2}}b^3}-\frac{35\left(bx^2+a\right)^2-42\left(bx^2+a\right)a+15\,a^2}{105\left(bx^2+a\right)^{\frac{7}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 8/105/(a^(3/2)*b^3) - 1/105*(35*(b*x^2 + a)^2 - 42*(b*x^2 + a)*a + 15*a^2)/
((b*x^2 + a)^(7/2)*b^3)
```

maple [A] time = 0.05, size = 48, normalized size = 0.63

$$-\frac{\left(b\,x^2+a\right)\left(35b^2x^4+28ab\,x^2+8a^2\right)x^{\frac{9}{2}}}{105\left(b\,x^3+ax\right)^{\frac{9}{2}}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/105*(b*x^2+a)*(35*b^2*x^4+28*a*b*x^2+8*a^2)*x^(9/2)/b^3/(b*x^3+a*x)^(9/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{19}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(19/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{19/2}}{\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(19/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(19/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

3.44 $\int \frac{x^{17/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=51

$$\frac{2x^{15/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{17/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Rubi [A] time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 2014}

$$\frac{2x^{15/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{17/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(17/2)/(a*x + b*x^3)^(9/2),x]

[Out] x^(17/2)/(7*a*(a*x + b*x^3)^(7/2)) + (2*x^(15/2))/(35*a^2*(a*x + b*x^3)^(5/
2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{17/2}}{\left(ax+bx^3\right)^{9/2}}\,dx &= \frac{x^{17/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{2\int \frac{x^{15/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7a}\\ &= \frac{x^{17/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{2x^{15/2}}{35a^2\left(ax+bx^3\right)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.86

$$\frac{x^{9/2}\sqrt{x\left(a+bx^2\right)}\left(7a+2bx^2\right)}{35a^2\left(a+bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(17/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(9/2)*Sqrt[x*(a + b*x^2)]*(7*a + 2*b*x^2))/(35*a^2*(a + b*x^2)^4)
```

IntegrateAlgebraic [A] time = 0.11, size = 42, normalized size = 0.82

$$\frac{x^{19/2}\left(a+bx^2\right)\left(7a+2bx^2\right)}{35a^2\left(x\left(a+bx^2\right)\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(17/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(19/2)*(a + b*x^2)*(7*a + 2*b*x^2))/(35*a^2*(x*(a + b*x^2))^(9/2))
```

fricas [A] time = 0.43, size = 76, normalized size = 1.49

$$\frac{\left(2\,bx^6+7\,ax^4\right)\sqrt{bx^3+ax}\,\sqrt{x}}{35\left(a^2b^4x^8+4\,a^3b^3x^6+6\,a^4b^2x^4+4\,a^5bx^2+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(2*b*x^6 + 7*a*x^4)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^2*b^4*x^8 + 4*a^3*b^3
*x^6 + 6*a^4*b^2*x^4 + 4*a^5*b*x^2 + a^6)
```

giac [A] time = 0.29, size = 29, normalized size = 0.57

$$\frac{x^5\left(\frac{2\,bx^2}{a^2}+\frac{7}{a}\right)}{35\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*x^5*(2*b*x^2/a^2 + 7/a)/(b*x^2 + a)^(7/2)
```

maple [A] time = 0.06, size = 37, normalized size = 0.73

$$\frac{\left(b\,x^2+a\right)\left(2b\,x^2+7a\right)x^{\frac{19}{2}}}{35\left(b\,x^3+ax\right)^{\frac{9}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] 1/35*(b*x^2+a)*x^(19/2)*(2*b*x^2+7*a)/a^2/(b*x^3+a*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{17}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(17/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{17/2}}{\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(a*x + b*x^3)^(9/2),x)
[Out] int(x^(17/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(17/2)/(b*x**3+a*x)**(9/2),x)
[Out] Timed out
```

3.45 $\int \frac{x^{15/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=51

$$-\frac{2x^{5/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{11/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Rubi [A] time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 2014}

$$-\frac{2x^{5/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{11/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(15/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -x^(11/2)/(7*b*(a*x + b*x^3)^(7/2)) - (2*x^(5/2))/(35*b^2*(a*x + b*x^3)^(5/
2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{15/2}}{\left(ax+bx^3\right)^{9/2}}\,dx &= -\frac{x^{11/2}}{7b\left(ax+bx^3\right)^{7/2}}+\frac{2\int \frac{x^{9/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7b}\\ &= -\frac{x^{11/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{2x^{5/2}}{35b^2\left(ax+bx^3\right)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.86

$$-\frac{\sqrt{x}\left(2a+7bx^2\right)}{35b^2\left(a+bx^2\right)^3\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(15/2)/(a*x + b*x^3)^(9/2),x]
[Out] -1/35*(Sqrt[x]*(2*a + 7*b*x^2))/(b^2*(a + b*x^2)^3*Sqrt[x*(a + b*x^2)])
```

IntegrateAlgebraic [A] time = 0.63, size = 35, normalized size = 0.69

$$-\frac{x^{7/2}\left(2a+7bx^2\right)}{35b^2\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(15/2)/(a*x + b*x^3)^(9/2),x]
[Out] -1/35*(x^(7/2)*(2*a + 7*b*x^2))/(b^2*(a*x + b*x^3)^(7/2))
```

fricas [A] time = 0.43, size = 75, normalized size = 1.47

$$-\frac{\sqrt{bx^3+ax}\left(7\,bx^2+2\,a\right)\sqrt{x}}{35\left(b^6x^9+4\,ab^5x^7+6\,a^2b^4x^5+4\,a^3b^3x^3+a^4b^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
[Out] -1/35*sqrt(b*x^3 + a*x)*(7*b*x^2 + 2*a)*sqrt(x)/(b^6*x^9 + 4*a*b^5*x^7 + 6*
a^2*b^4*x^5 + 4*a^3*b^3*x^3 + a^4*b^2*x)
```

giac [A] time = 0.30, size = 33, normalized size = 0.65

$$-\frac{7\,bx^2+2\,a}{35\left(bx^2+a\right)^{\frac{7}{2}}b^2}+\frac{2}{35\,a^{\frac{5}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
[Out] -1/35*(7*b*x^2 + 2*a)/((b*x^2 + a)^(7/2)*b^2) + 2/35/(a^(5/2)*b^2)
```

maple [A] time = 0.06, size = 37, normalized size = 0.73

$$-\frac{\left(b\,x^2+a\right)\left(7b\,x^2+2a\right)x^{\frac{9}{2}}}{35\left(b\,x^3+ax\right)^{\frac{9}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(b*x^3+a*x)^(9/2),x)
[Out] -1/35*(b*x^2+a)*(7*b*x^2+2*a)*x^(9/2)/b^2/(b*x^3+a*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{15}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
[Out] integrate(x^(15/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{15/2}}{\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(a*x + b*x^3)^(9/2),x)
[Out] int(x^(15/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)/(b*x**3+a*x)**(9/2),x)
[Out] Timed out
```

3.46 $\int \frac{x^{13/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=76

$$\frac{8x^{9/2}}{105a^3\left(ax+bx^3\right)^{3/2}}+\frac{4x^{11/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Rubi [A] time = 0.11, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 2014}

$$\frac{4x^{11/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{8x^{9/2}}{105a^3\left(ax+bx^3\right)^{3/2}}+\frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(13/2)/(7*a*(a*x + b*x^3)^(7/2)) + (4*x^(11/2))/(35*a^2*(a*x + b*x^3)^(5/
2)) + (8*x^(9/2))/(105*a^3*(a*x + b*x^3)^(3/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{13/2}}{\left(ax+bx^3\right)^{9/2}}\,dx &= \frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{4\int \frac{x^{11/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7a}\\
&= \frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{4x^{11/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{8\int \frac{x^{9/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{35a^2}\\
&= \frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{4x^{11/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{8x^{9/2}}{105a^3\left(ax+bx^3\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.72

$$\frac{x^{5/2}\sqrt{x\left(a+bx^2\right)}\left(35a^2+28abx^2+8b^2x^4\right)}{105a^3\left(a+bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(5/2)*Sqrt[x*(a + b*x^2)]*(35*a^2 + 28*a*b*x^2 + 8*b^2*x^4))/(105*a^3*(a
 + b*x^2)^4)
```

IntegrateAlgebraic [A] time = 0.92, size = 46, normalized size = 0.61

$$\frac{x^{13/2}\left(35a^2+28abx^2+8b^2x^4\right)}{105a^3\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(13/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(13/2)*(35*a^2 + 28*a*b*x^2 + 8*b^2*x^4))/(105*a^3*(a*x + b*x^3)^(7/2))
```

fricas [A] time = 0.41, size = 87, normalized size = 1.14

$$\frac{\left(8\,b^2x^6+28\,abx^4+35\,a^2x^2\right)\sqrt{bx^3+ax}\,\sqrt{x}}{105\left(a^3b^4x^8+4\,a^4b^3x^6+6\,a^5b^2x^4+4\,a^6bx^2+a^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*b^2*x^6 + 28*a*b*x^4 + 35*a^2*x^2)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^3*
b^4*x^8 + 4*a^4*b^3*x^6 + 6*a^5*b^2*x^4 + 4*a^6*b*x^2 + a^7)
```

giac [A] time = 0.28, size = 43, normalized size = 0.57

$$\frac{\left(4\,x^2\left(\frac{2\,b^2x^2}{a^3}+\frac{7\,b}{a^2}\right)+\frac{35}{a}\right)x^3}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(4*x^2*(2*b^2*x^2/a^3 + 7*b/a^2) + 35/a)*x^3/(b*x^2 + a)^(7/2)
```

maple [A] time = 0.05, size = 48, normalized size = 0.63

$$\frac{\left(b\,x^2+a\right)\left(8b^2x^4+28ab\,x^2+35a^2\right)x^{\frac{15}{2}}}{105\left(b\,x^3+ax\right)^{\frac{9}{2}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] 1/105*(b*x^2+a)*x^(15/2)*(8*b^2*x^4+28*a*b*x^2+35*a^2)/a^3/(b*x^3+a*x)^(9/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{13}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
[Out] integrate(x^(13/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{13/2}}{\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(a*x + b*x^3)^(9/2),x)
[Out] int(x^(13/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(b*x**3+a*x)**(9/2),x)
[Out] Timed out
```

3.47 $\int \frac{x^{11/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=25

$$-\frac{x^{7/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$-\frac{x^{7/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -x^(7/2)/(7*b*(a*x + b*x^3)^(7/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{11/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = -\frac{x^{7/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.00

$$-\frac{x^{7/2}}{7b\left(x\left(a+bx^2\right)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/7*x^(7/2)/(b*(x*(a + b*x^2))^(7/2))
```

IntegrateAlgebraic [A] time = 0.50, size = 25, normalized size = 1.00

$$-\frac{x^{7/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(11/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/7*x^(7/2)/(b*(a*x + b*x^3)^(7/2))
```

fricas [B] time = 0.43, size = 63, normalized size = 2.52

$$-\frac{\sqrt{bx^3+ax}\sqrt{x}}{7\left(b^5x^9+4\,ab^4x^7+6\,a^2b^3x^5+4\,a^3b^2x^3+a^4bx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/7*sqrt(b*x^3 + a*x)*sqrt(x)/(b^5*x^9 + 4*a*b^4*x^7 + 6*a^2*b^3*x^5 + 4*a
^3*b^2*x^3 + a^4*b*x)
```

giac [A] time = 0.20, size = 23, normalized size = 0.92

$$-\frac{1}{7\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{1}{7\,a^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/7/((b*x^2 + a)^(7/2)*b) + 1/7/(a^(7/2)*b)
```

maple [A] time = 0.04, size = 27, normalized size = 1.08

$$-\frac{\left(b\,x^2+a\right)x^{\frac{9}{2}}}{7\left(b\,x^3+ax\right)^{\frac{9}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/7*(b*x^2+a)/b*x^(9/2)/(b*x^3+a*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{11}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(11/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{x^{11/2}}{\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(11/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{11}{2}}}{\left(x\left(a+bx^2\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Integral(x**(11/2)/(x*(a + b*x**2))**(9/2), x)
```

3.48 $\int \frac{x^{9/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=101

$$\frac{16x^{3/2}}{35a^4\sqrt{ax+bx^3}}+\frac{8x^{5/2}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{6x^{7/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Rubi [A] time = 0.16, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 2014}

$$\frac{6x^{7/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{8x^{5/2}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{16x^{3/2}}{35a^4\sqrt{ax+bx^3}}+\frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(a*x + b*x^3)^(9/2),x]

[Out] x^(9/2)/(7*a*(a*x + b*x^3)^(7/2)) + (6*x^(7/2))/(35*a^2*(a*x + b*x^3)^(5/2)
) + (8*x^(5/2))/(35*a^3*(a*x + b*x^3)^(3/2)) + (16*x^(3/2))/(35*a^4*Sqrt[a*
x + b*x^3])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{9/2}}{\left(ax+bx^3\right)^{9/2}}\,dx &= \frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{6\int \frac{x^{7/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7a}\\
&= \frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{6x^{7/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{24\int \frac{x^{5/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{35a^2}\\
&= \frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{6x^{7/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{8x^{5/2}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{16\int \frac{x^{3/2}}{\left(ax+bx^3\right)^{3/2}}\,dx}{35a^3}\\
&= \frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{6x^{7/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{8x^{5/2}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{16x^{3/2}}{35a^4\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.65

$$\frac{\sqrt{x}\sqrt{x\left(a+bx^2\right)}\left(35a^3+70a^2bx^2+56ab^2x^4+16b^3x^6\right)}{35a^4\left(a+bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[x*(a + b*x^2)]*(35*a^3 + 70*a^2*b*x^2 + 56*a*b^2*x^4 + 16*b^3
*x^6))/(35*a^4*(a + b*x^2)^4)
```

IntegrateAlgebraic [A] time = 0.89, size = 57, normalized size = 0.56

$$\frac{x^{9/2}\left(35a^3+70a^2bx^2+56ab^2x^4+16b^3x^6\right)}{35a^4\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(9/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(9/2)*(35*a^3 + 70*a^2*b*x^2 + 56*a*b^2*x^4 + 16*b^3*x^6))/(35*a^4*(a*x
+ b*x^3)^(7/2))
```

fricas [A] time = 0.40, size = 95, normalized size = 0.94

$$\frac{\left(16\,b^3x^6+56\,ab^2x^4+70\,a^2bx^2+35\,a^3\right)\sqrt{bx^3+ax}\,\sqrt{x}}{35\left(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(16*b^3*x^6 + 56*a*b^2*x^4 + 70*a^2*b*x^2 + 35*a^3)*sqrt(b*x^3 + a*x)*
sqrt(x)/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)
```

giac [A] time = 0.25, size = 55, normalized size = 0.54

$$\frac{\left(2\left(4\,x^2\left(\frac{2\,b^3x^2}{a^4}+\frac{7\,b^2}{a^3}\right)+\frac{35\,b}{a^2}\right)x^2+\frac{35}{a}\right)x}{35\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*(2*(4*x^2*(2*b^3*x^2/a^4 + 7*b^2/a^3) + 35*b/a^2)*x^2 + 35/a)*x/(b*x^2
 + a)^(7/2)
```

maple [A] time = 0.04, size = 59, normalized size = 0.58

$$\frac{\left(b\,x^2+a\right)\left(16b^3x^6+56a\,b^2x^4+70a^2b\,x^2+35a^3\right)x^{\frac{11}{2}}}{35\left(b\,x^3+ax\right)^{\frac{9}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] 1/35*(b*x^2+a)*x^(11/2)*(16*b^3*x^6+56*a*b^2*x^4+70*a^2*b*x^2+35*a^3)/a^4/(
b*x^3+a*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{9}{2}}}{\left(bx^3 + ax\right)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(9/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{9/2}}{\left(b\,x^3 + a\,x\right)^{9/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(9/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

3.49 $\int \frac{x^{7/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=130

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{a^{9/2}}+\frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}}+\frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}}+\frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Rubi [A] time = 0.20, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2023, 2029, 206}

$$\frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}}+\frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{a^{9/2}}+\frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(7/2)/(7*a*(a*x + b*x^3)^(7/2)) + x^(5/2)/(5*a^2*(a*x + b*x^3)^(5/2)) + x
^(3/2)/(3*a^3*(a*x + b*x^3)^(3/2)) + Sqrt[x]/(a^4*Sqrt[a*x + b*x^3]) - ArcT
anh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^3]]/a^(9/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = \frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{\int \frac{x^{5/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{a}$$

$$= \frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}} + \frac{\int \frac{x^{3/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{a^2}$$

$$= \frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}} + \frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}} + \frac{\int \frac{\sqrt{x}}{\left(ax+bx^3\right)^{3/2}}\,dx}{a^3}$$

$$= \frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}} + \frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}} + \frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{ax+bx\ldots}}}{a^4}$$

$$= \frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}} + \frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}} + \frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}} - \frac{\text{Subst}\left(\int \frac{}{1\ldots}\right.}{}$$

$$= \frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}} + \frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}} + \frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\ldots}}{\sqrt{a\ldots}}\right.}{a^{9/2}}$$

Mathematica [C] time = 0.01, size = 43, normalized size = 0.33

$$\frac{x^{7/2}\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};\frac{bx^2}{a}+1\right)}{7a\left(x\left(a+bx^2\right)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(7/2)*Hypergeometric2F1[-7/2, 1, -5/2, 1 + (b*x^2)/a])/(7*a*(x*(a + b*x^
2))^(7/2))
```

IntegrateAlgebraic [A] time = 1.04, size = 99, normalized size = 0.76

$$\frac{\sqrt{ax+bx^3}\left(176a^3+406a^2bx^2+350ab^2x^4+105b^3x^6\right)}{105a^4\sqrt{x}\left(a+bx^2\right)^4} - \frac{\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (Sqrt[a*x + b*x^3]*(176*a^3 + 406*a^2*b*x^2 + 350*a*b^2*x^4 + 105*b^3*x^6))
/(105*a^4*Sqrt[x]*(a + b*x^2)^4) - ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x
^3]]/a^(9/2)
```

fricas [A] time = 0.44, size = 360, normalized size = 2.77

$$\left[\frac{105\left(b^4x^9+4ab^3x^7+6a^2b^2x^5+4a^3bx^3+a^4x\right)\sqrt{a}\log\left(\frac{bx^3+2ax-2\sqrt{bx^3+ax}\sqrt{a}\sqrt{x}}{x^3}\right)+2\left(105ab^3x^6+350a^2b^2x^4+406a^3bx^2+176a^4\right)\sqrt{bx^3+ax}\sqrt{x}}{210\left(a^5b^4x^9+4a^6b^3x^7+6a^7b^2x^5+4a^8bx^3+a^9x\right)}, \frac{105\left(b^4x^9+4ab^3x^7+6a^2b^2x^5+4a^3bx^3+a^4x\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax}\sqrt{-a}}{a\sqrt{x}}\right)+\left(105ab^3x^6+350a^2b^2x^4+406a^3bx^2+176a^4\right)\sqrt{bx^3+ax}\sqrt{x}}{105\left(a^5b^4x^9+4a^6b^3x^7+6a^7b^2x^5+4a^8bx^3+a^9x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*x)*s
qrt(a)*log((b*x^3 + 2*a*x - 2*sqrt(b*x^3 + a*x)*sqrt(a)*sqrt(x))/x^3) + 2*(
105*a*b^3*x^6 + 350*a^2*b^2*x^4 + 406*a^3*b*x^2 + 176*a^4)*sqrt(b*x^3 + a*x
)*sqrt(x))/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9
*x), 1/105*(105*(b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*
x)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x)*sqrt(-a)/(a*sqrt(x))) + (105*a*b^3*x^6
 + 350*a^2*b^2*x^4 + 406*a^3*b*x^2 + 176*a^4)*sqrt(b*x^3 + a*x)*sqrt(x))/(a
^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)]
```

giac [A] time = 0.27, size = 114, normalized size = 0.88

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^4} - \frac{105\,\sqrt{a}\,\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) + 176\,\sqrt{-a}}{105\,\sqrt{-a}\,a^{\frac{9}{2}}} + \frac{105\,\left(bx^2+a\right)^3 + 35\,\left(bx^2+a\right)^2 a + 21\,\left(bx^2+a\right)a^2 + 15\,a^3}{105\,\left(bx^2+a\right)^{\frac{7}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4) - 1/105*(105*sqrt(a)*arctan
(sqrt(a)/sqrt(-a)) + 176*sqrt(-a))/(sqrt(-a)*a^(9/2)) + 1/105*(105*(b*x^2 +
 a)^3 + 35*(b*x^2 + a)^2*a + 21*(b*x^2 + a)*a^2 + 15*a^3)/((b*x^2 + a)^(7/2
)*a^4)
```

maple [B] time = 0.05, size = 217, normalized size = 1.67

$$-\frac{\sqrt{\left(b\,x^2+a\right)x}\,\left(105\sqrt{b\,x^2+a}\;b^3x^6\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\;\sqrt{a}}{x}\right) - 105\sqrt{a}\;b^3x^6 + 315\sqrt{b\,x^2+a}\;a\,b^2x^4\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\;\sqrt{a}}{x}\right) - 350a^{\frac{3}{2}}b^2x^4 + 315\sqrt{b\,x^2+a}\;a^2b\,x^2\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\;\sqrt{a}}{x}\right) - 406a^{\frac{5}{2}}b\,x^2 + 105\sqrt{b\,x^2+a}\;a^3\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\;\sqrt{a}}{x}\right) - 176a^{\frac{7}{2}}\right)}{105\left(b\,x^2+a\right)^4 a^{\frac{9}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/105*((b*x^2+a)*x)^(1/2)/a^(9/2)*(105*ln(2*((b*x^2+a)^(1/2)*a^(1/2)+a)/x)
*x^6*b^3*(b*x^2+a)^(1/2)-105*a^(1/2)*x^6*b^3+315*ln(2*((b*x^2+a)^(1/2)*a^(1
/2)+a)/x)*x^4*a*b^2*(b*x^2+a)^(1/2)-350*a^(3/2)*x^4*b^2+315*ln(2*((b*x^2+a)
^(1/2)*a^(1/2)+a)/x)*x^2*a^2*b*(b*x^2+a)^(1/2)-406*a^(5/2)*x^2*b+105*ln(2*(
(b*x^2+a)^(1/2)*a^(1/2)+a)/x)*a^3*(b*x^2+a)^(1/2)-176*a^(7/2))/x^(1/2)/(b*x
^2+a)^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{7}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{7/2}}{{\left(b\,x^3+a\,x\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(7/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{7}{2}}}{\left(x\left(a+bx^2\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Integral(x**(7/2)/(x*(a + b*x**2))**(9/2), x)
```

3.50 $\int \frac{x^{5/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{128\sqrt{ax+bx^3}}{35a^5x^{3/2}}+\frac{64}{35a^4\sqrt{x}\,\sqrt{ax+bx^3}}+\frac{16\sqrt{x}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{8x^{3/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Rubi [A] time = 0.19, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 2014}

$$\frac{8x^{3/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{16\sqrt{x}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{64}{35a^4\sqrt{x}\,\sqrt{ax+bx^3}}-\frac{128\sqrt{ax+bx^3}}{35a^5x^{3/2}}+\frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(5/2)/(7*a*(a*x + b*x^3)^(7/2)) + (8*x^(3/2))/(35*a^2*(a*x + b*x^3)^(5/2)
) + (16*Sqrt[x])/(35*a^3*(a*x + b*x^3)^(3/2)) + 64/(35*a^4*Sqrt[x]*Sqrt[a*x
 + b*x^3]) - (128*Sqrt[a*x + b*x^3])/(35*a^5*x^(3/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = \frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{8\int \frac{x^{3/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7a}$$

$$= \frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{8x^{3/2}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{48\int \frac{\sqrt{x}}{\left(ax+bx^3\right)^{5/2}}\,dx}{35a^2}$$

$$= \frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{8x^{3/2}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{16\sqrt{x}}{35a^3\left(ax+bx^3\right)^{3/2}} + \frac{64\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{3/2}}\,dx}{35a^3}$$

$$= \frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{8x^{3/2}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{16\sqrt{x}}{35a^3\left(ax+bx^3\right)^{3/2}} + \frac{64}{35a^4\sqrt{x}\sqrt{ax+bx^3}} + 12$$

$$= \frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{8x^{3/2}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{16\sqrt{x}}{35a^3\left(ax+bx^3\right)^{3/2}} + \frac{64}{35a^4\sqrt{x}\sqrt{ax+bx^3}} - 128$$

Mathematica [A] time = 0.04, size = 77, normalized size = 0.61

$$-\frac{\sqrt{x\left(a+bx^2\right)}\left(35a^4+280a^3bx^2+560a^2b^2x^4+448ab^3x^6+128b^4x^8\right)}{35a^5x^{3/2}\left(a+bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/35*(Sqrt[x*(a + b*x^2)]*(35*a^4 + 280*a^3*b*x^2 + 560*a^2*b^2*x^4 + 448*
a*b^3*x^6 + 128*b^4*x^8))/(a^5*x^(3/2)*(a + b*x^2)^4)
```

IntegrateAlgebraic [A] time = 1.35, size = 68, normalized size = 0.54

$$-\frac{x^{5/2}\left(35a^4+280a^3bx^2+560a^2b^2x^4+448ab^3x^6+128b^4x^8\right)}{35a^5\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/35*(x^(5/2)*(35*a^4 + 280*a^3*b*x^2 + 560*a^2*b^2*x^4 + 448*a*b^3*x^6 +
128*b^4*x^8))/(a^5*(a*x + b*x^3)^(7/2))
```

fricas [A] time = 0.47, size = 110, normalized size = 0.87

$$-\frac{\left(128\,b^4x^8+448\,ab^3x^6+560\,a^2b^2x^4+280\,a^3bx^2+35\,a^4\right)\sqrt{bx^3+ax}\,\sqrt{x}}{35\left(a^5b^4x^{10}+4\,a^6b^3x^8+6\,a^7b^2x^6+4\,a^8bx^4+a^9x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(128*b^4*x^8 + 448*a*b^3*x^6 + 560*a^2*b^2*x^4 + 280*a^3*b*x^2 + 35*a
^4)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^5*b^4*x^10 + 4*a^6*b^3*x^8 + 6*a^7*b^2*x^6
 + 4*a^8*b*x^4 + a^9*x^2)
```

giac [A] time = 0.27, size = 90, normalized size = 0.71

$$-\frac{\left(\left(x^2\left(\frac{93\,b^4x^2}{a^5}+\frac{308\,b^3}{a^4}\right)+\frac{350\,b^2}{a^3}\right)x^2+\frac{140\,b}{a^2}\right)x}{35\,\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{2\,\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*((x^2*(93*b^4*x^2/a^5 + 308*b^3/a^4) + 350*b^2/a^3)*x^2 + 140*b/a^2)*
x/(b*x^2 + a)^(7/2) + 2*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

maple [A] time = 0.05, size = 70, normalized size = 0.56

$$-\frac{\left(b\,x^2+a\right)\left(128x^8b^4+448a\,x^6b^3+560a^2x^4b^2+280a^3x^2b+35a^4\right)x^{\frac{7}{2}}}{35\left(b\,x^3+ax\right)^{\frac{9}{2}}a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/35*(b*x^2+a)*x^(7/2)*(128*b^4*x^8+448*a*b^3*x^6+560*a^2*b^2*x^4+280*a^3*
b*x^2+35*a^4)/a^5/(b*x^3+a*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{{\left(b\,x^3+a\,x\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(5/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\left(x\left(a+bx^2\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Integral(x**(5/2)/(x*(a + b*x**2))**(9/2), x)
```

3.51 $\int \frac{x^{3/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=159

$$\frac{9b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{2a^{11/2}}-\frac{9\sqrt{ax+bx^3}}{2a^5x^{5/2}}+\frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}}+\frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}}+\frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Rubi [A] time = 0.24, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2023, 2025, 2029, 206}

$$\frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}}+\frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}}-\frac{9\sqrt{ax+bx^3}}{2a^5x^{5/2}}+\frac{9b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{2a^{11/2}}+\frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(3/2)/(7*a*(a*x + b*x^3)^(7/2)) + (9*Sqrt[x])/(35*a^2*(a*x + b*x^3)^(5/2)
) + 3/(5*a^3*Sqrt[x]*(a*x + b*x^3)^(3/2)) + 3/(a^4*x^(3/2)*Sqrt[a*x + b*x^3
]) - (9*Sqrt[a*x + b*x^3])/(2*a^5*x^(5/2)) + (9*b*ArcTanh[(Sqrt[a]*Sqrt[x])
/Sqrt[a*x + b*x^3]])/(2*a^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{9\int \frac{\sqrt{x}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7a}$$

$$= \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{9\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{5/2}}\,dx}{5a^2}$$

$$= \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}} + \frac{3\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{3/2}}\,dx}{a^3}$$

$$= \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}} + \frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}} + \frac{9\int \frac{}{x^5}}{}$$

$$= \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}} + \frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}} - \frac{9\sqrt{ax}}{2a^5}$$

$$= \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}} + \frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}} - \frac{9\sqrt{ax}}{2a^5}$$

$$= \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}} + \frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}} - \frac{9\sqrt{ax}}{2a^5}$$

Mathematica [C] time = 0.02, size = 44, normalized size = 0.28

$$-\frac{bx^{7/2}\,{}_2F_1\left(-\frac{7}{2},2;-\frac{5}{2};\frac{bx^2}{a}+1\right)}{7a^2\left(x\left(a+bx^2\right)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/7*(b*x^(7/2)*Hypergeometric2F1[-7/2, 2, -5/2, 1 + (b*x^2)/a])/(a^2*(x*(a
 + b*x^2))^(7/2))
```

IntegrateAlgebraic [A] time = 1.95, size = 113, normalized size = 0.71

$$\frac{9b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{2a^{11/2}} + \frac{\sqrt{ax+bx^3}\left(-35a^4-528a^3bx^2-1218a^2b^2x^4-1050ab^3x^6-315b^4x^8\right)}{70a^5x^{5/2}\left(a+bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (Sqrt[a*x + b*x^3]*(-35*a^4 - 528*a^3*b*x^2 - 1218*a^2*b^2*x^4 - 1050*a*b^3
*x^6 - 315*b^4*x^8))/(70*a^5*x^(5/2)*(a + b*x^2)^4) + (9*b*ArcTanh[(Sqrt[a]
*Sqrt[x])/Sqrt[a*x + b*x^3]])/(2*a^(11/2))
```

fricas [A] time = 0.43, size = 396, normalized size = 2.49

$$\left[\frac{315\left(b^5x^{11}+4ab^4x^9+6a^2b^3x^7+4a^3b^2x^5+a^4bx^3\right)\sqrt{a}\log\left(\frac{bx^3+2ax+2\sqrt{bx^3+ax}\sqrt{a}\sqrt{x}}{x^3}\right)-2\left(315ab^4x^8+1050a^2b^3x^6+1218a^3b^2x^4+528a^4bx^2+35a^5\right)\sqrt{bx^3+ax}\sqrt{x}}{140\left(a^6b^4x^{11}+4a^7b^3x^9+6a^8b^2x^7+4a^9bx^5+a^{10}x^3\right)}, -\frac{315\left(b^5x^{11}+4ab^4x^9+6a^2b^3x^7+4a^3b^2x^5+a^4bx^3\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax}\sqrt{-a}}{a\sqrt{x}}\right)+\left(315ab^4x^8+1050a^2b^3x^6+1218a^3b^2x^4+528a^4bx^2+35a^5\right)\sqrt{bx^3+ax}\sqrt{x}}{70\left(a^6b^4x^{11}+4a^7b^3x^9+6a^8b^2x^7+4a^9bx^5+a^{10}x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/140*(315*(b^5*x^11 + 4*a*b^4*x^9 + 6*a^2*b^3*x^7 + 4*a^3*b^2*x^5 + a^4*b
*x^3)*sqrt(a)*log((b*x^3 + 2*a*x + 2*sqrt(b*x^3 + a*x)*sqrt(a)*sqrt(x))/x^3
) - 2*(315*a*b^4*x^8 + 1050*a^2*b^3*x^6 + 1218*a^3*b^2*x^4 + 528*a^4*b*x^2
+ 35*a^5)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*
b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3), -1/70*(315*(b^5*x^11 + 4*a*b^4*x^9 + 6*a
^2*b^3*x^7 + 4*a^3*b^2*x^5 + a^4*b*x^3)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x)*s
qrt(-a)/(a*sqrt(x))) + (315*a*b^4*x^8 + 1050*a^2*b^3*x^6 + 1218*a^3*b^2*x^4
 + 528*a^4*b*x^2 + 35*a^5)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^6*b^4*x^11 + 4*a^7
*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3)]
```

giac [A] time = 0.25, size = 104, normalized size = 0.65

$$-\frac{9\,b\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}\,a^5}-\frac{\sqrt{bx^2+a}}{2\,a^5x^2}-\frac{140\,\left(bx^2+a\right)^3b+35\,\left(bx^2+a\right)^2ab+14\,\left(bx^2+a\right)a^2b+5\,a^3b}{35\,\left(bx^2+a\right)^{\frac{7}{2}}a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -9/2*b*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^5) - 1/2*sqrt(b*x^2 + a
)/(a^5*x^2) - 1/35*(140*(b*x^2 + a)^3*b + 35*(b*x^2 + a)^2*a*b + 14*(b*x^2
+ a)*a^2*b + 5*a^3*b)/((b*x^2 + a)^(7/2)*a^5)
```

maple [A] time = 0.06, size = 234, normalized size = 1.47

$$\frac{\sqrt{\left(b\,x^2+a\right)x}\,\left(315\sqrt{b\,x^2+a}\,b^4x^8\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)-315\sqrt{a}\,b^4x^8+945\sqrt{b\,x^2+a}\,a\,b^3x^6\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)-1050a^{\frac{3}{2}}b^3x^6+945\sqrt{b\,x^2+a}\,a^2b^2x^4\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)-1218a^{\frac{5}{2}}b^2x^4+315\sqrt{b\,x^2+a}\,a^3b\,x^2\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)-528a^{\frac{7}{2}}b\,x^2-35a^{\frac{9}{2}}\right)}{70\left(b\,x^2+a\right)^4a^{\frac{11}{2}}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] 1/70*((b*x^2+a)*x)^(1/2)/a^(11/2)*(315*ln(2*(a+(b*x^2+a)^(1/2)*a^(1/2))/x)*
x^8*b^4*(b*x^2+a)^(1/2)-315*a^(1/2)*x^8*b^4+945*ln(2*(a+(b*x^2+a)^(1/2)*a^(
1/2))/x)*x^6*a*b^3*(b*x^2+a)^(1/2)-1050*a^(3/2)*x^6*b^3+945*ln(2*(a+(b*x^2+
a)^(1/2)*a^(1/2))/x)*x^4*a^2*b^2*(b*x^2+a)^(1/2)-1218*a^(5/2)*x^4*b^2+315*l
n(2*(a+(b*x^2+a)^(1/2)*a^(1/2))/x)*x^2*a^3*b*(b*x^2+a)^(1/2)-528*a^(7/2)*x^
2*b-35*a^(9/2))/x^(5/2)/(b*x^2+a)^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{{\left(b\,x^3+a\,x\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(3/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\left(x\left(a+bx^2\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Integral(x**(3/2)/(x*(a + b*x**2))**(9/2), x)
```

3.52 $\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}}\,dx$

Optimal. Leaf size=152

$$\frac{256b\sqrt{ax+bx^3}}{21a^6x^{3/2}}-\frac{128\sqrt{ax+bx^3}}{21a^5x^{7/2}}+\frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}}+\frac{16}{21a^3x^{3/2}\left(ax+bx^3\right)^{3/2}}+\frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}}+\frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}}$$

Rubi [A] time = 0.23, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 2014}

$$\frac{256b\sqrt{ax+bx^3}}{21a^6x^{3/2}}-\frac{128\sqrt{ax+bx^3}}{21a^5x^{7/2}}+\frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}}+\frac{16}{21a^3x^{3/2}\left(ax+bx^3\right)^{3/2}}+\frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}}+\frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a*x + b*x^3)^(9/2),x]
```

```
[Out] Sqrt[x]/(7*a*(a*x + b*x^3)^(7/2)) + 2/(7*a^2*Sqrt[x]*(a*x + b*x^3)^(5/2)) +
 16/(21*a^3*x^(3/2)*(a*x + b*x^3)^(3/2)) + 32/(7*a^4*x^(5/2)*Sqrt[a*x + b*x
^3]) - (128*Sqrt[a*x + b*x^3])/(21*a^5*x^(7/2)) + (256*b*Sqrt[a*x + b*x^3])
/(21*a^6*x^(3/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(ax+bx^3\right)^{9/2}}\,dx = \frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{10\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{7/2}}\,dx}{7a}$$

$$= \frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}} + \frac{16\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{5/2}}\,dx}{7a^2}$$

$$= \frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}} + \frac{16}{21a^3x^{3/2}\left(ax+bx^3\right)^{3/2}} + \frac{32\int \frac{1}{x^{5/2}\left(ax+bx^3\right)^{3/2}}\,dx}{7a^3}$$

$$= \frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}} + \frac{16}{21a^3x^{3/2}\left(ax+bx^3\right)^{3/2}} + \frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}} + \frac{1}{\cdots}$$

$$= \frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}} + \frac{16}{21a^3x^{3/2}\left(ax+bx^3\right)^{3/2}} + \frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}} - \frac{1}{\cdots}$$

$$= \frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}} + \frac{16}{21a^3x^{3/2}\left(ax+bx^3\right)^{3/2}} + \frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}} - \frac{1}{\cdots}$$

Mathematica [A] time = 0.05, size = 88, normalized size = 0.58

$$\frac{\sqrt{x\left(a+bx^2\right)}\left(-7a^5+70a^4bx^2+560a^3b^2x^4+1120a^2b^3x^6+896ab^4x^8+256b^5x^{10}\right)}{21a^6x^{7/2}\left(a+bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (Sqrt[x*(a + b*x^2)]*(-7*a^5 + 70*a^4*b*x^2 + 560*a^3*b^2*x^4 + 1120*a^2*b^
3*x^6 + 896*a*b^4*x^8 + 256*b^5*x^10))/(21*a^6*x^(7/2)*(a + b*x^2)^4)
```

IntegrateAlgebraic [A] time = 2.79, size = 79, normalized size = 0.52

$$\frac{\sqrt{x}\left(-7a^5+70a^4bx^2+560a^3b^2x^4+1120a^2b^3x^6+896ab^4x^8+256b^5x^{10}\right)}{21a^6\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (Sqrt[x]*(-7*a^5 + 70*a^4*b*x^2 + 560*a^3*b^2*x^4 + 1120*a^2*b^3*x^6 + 896*
a*b^4*x^8 + 256*b^5*x^10))/(21*a^6*(a*x + b*x^3)^(7/2))
```

fricas [A] time = 0.52, size = 121, normalized size = 0.80

$$\frac{\left(256\,b^5x^{10}+896\,ab^4x^8+1120\,a^2b^3x^6+560\,a^3b^2x^4+70\,a^4bx^2-7\,a^5\right)\sqrt{bx^3+ax}\,\sqrt{x}}{21\left(a^6b^4x^{12}+4\,a^7b^3x^{10}+6\,a^8b^2x^8+4\,a^9bx^6+a^{10}x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/21*(256*b^5*x^10 + 896*a*b^4*x^8 + 1120*a^2*b^3*x^6 + 560*a^3*b^2*x^4 + 7
0*a^4*b*x^2 - 7*a^5)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^6*b^4*x^12 + 4*a^7*b^3*x^
10 + 6*a^8*b^2*x^8 + 4*a^9*b*x^6 + a^10*x^4)
```

giac [A] time = 0.32, size = 147, normalized size = 0.97

$$\frac{\left(\left(x^2\left(\frac{158\,b^5x^2}{a^6}+\frac{511\,b^4}{a^5}\right)+\frac{560\,b^3}{a^4}\right)x^2+\frac{210\,b^2}{a^3}\right)x}{21\left(bx^2+a\right)^{\frac{7}{2}}}-\frac{4\left(6\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4b^{\frac{3}{2}}-15\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2ab^{\frac{3}{2}}+7\,a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/21*((x^2*(158*b^5*x^2/a^6 + 511*b^4/a^5) + 560*b^3/a^4)*x^2 + 210*b^2/a^3
)*x/(b*x^2 + a)^(7/2) - 4/3*(6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(3/2) - 15
*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2) + 7*a^2*b^(3/2))/(((sqrt(b)*x -
sqrt(b*x^2 + a))^2 - a)^3*a^5)
```

maple [A] time = 0.04, size = 81, normalized size = 0.53

$$-\frac{\left(b\,x^2+a\right)\left(-256b^5x^{10}-896a\,b^4x^8-1120a^2b^3x^6-560a^3b^2x^4-70a^4b\,x^2+7a^5\right)x^{\frac{3}{2}}}{21\left(b\,x^3+ax\right)^{\frac{9}{2}}a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/21*(b*x^2+a)*x^(3/2)*(-256*b^5*x^10-896*a*b^4*x^8-1120*a^2*b^3*x^6-560*a
^3*b^2*x^4-70*a^4*b*x^2+7*a^5)/a^6/(b*x^3+a*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x}}{\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(1/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\left(x\left(a+bx^2\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Integral(sqrt(x)/(x*(a + b*x**2))**(9/2), x)
```

3.53 $\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=189

$$-\frac{99b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{8a^{13/2}}+\frac{99b\sqrt{ax+bx^3}}{8a^6x^{5/2}}-\frac{33\sqrt{ax+bx^3}}{4a^5x^{9/2}}+\frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}}+\frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}}+\frac{}{35a^2x^3$$

Rubi [A] time = 0.29, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2023, 2025, 2029, 206}

$$-\frac{99b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{8a^{13/2}}+\frac{99b\sqrt{ax+bx^3}}{8a^6x^{5/2}}-\frac{33\sqrt{ax+bx^3}}{4a^5x^{9/2}}+\frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}}+\frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}}+\frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}}+\frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a*x + b*x^3)^(9/2)),x]
```

```
[Out] 1/(7*a*Sqrt[x]*(a*x + b*x^3)^(7/2)) + 11/(35*a^2*x^(3/2)*(a*x + b*x^3)^(5/2
)) + 33/(35*a^3*x^(5/2)*(a*x + b*x^3)^(3/2)) + 33/(5*a^4*x^(7/2)*Sqrt[a*x +
 b*x^3]) - (33*Sqrt[a*x + b*x^3])/(4*a^5*x^(9/2)) + (99*b*Sqrt[a*x + b*x^3]
)/(8*a^6*x^(5/2)) - (99*b^2*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^3]])/(
8*a^(13/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{9/2}}\,dx = \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{7/2}}\,dx}{7a}$$

$$= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} + \frac{99\int \frac{1}{x^{5/2}\left(ax+bx^3\right)^{5/2}}\,dx}{35a^2}$$

$$= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} + \frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}} + \frac{33\int \frac{1}{x^{7/2}(ax+bx}}{5a^3}$$

$$= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} + \frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}} + \frac{33}{5a^4x^{7/2}\sqrt{ax+}}$$

$$= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} + \frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}} + \frac{33}{5a^4x^{7/2}\sqrt{ax+}}$$

$$= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} + \frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}} + \frac{33}{5a^4x^{7/2}\sqrt{ax+}}$$

$$= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} + \frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}} + \frac{33}{5a^4x^{7/2}\sqrt{ax+}}$$

$$= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} + \frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}} + \frac{33}{5a^4x^{7/2}\sqrt{ax+}}$$

Mathematica [C]   time = 0.03, size = 46, normalized size = 0.24

$$\frac{b^2x^{7/2}\,{}_2F_1\left(-\frac{7}{2},3;-\frac{5}{2};\frac{bx^2}{a}+1\right)}{7a^3\left(x\left(a+bx^2\right)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a*x + b*x^3)^(9/2)),x]
```

```
[Out] (b^2*x^(7/2)*Hypergeometric2F1[-7/2, 3, -5/2, 1 + (b*x^2)/a])/(7*a^3*(x*(a
+ b*x^2))^(7/2))
```

IntegrateAlgebraic [A]   time = 3.42, size = 126, normalized size = 0.67

$$\frac{\sqrt{ax+bx^3}\left(-70a^5+385a^4bx^2+5808a^3b^2x^4+13398a^2b^3x^6+11550ab^4x^8+3465b^5x^{10}\right)}{280a^6x^{9/2}\left(a+bx^2\right)^4} - \frac{99b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{8a^{13/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a*x + b*x^3)^(9/2)),x]
```

```
[Out] (Sqrt[a*x + b*x^3]*(-70*a^5 + 385*a^4*b*x^2 + 5808*a^3*b^2*x^4 + 13398*a^2*
b^3*x^6 + 11550*a*b^4*x^8 + 3465*b^5*x^10))/(280*a^6*x^(9/2)*(a + b*x^2)^4)
 - (99*b^2*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^3]])/(8*a^(13/2))
```

fricas [A]   time = 0.43, size = 422, normalized size = 2.23

$$\left[\frac{3465\left(b^{6}x^{13}+4\,ab^{5}x^{11}+6\,a^{2}b^{4}x^{9}+4\,a^{3}b^{3}x^{7}+a^{4}b^{2}x^{5}\right)\sqrt{a}\log\left(\frac{bx^{3}+2\,ax-2\,\sqrt{bx^{3}+ax}\,\sqrt{a}\,\sqrt{x}}{x^{3}}\right)+2\left(3465\,ab^{5}x^{10}+11550\,a^{2}b^{4}x^{8}+13398\,a^{3}b^{3}x^{6}+5808\,a^{4}b^{2}x^{4}+385\,a^{5}bx^{2}-70\,a^{6}\right)\sqrt{bx^{3}+ax}\,\sqrt{x}}{560\left(a^{7}b^{4}x^{13}+4\,a^{8}b^{3}x^{11}+6\,a^{9}b^{2}x^{9}+4\,a^{10}bx^{7}+a^{11}x^{5}\right)},\ \frac{3465\left(b^{6}x^{13}+4\,ab^{5}x^{11}+6\,a^{2}b^{4}x^{9}+4\,a^{3}b^{3}x^{7}+a^{4}b^{2}x^{5}\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^{3}+ax}\,\sqrt{-a}}{a\sqrt{x}}\right)+\left(3465\,ab^{5}x^{10}+11550\,a^{2}b^{4}x^{8}+13398\,a^{3}b^{3}x^{6}+5808\,a^{4}b^{2}x^{4}+385\,a^{5}bx^{2}-70\,a^{6}\right)\sqrt{bx^{3}+ax}\,\sqrt{x}}{280\left(a^{7}b^{4}x^{13}+4\,a^{8}b^{3}x^{11}+6\,a^{9}b^{2}x^{9}+4\,a^{10}bx^{7}+a^{11}x^{5}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/560*(3465*(b^6*x^13 + 4*a*b^5*x^11 + 6*a^2*b^4*x^9 + 4*a^3*b^3*x^7 + a^4
*b^2*x^5)*sqrt(a)*log((b*x^3 + 2*a*x - 2*sqrt(b*x^3 + a*x)*sqrt(a)*sqrt(x))
/x^3) + 2*(3465*a*b^5*x^10 + 11550*a^2*b^4*x^8 + 13398*a^3*b^3*x^6 + 5808*a
^4*b^2*x^4 + 385*a^5*b*x^2 - 70*a^6)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^7*b^4*x^
13 + 4*a^8*b^3*x^11 + 6*a^9*b^2*x^9 + 4*a^10*b*x^7 + a^11*x^5), 1/280*(3465
*(b^6*x^13 + 4*a*b^5*x^11 + 6*a^2*b^4*x^9 + 4*a^3*b^3*x^7 + a^4*b^2*x^5)*sq
rt(-a)*arctan(sqrt(b*x^3 + a*x)*sqrt(-a)/(a*sqrt(x))) + (3465*a*b^5*x^10 +
11550*a^2*b^4*x^8 + 13398*a^3*b^3*x^6 + 5808*a^4*b^2*x^4 + 385*a^5*b*x^2 -
70*a^6)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^7*b^4*x^13 + 4*a^8*b^3*x^11 + 6*a^9*b
^2*x^9 + 4*a^10*b*x^7 + a^11*x^5)]
```

giac [A]   time = 0.27, size = 138, normalized size = 0.73

$$\frac{99\,b^{2}\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}\,a^{6}}+\frac{350\left(bx^{2}+a\right)^{3}b^{2}+70\left(bx^{2}+a\right)^{2}ab^{2}+21\left(bx^{2}+a\right)a^{2}b^{2}+5\,a^{3}b^{2}}{35\left(bx^{2}+a\right)^{\frac{7}{2}}a^{6}}+\frac{19\left(bx^{2}+a\right)^{\frac{3}{2}}b^{2}-21\,\sqrt{bx^{2}+a}\,ab^{2}}{8\,a^{6}b^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 99/8*b^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^6) + 1/35*(350*(b*x^2
 + a)^3*b^2 + 70*(b*x^2 + a)^2*a*b^2 + 21*(b*x^2 + a)*a^2*b^2 + 5*a^3*b^2)/
((b*x^2 + a)^(7/2)*a^6) + 1/8*(19*(b*x^2 + a)^(3/2)*b^2 - 21*sqrt(b*x^2 + a
)*a*b^2)/(a^6*b^2*x^4)
```

maple [A]   time = 0.06, size = 247, normalized size = 1.31

$$-\frac{\sqrt{\left(b\,x^{2}+a\right)x}\,\left(3465\sqrt{b\,x^{2}+a}\,b^{5}x^{10}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)-3465\sqrt{a}\,b^{5}x^{10}+10395\sqrt{b\,x^{2}+a}\,a\,b^{4}x^{8}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)-11550a^{\frac{3}{2}}b^{4}x^{8}+10395\sqrt{b\,x^{2}+a}\,a^{2}b^{3}x^{6}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)-13398a^{\frac{5}{2}}b^{3}x^{6}+3465\sqrt{b\,x^{2}+a}\,a^{3}b^{2}x^{4}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)-5808a^{\frac{7}{2}}b^{2}x^{4}-385a^{\frac{9}{2}}b\,x^{2}+70a^{\frac{11}{2}}\right)}{280\left(b\,x^{2}+a\right)^{4}a^{\frac{13}{2}}x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/280*((b*x^2+a)*x)^(1/2)/a^(13/2)*(3465*ln(2*(a+(b*x^2+a)^(1/2)*a^(1/2))/
x)*x^10*b^5*(b*x^2+a)^(1/2)-3465*a^(1/2)*x^10*b^5+10395*ln(2*(a+(b*x^2+a)^(
1/2)*a^(1/2))/x)*x^8*a*b^4*(b*x^2+a)^(1/2)-11550*a^(3/2)*x^8*b^4+10395*ln(2
*(a+(b*x^2+a)^(1/2)*a^(1/2))/x)*x^6*a^2*b^3*(b*x^2+a)^(1/2)-13398*a^(5/2)*x
^6*b^3+3465*ln(2*(a+(b*x^2+a)^(1/2)*a^(1/2))/x)*x^4*a^3*b^2*(b*x^2+a)^(1/2)
-5808*a^(7/2)*x^4*b^2-385*a^(9/2)*x^2*b+70*a^(11/2))/x^(9/2)/(b*x^2+a)^4
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+ax\right)^{\frac{9}{2}}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x)^(9/2)*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{x}\,\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a*x + b*x^3)^(9/2)),x)
```

```
[Out] int(1/(x^(1/2)*(a*x + b*x^3)^(9/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\left(x\left(a+bx^2\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Integral(1/(sqrt(x)*(x*(a + b*x**2))**(9/2)), x)
```

3.54 $\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}} dx$

Optimal. Leaf size=180

$$-\frac{1024b^2\sqrt{ax+bx^3}}{35a^7x^{3/2}}+\frac{512b\sqrt{ax+bx^3}}{35a^6x^{7/2}}-\frac{384\sqrt{ax+bx^3}}{35a^5x^{11/2}}+\frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}}+\frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}}+\frac{1}{35a^2x^{5/2}\left(a\right.}$$

Rubi [A] time = 0.28, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 2014}

$$-\frac{1024b^2\sqrt{ax+bx^3}}{35a^7x^{3/2}}+\frac{512b\sqrt{ax+bx^3}}{35a^6x^{7/2}}-\frac{384\sqrt{ax+bx^3}}{35a^5x^{11/2}}+\frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}}+\frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}}+\frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}}+\frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a*x + b*x^3)^(9/2)),x]
```

```
[Out] 1/(7*a*x^(3/2)*(a*x + b*x^3)^(7/2)) + 12/(35*a^2*x^(5/2)*(a*x + b*x^3)^(5/2
)) + 8/(7*a^3*x^(7/2)*(a*x + b*x^3)^(3/2)) + 64/(7*a^4*x^(9/2)*Sqrt[a*x + b
*x^3]) - (384*Sqrt[a*x + b*x^3])/(35*a^5*x^(11/2)) + (512*b*Sqrt[a*x + b*x^
3])/(35*a^6*x^(7/2)) - (1024*b^2*Sqrt[a*x + b*x^3])/(35*a^7*x^(3/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}}\,dx = \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12\int \frac{1}{x^{5/2}\left(ax+bx^3\right)^{7/2}}\,dx}{7a}$$

$$= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} + \frac{24\int \frac{1}{x^{7/2}\left(ax+bx^3\right)^{5/2}}\,dx}{7a^2}$$

$$= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} + \frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}} + \frac{64\int \frac{1}{x^{9/2}\left(ax+bx^3\right.}}{7a^3}$$

$$= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} + \frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}} + \frac{64}{7a^4x^{9/2}\sqrt{ax+}}$$

$$= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} + \frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}} + \frac{64}{7a^4x^{9/2}\sqrt{ax+}}$$

$$= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} + \frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}} + \frac{64}{7a^4x^{9/2}\sqrt{ax+}}$$

$$= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} + \frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}} + \frac{64}{7a^4x^{9/2}\sqrt{ax+}}$$

Mathematica [A] time = 0.04, size = 99, normalized size = 0.55

$$-\frac{\sqrt{x\left(a+bx^2\right)}\left(7a^6-28a^5bx^2+280a^4b^2x^4+2240a^3b^3x^6+4480a^2b^4x^8+3584ab^5x^{10}+1024b^6x^{12}\right)}{35a^7x^{11/2}\left(a+bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a*x + b*x^3)^(9/2)),x]
```

```
[Out] -1/35*(Sqrt[x*(a + b*x^2)]*(7*a^6 - 28*a^5*b*x^2 + 280*a^4*b^2*x^4 + 2240*a
^3*b^3*x^6 + 4480*a^2*b^4*x^8 + 3584*a*b^5*x^10 + 1024*b^6*x^12))/(a^7*x^(1
1/2)*(a + b*x^2)^4)
```

IntegrateAlgebraic [A] time = 5.05, size = 90, normalized size = 0.50

$$\frac{-7a^6+28a^5bx^2-280a^4b^2x^4-2240a^3b^3x^6-4480a^2b^4x^8-3584ab^5x^{10}-1024b^6x^{12}}{35a^7x^{3/2}\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(a*x + b*x^3)^(9/2)),x]
```

```
[Out] (-7*a^6 + 28*a^5*b*x^2 - 280*a^4*b^2*x^4 - 2240*a^3*b^3*x^6 - 4480*a^2*b^4*
x^8 - 3584*a*b^5*x^10 - 1024*b^6*x^12)/(35*a^7*x^(3/2)*(a*x + b*x^3)^(7/2))
```

fricas [A] time = 0.55, size = 132, normalized size = 0.73

$$-\frac{\left(1024\,b^6x^{12}+3584\,ab^5x^{10}+4480\,a^2b^4x^8+2240\,a^3b^3x^6+280\,a^4b^2x^4-28\,a^5bx^2+7\,a^6\right)\sqrt{bx^3+ax}\,\sqrt{x}}{35\left(a^7b^4x^{14}+4\,a^8b^3x^{12}+6\,a^9b^2x^{10}+4\,a^{10}bx^8+a^{11}x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(1024*b^6*x^12 + 3584*a*b^5*x^10 + 4480*a^2*b^4*x^8 + 2240*a^3*b^3*x^
6 + 280*a^4*b^2*x^4 - 28*a^5*b*x^2 + 7*a^6)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^7*
b^4*x^14 + 4*a^8*b^3*x^12 + 6*a^9*b^2*x^10 + 4*a^10*b*x^8 + a^11*x^6)
```

giac [A] time = 0.41, size = 202, normalized size = 1.12

$$-\frac{\left(\left(2\,x^{2}\left(\frac{281\,b^{6}x^{2}}{a^{7}}+\frac{896\,b^{5}}{a^{6}}\right)+\frac{1925\,b^{4}}{a^{5}}\right)x^{2}+\frac{700\,b^{3}}{a^{4}}\right)x}{35\left(bx^{2}+a\right)^{\frac{7}{2}}}+\frac{4\left(25\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{8}b^{\frac{5}{2}}-120\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{6}ab^{\frac{5}{2}}+210\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}a^{2}b^{\frac{5}{2}}-140\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}a^{3}b^{\frac{5}{2}}+33\,a^{4}b^{\frac{5}{2}}\right)}{5\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}-a\right)^{5}a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*((2*x^2*(281*b^6*x^2/a^7 + 896*b^5/a^6) + 1925*b^4/a^5)*x^2 + 700*b^3
/a^4)*x/(b*x^2 + a)^(7/2) + 4/5*(25*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(5/2)
 - 120*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(5/2) + 210*(sqrt(b)*x - sqrt(b*
x^2 + a))^4*a^2*b^(5/2) - 140*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(5/2) +
 33*a^4*b^(5/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5*a^6)
```

maple [A] time = 0.05, size = 92, normalized size = 0.51

$$-\frac{\left(b\,x^{2}+a\right)\left(1024b^{6}x^{12}+3584b^{5}x^{10}a+4480x^{8}b^{4}a^{2}+2240b^{3}x^{6}a^{3}+280b^{2}x^{4}a^{4}-28b\,x^{2}a^{5}+7a^{6}\right)}{35\left(b\,x^{3}+ax\right)^{\frac{9}{2}}a^{7}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/35*(b*x^2+a)*(1024*b^6*x^12+3584*a*b^5*x^10+4480*a^2*b^4*x^8+2240*a^3*b^
3*x^6+280*a^4*b^2*x^4-28*a^5*b*x^2+7*a^6)/x^(1/2)/a^7/(b*x^3+a*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+ax\right)^{\frac{9}{2}}x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x)^(9/2)*x^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3/2}\left(b\,x^{3}+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a*x + b*x^3)^(9/2)),x)
```

```
[Out] int(1/(x^(3/2)*(a*x + b*x^3)^(9/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\left(x\left(a+bx^{2}\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Integral(1/(x**(3/2)*(x*(a + b*x**2))**(9/2)), x)
```

3.55 $\int \frac{x^4}{\sqrt{ax+bx^4}} \, dx$

Optimal. Leaf size=55

$$\frac{x\sqrt{ax+bx^4}}{3b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax+bx^4}}\right)}{3b^{3/2}}$$

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2024, 2029, 206}

$$\frac{x\sqrt{ax+bx^4}}{3b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax+bx^4}}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a*x + b*x^4],x]
```

```
[Out] (x*Sqrt[a*x + b*x^4])/(3*b) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x + b*x^4]])/
(3*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{ax+bx^4}} \, dx &= \frac{x\sqrt{ax+bx^4}}{3b} - \frac{a\int \frac{x}{\sqrt{ax+bx^4}} \, dx}{2b} \\
&= \frac{x\sqrt{ax+bx^4}}{3b} - \frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2} \, dx, x, \frac{x^2}{\sqrt{ax+bx^4}}\right)}{3b} \\
&= \frac{x\sqrt{ax+bx^4}}{3b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax+bx^4}}\right)}{3b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 81, normalized size = 1.47

$$\frac{\sqrt{b}x^2\left(a+bx^3\right)-a\sqrt{x}\sqrt{a+bx^3}\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx^3}}\right)}{3b^{3/2}\sqrt{x\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a*x + b*x^4],x]
```

```
[Out] (Sqrt[b]*x^2*(a + b*x^3) - a*Sqrt[x]*Sqrt[a + b*x^3]*ArcTanh[(Sqrt[b]*x^(3/
2))/Sqrt[a + b*x^3]])/(3*b^(3/2)*Sqrt[x*(a + b*x^3)])
```

IntegrateAlgebraic [A] time = 0.54, size = 62, normalized size = 1.13

$$\frac{x\sqrt{ax+bx^4}}{3b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x\sqrt{ax+bx^4}}{a+bx^3}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/Sqrt[a*x + b*x^4],x]
```

```
[Out] (x*Sqrt[a*x + b*x^4])/(3*b) - (a*ArcTanh[(Sqrt[b]*x*Sqrt[a*x + b*x^4])/(a +
 b*x^3)])/(3*b^(3/2))
```

fricas [A] time = 0.57, size = 133, normalized size = 2.42

$$\left[\frac{4\sqrt{bx^4+ax}\,bx+a\sqrt{b}\log\left(-8\,b^2x^6-8\,abx^3-a^2+4\left(2\,bx^4+ax\right)\sqrt{bx^4+ax}\sqrt{b}\right)}{12\,b^2},\ \frac{2\sqrt{bx^4+ax}\,bx+a\sqrt{-b}\arctan\left(\frac{2\sqrt{bx^4+ax}\sqrt{-b}\,x}{2\,bx^3+a}\right)}{6\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*sqrt(b*x^4 + a*x)*b*x + a*sqrt(b)*log(-8*b^2*x^6 - 8*a*b*x^3 - a^2
 + 4*(2*b*x^4 + a*x)*sqrt(b*x^4 + a*x)*sqrt(b)))/b^2, 1/6*(2*sqrt(b*x^4 + a
*x)*b*x + a*sqrt(-b)*arctan(2*sqrt(b*x^4 + a*x)*sqrt(-b)*x/(2*b*x^3 + a)))/
b^2]
```

giac [A] time = 0.30, size = 45, normalized size = 0.82

$$\frac{\sqrt{bx^4+ax}\,x}{3\,b}+\frac{a\arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\sqrt{-b}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(b*x^4 + a*x)*x/b + 1/3*a*arctan(sqrt(b + a/x^3)/sqrt(-b))/(sqrt(-b
)*b)
```

maple [C] time = 0.09, size = 997, normalized size = 18.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^4+a*x)^(1/2),x)
```

```
[Out] 1/3*x*(b*x^4+a*x)^(1/2)/b-a*(1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1
/3)/b)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*x/(-1/2*(-a*
b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b))^(1/2)*(x
-(-a*b^2)^(1/3)/b)^2*((-a*b^2)^(1/3)/b*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2
)*(-a*b^2)^(1/3)/b)/(-1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/
(x-(-a*b^2)^(1/3)/b))^(1/2)*((-a*b^2)^(1/3)/b*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I
*3^(1/2)*(-a*b^2)^(1/3)/b)/(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1
/3)/b)/(x-(-a*b^2)^(1/3)/b))^(1/2)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a
*b^2)^(1/3)/b)/(-a*b^2)^(1/3)/(b*x*(x-(-a*b^2)^(1/3)/b)*(x+1/2*(-a*b^2)^(1/
3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*
(-a*b^2)^(1/3)/b))^(1/2)*((-a*b^2)^(1/3)/b*EllipticF(((-3/2*(-a*b^2)^(1/3)/
b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*x/(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-
a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b))^(1/2),((3/2*(-a*b^2)^(1/3)/b+1/2*I*3^
(1/2)*(-a*b^2)^(1/3)/b)*(1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/
b)/(1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(3/2*(-a*b^2)^(1/3
)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2))-(-a*b^2)^(1/3)/b*EllipticPi(((-
3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*x/(-1/2*(-a*b^2)^(1/3)
/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b))^(1/2),(-1/2*(-a*b^
2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(
1/2)*(-a*b^2)^(1/3)/b),((3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/
b)*(1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(1/2*(-a*b^2)^(1/3
)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(3/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a
*b^2)^(1/3)/b))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{bx^4 + ax}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(b*x^4 + a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\sqrt{b\,x^4 + a\,x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x + b*x^4)^(1/2),x)
```

```
[Out] int(x^4/(a*x + b*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x\left(a + bx^3\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x*(a + b*x**3)), x)
```

3.56 $\int \frac{x}{\sqrt{ax+bx^4}}\,dx$

Optimal. Leaf size=32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax+bx^4}}\right)}{3\sqrt{b}}$$

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2029, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax+bx^4}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a*x + b*x^4],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x + b*x^4]])/(3*Sqrt[b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{ax+bx^4}}\,dx &= \frac{2}{3}\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{ax+bx^4}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax+bx^4}}\right)}{3\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 1.91

$$\frac{2\sqrt{x}\,\sqrt{a+bx^3}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a+bx^3}}\right)}{3\sqrt{b}\,\sqrt{x\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a*x + b*x^4],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[a + b*x^3]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]])/(3*S
qrt[b]*Sqrt[x*(a + b*x^3)])
```

IntegrateAlgebraic [A] time = 0.41, size = 39, normalized size = 1.22

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x\sqrt{ax+bx^4}}{a+bx^3}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[a*x + b*x^4],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x*Sqrt[a*x + b*x^4])/(a + b*x^3)])/(3*Sqrt[b])
```

fricas [A] time = 0.57, size = 94, normalized size = 2.94

$$\left[\frac{\log\left(-8\,b^2x^6-8\,abx^3-a^2-4\left(2\,bx^4+ax\right)\sqrt{bx^4+ax}\,\sqrt{b}\right)}{6\,\sqrt{b}}, -\frac{\sqrt{-b}\,\arctan\left(\frac{2\,\sqrt{bx^4+ax}\,\sqrt{-b}\,x}{2\,bx^3+a}\right)}{3\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log(-8*b^2*x^6 - 8*a*b*x^3 - a^2 - 4*(2*b*x^4 + a*x)*sqrt(b*x^4 + a*x)
*sqrt(b))/sqrt(b), -1/3*sqrt(-b)*arctan(2*sqrt(b*x^4 + a*x)*sqrt(-b)*x/(2*b
*x^3 + a))/b]
```

giac [A] time = 0.21, size = 23, normalized size = 0.72

$$-\frac{2\,\arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\,\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*arctan(sqrt(b + a/x^3)/sqrt(-b))/sqrt(-b)
```

maple [C] time = 0.09, size = 979, normalized size = 30.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a*x)^(1/2),x)
```

```
[Out] 2*(1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*((-3/2*(-a*b^2)^(1/
3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(
-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b)*x)^(1/2)*(x-(-a*b^2)^(1/3)/b)^2*((-a*
b^2)^(1/3)*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-1/2*(-
a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b)/b)^(1/2
)*((-a*b^2)^(1/3)*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(
-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b)/
b)^(1/2)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-a*b^2)^(1
/3)*b/((x-(-a*b^2)^(1/3)/b)*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^
(1/3)/b)*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*b*x)^(1/2)
*((-a*b^2)^(1/3)/b*EllipticF(((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)
^(1/3)/b)/(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(x-(-a*b^2
)^(1/3)/b)*x)^(1/2),((3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*
(1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(1/2*(-a*b^2)^(1/3)/b
+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(3/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^
2)^(1/3)/b))^(1/2))-(-a*b^2)^(1/3)/b*EllipticPi(((-3/2*(-a*b^2)^(1/3)/b+1/2
*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^
(1/3)/b)/(x-(-a*b^2)^(1/3)/b)*x)^(1/2),(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)
*(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b),(
(3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*(1/2*(-a*b^2)^(1/3)/b
-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^
2)^(1/3)/b)/(3/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{bx^4 + ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(b*x^4 + a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\sqrt{b\,x^4 + a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^4)^(1/2),x)
[Out] int(x/(a*x + b*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x\left(a + bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a*x)**(1/2),x)
[Out] Integral(x/sqrt(x*(a + b*x**3)), x)
```

3.57 $\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx$

Optimal. Leaf size=23

$$-\frac{2\sqrt{ax+bx^4}}{3ax^2}$$

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2014}

$$-\frac{2\sqrt{ax+bx^4}}{3ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a*x + b*x^4]),x]
[Out] (-2*Sqrt[a*x + b*x^4])/(3*a*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{ax+bx^4}}{3ax^2}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 1.00

$$-\frac{2\sqrt{x\left(a+bx^3\right)}}{3ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a*x + b*x^4]),x]
[Out] (-2*Sqrt[x*(a + b*x^3)])/(3*a*x^2)
```

IntegrateAlgebraic [A] time = 0.44, size = 23, normalized size = 1.00

$$-\frac{2\sqrt{ax+bx^4}}{3ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[a*x + b*x^4]),x]
[Out] (-2*Sqrt[a*x + b*x^4])/(3*a*x^2)
```

fricas [A] time = 0.42, size = 19, normalized size = 0.83

$$-\frac{2\,\sqrt{bx^4+ax}}{3\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(b*x^4 + a*x)/(a*x^2)
```

giac [A] time = 0.22, size = 14, normalized size = 0.61

$$-\frac{2\sqrt{b+\frac{a}{x^3}}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(b + a/x^3)/a
```

maple [A] time = 0.04, size = 27, normalized size = 1.17

$$-\frac{2\left(b\,x^{3}+a\right)}{3\sqrt{b\,x^{4}+ax}\;ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^4+a*x)^(1/2),x)
```

```
[Out] -2/3/x*(b*x^3+a)/a/(b*x^4+a*x)^(1/2)
```

maxima [A] time = 1.46, size = 26, normalized size = 1.13

$$-\frac{2\left(bx^{4}+ax\right)}{3\sqrt{bx^{3}+a}\,ax^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*(b*x^4 + a*x)/(sqrt(b*x^3 + a)*a*x^(5/2))
```

mupad [B] time = 5.13, size = 19, normalized size = 0.83

$$-\frac{2\sqrt{b\,x^{4}+a\,x}}{3\,a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^4)^(1/2)),x)
```

```
[Out] -(2*(a*x + b*x^4)^(1/2))/(3*a*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{2}\sqrt{x\left(a+bx^{3}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x*(a + b*x**3))), x)
```

3.58 $\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx$

Optimal. Leaf size=48

$$\frac{4b\sqrt{ax+bx^4}}{9a^2x^2}-\frac{2\sqrt{ax+bx^4}}{9ax^5}$$

Rubi [A] time = 0.07, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2016, 2014}

$$\frac{4b\sqrt{ax+bx^4}}{9a^2x^2}-\frac{2\sqrt{ax+bx^4}}{9ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x + b*x^4])/(9*a*x^5) + (4*b*Sqrt[a*x + b*x^4])/(9*a^2*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx &= -\frac{2\sqrt{ax+bx^4}}{9ax^5}-\frac{(2b)\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx}{3a}\\ &= -\frac{2\sqrt{ax+bx^4}}{9ax^5}+\frac{4b\sqrt{ax+bx^4}}{9a^2x^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 0.65

$$-\frac{2\left(a-2bx^3\right)\sqrt{x\left(a+bx^3\right)}}{9a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*(a - 2*b*x^3)*Sqrt[x*(a + b*x^3)])/(9*a^2*x^5)
```

IntegrateAlgebraic [A] time = 0.46, size = 33, normalized size = 0.69

$$\frac{2\left(2bx^3-a\right)\sqrt{ax+bx^4}}{9a^2x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*Sqrt[a*x + b*x^4]),x]
[Out] (2*(-a + 2*b*x^3)*Sqrt[a*x + b*x^4])/(9*a^2*x^5)
```

fricas [A] time = 0.42, size = 29, normalized size = 0.60

$$\frac{2\sqrt{bx^4+ax}\left(2\,bx^3-a\right)}{9\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
[Out] 2/9*sqrt(b*x^4 + a*x)*(2*b*x^3 - a)/(a^2*x^5)
```

giac [A] time = 0.19, size = 30, normalized size = 0.62

$$-\frac{2\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}}{9\,a^2}+\frac{2\sqrt{b+\frac{a}{x^3}}\,b}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a*x)^(1/2),x, algorithm="giac")
[Out] -2/9*(b + a/x^3)^(3/2)/a^2 + 2/3*sqrt(b + a/x^3)*b/a^2
```

maple [A] time = 0.04, size = 35, normalized size = 0.73

$$-\frac{2\left(b\,x^3+a\right)\left(-2b\,x^3+a\right)}{9\sqrt{b\,x^4+ax}\;a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^4+a*x)^(1/2),x)
[Out] -2/9*(b*x^3+a)*(-2*b*x^3+a)/x^4/a^2/(b*x^4+a*x)^(1/2)
```

maxima [A] time = 1.51, size = 38, normalized size = 0.79

$$\frac{2\left(2\,b^2x^7+abx^4-a^2x\right)}{9\,\sqrt{bx^3+a}\,a^2x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
[Out] 2/9*(2*b^2*x^7 + a*b*x^4 - a^2*x)/(sqrt(b*x^3 + a)*a^2*x^(11/2))
```

mupad [B] time = 5.13, size = 27, normalized size = 0.56

$$-\frac{2\sqrt{b\,x^4+a\,x}\;\left(a-2\,b\,x^3\right)}{9\,a^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a*x + b*x^4)^(1/2)),x)
[Out] -(2*(a*x + b*x^4)^(1/2)*(a - 2*b*x^3))/(9*a^2*x^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\sqrt{x\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(x*(a + b*x**3))), x)
```

3.59 $\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx$

Optimal. Leaf size=74

$$-\frac{16b^2\sqrt{ax+bx^4}}{45a^3x^2}+\frac{8b\sqrt{ax+bx^4}}{45a^2x^5}-\frac{2\sqrt{ax+bx^4}}{15ax^8}$$

Rubi [A] time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2016, 2014}

$$-\frac{16b^2\sqrt{ax+bx^4}}{45a^3x^2}+\frac{8b\sqrt{ax+bx^4}}{45a^2x^5}-\frac{2\sqrt{ax+bx^4}}{15ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x + b*x^4])/(15*a*x^8) + (8*b*Sqrt[a*x + b*x^4])/(45*a^2*x^5) -
(16*b^2*Sqrt[a*x + b*x^4])/(45*a^3*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx &= -\frac{2\sqrt{ax+bx^4}}{15ax^8}-\frac{(4b)\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx}{5a}\\ &= -\frac{2\sqrt{ax+bx^4}}{15ax^8}+\frac{8b\sqrt{ax+bx^4}}{45a^2x^5}+\frac{\left(8b^2\right)\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx}{15a^2}\\ &= -\frac{2\sqrt{ax+bx^4}}{15ax^8}+\frac{8b\sqrt{ax+bx^4}}{45a^2x^5}-\frac{16b^2\sqrt{ax+bx^4}}{45a^3x^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.59

$$-\frac{2\sqrt{x\left(a+bx^3\right)}\left(3a^2-4abx^3+8b^2x^6\right)}{45a^3x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*Sqrt[x*(a + b*x^3)]*(3*a^2 - 4*a*b*x^3 + 8*b^2*x^6))/(45*a^3*x^8)
```

IntegrateAlgebraic [A] time = 0.50, size = 44, normalized size = 0.59

$$-\frac{2\sqrt{ax+bx^4}\left(3a^2-4abx^3+8b^2x^6\right)}{45a^3x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^8*Sqrt[a*x + b*x^4]),x]
[Out] (-2*Sqrt[a*x + b*x^4]*(3*a^2 - 4*a*b*x^3 + 8*b^2*x^6))/(45*a^3*x^8)
```

fricas [A] time = 0.43, size = 40, normalized size = 0.54

$$-\frac{2\left(8\,b^2x^6-4\,abx^3+3\,a^2\right)\sqrt{bx^4+ax}}{45\,a^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
[Out] -2/45*(8*b^2*x^6 - 4*a*b*x^3 + 3*a^2)*sqrt(b*x^4 + a*x)/(a^3*x^8)
```

giac [A] time = 0.20, size = 47, normalized size = 0.64

$$-\frac{2\sqrt{b+\frac{a}{x^3}}\,b^2}{3\,a^3}-\frac{2\left(3\left(b+\frac{a}{x^3}\right)^{\frac{5}{2}}-10\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}b\right)}{45\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a*x)^(1/2),x, algorithm="giac")
[Out] -2/3*sqrt(b + a/x^3)*b^2/a^3 - 2/45*(3*(b + a/x^3)^(5/2) - 10*(b + a/x^3)^(
3/2)*b)/a^3
```

maple [A] time = 0.05, size = 48, normalized size = 0.65

$$-\frac{2\left(b\,x^3+a\right)\left(8b^2x^6-4ab\,x^3+3a^2\right)}{45\sqrt{b\,x^4+ax}\;a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^4+a*x)^(1/2),x)
[Out] -2/45*(b*x^3+a)*(8*b^2*x^6-4*a*b*x^3+3*a^2)/x^7/a^3/(b*x^4+a*x)^(1/2)
```

maxima [A] time = 1.49, size = 50, normalized size = 0.68

$$-\frac{2\left(8\,b^3x^{10}+4\,ab^2x^7-a^2bx^4+3\,a^3x\right)}{45\,\sqrt{bx^3+a}\,a^3x^{\frac{17}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
[Out] -2/45*(8*b^3*x^10 + 4*a*b^2*x^7 - a^2*b*x^4 + 3*a^3*x)/(sqrt(b*x^3 + a)*a^3
*x^(17/2))
```

mupad [B] time = 5.27, size = 40, normalized size = 0.54

$$-\frac{2\,\sqrt{b\,x^4+a\,x}\,\left(3\,a^2-4\,a\,b\,x^3+8\,b^2\,x^6\right)}{45\,a^3\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a*x + b*x^4)^(1/2)),x)
```

```
[Out] -(2*(a*x + b*x^4)^(1/2)*(3*a^2 + 8*b^2*x^6 - 4*a*b*x^3))/(45*a^3*x^8)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8\sqrt{x\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**8*sqrt(x*(a + b*x**3))), x)
```

3.60 $\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=174

$$-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{64a^{11/2}}+\frac{63b^4\sqrt{ax+b\sqrt{x}}}{64a^5}-\frac{21b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}}{32a^4}+\frac{21b^2x\sqrt{ax+b\sqrt{x}}}{40a^3}-\frac{9bx^{3/2}\sqrt{ax+b\sqrt{x}}}{20a^2}+$$

Rubi [A] time = 0.15, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 670, 640, 620, 206}

$$\frac{63b^4\sqrt{ax+b\sqrt{x}}}{64a^5}-\frac{21b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}}{32a^4}+\frac{21b^2x\sqrt{ax+b\sqrt{x}}}{40a^3}-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{64a^{11/2}}-\frac{9bx^{3/2}\sqrt{ax+b\sqrt{x}}}{20a^2}+\frac{2x^2\sqrt{ax+b\sqrt{x}}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (63*b^4*Sqrt[b*Sqrt[x] + a*x])/(64*a^5) - (21*b^3*Sqrt[x]*Sqrt[b*Sqrt[x] +
a*x])/(32*a^4) + (21*b^2*x*Sqrt[b*Sqrt[x] + a*x])/(40*a^3) - (9*b*x^(3/2)*S
qrt[b*Sqrt[x] + a*x])/(20*a^2) + (2*x^2*Sqrt[b*Sqrt[x] + a*x])/(5*a) - (63*
b^5*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(64*a^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\,\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)$$

$$= \frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a} - \frac{(9b)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{5a}$$

$$= -\frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} + \frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a} + \frac{\left(63b^2\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{40a^2}$$

$$= \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} - \frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} + \frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a} - \frac{\left(21b^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx,\right.}{16a^3}$$

$$= -\frac{21b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} - \frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} + \frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a} + \frac{(63}{}$$

$$= \frac{63b^4\sqrt{b\sqrt{x}+ax}}{64a^5} - \frac{21b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} - \frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} + \frac{2x^2}{}$$

$$= \frac{63b^4\sqrt{b\sqrt{x}+ax}}{64a^5} - \frac{21b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} - \frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} + \frac{2x^2}{}$$

$$= \frac{63b^4\sqrt{b\sqrt{x}+ax}}{64a^5} - \frac{21b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} - \frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} + \frac{2x^2}{}$$

Mathematica [A] time = 0.19, size = 151, normalized size = 0.87

$$\frac{\left(a\sqrt{x}+b\right)\left(\sqrt{a}\sqrt{x}\sqrt{\frac{a\sqrt{x}}{b}+1}\left(128a^4x^2-144a^3bx^{3/2}+168a^2b^2x-210ab^3\sqrt{x}+315b^4\right)-315b^{9/2}\sqrt[4]{x}\sinh^{-1}\left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}}\right)\right)}{320a^{11/2}\sqrt{\frac{a\sqrt{x}}{b}+1}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] ((b + a*Sqrt[x])*(Sqrt[a]*Sqrt[1 + (a*Sqrt[x])/b]*Sqrt[x]*(315*b^4 - 210*a*
b^3*Sqrt[x] + 168*a^2*b^2*x - 144*a^3*b*x^(3/2) + 128*a^4*x^2) - 315*b^(9/2
)*x^(1/4)*ArcSinh[(Sqrt[a]*x^(1/4))/Sqrt[b]]))/(320*a^(11/2)*Sqrt[1 + (a*Sq
rt[x])/b]*Sqrt[b*Sqrt[x] + a*x])
```

IntegrateAlgebraic [A] time = 0.31, size = 113, normalized size = 0.65

$$\frac{63b^5\log\left(-2\sqrt{a}\sqrt{ax+b\sqrt{x}}+2a\sqrt{x}+b\right)}{128a^{11/2}} + \frac{\sqrt{ax+b\sqrt{x}}\left(128a^4x^2-144a^3bx^{3/2}+168a^2b^2x-210ab^3\sqrt{x}+315b^4\right)}{320a^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (Sqrt[b*Sqrt[x] + a*x]*(315*b^4 - 210*a*b^3*Sqrt[x] + 168*a^2*b^2*x - 144*a
^3*b*x^(3/2) + 128*a^4*x^2))/(320*a^5) + (63*b^5*Log[b + 2*a*Sqrt[x] - 2*Sq
rt[a]*Sqrt[b*Sqrt[x] + a*x]])/(128*a^(11/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.29, size = 111, normalized size = 0.64

$$\frac{1}{320}\sqrt{ax+b\sqrt{x}}\left(2\left(4\left(2\sqrt{x}\left(\frac{8\sqrt{x}}{a}-\frac{9b}{a^2}\right)+\frac{21b^2}{a^3}\right)\sqrt{x}-\frac{105b^3}{a^4}\right)\sqrt{x}+\frac{315b^4}{a^5}\right)+\frac{63b^5\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{128a^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/320*sqrt(a*x + b*sqrt(x))*(2*(4*(2*sqrt(x)*(8*sqrt(x)/a - 9*b/a^2) + 21*b
^2/a^3)*sqrt(x) - 105*b^3/a^4)*sqrt(x) + 315*b^4/a^5) + 63/128*b^5*log(abs(
-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(11/2)
```

maple [A] time = 0.09, size = 223, normalized size = 1.28

$$\frac{\sqrt{ax+b\sqrt{x}}\left(-640ab^5\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\sqrt{a}}{2\sqrt{a}}\right)+325ab^5\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\sqrt{a}}{2\sqrt{a}}\right)-1300\sqrt{ax+b\sqrt{x}}\,a^{\frac{5}{2}}b^3\sqrt{x}+256\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{9}{2}}x+1280\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{3}{2}}b^4-650\sqrt{ax+b\sqrt{x}}\,a^{\frac{3}{2}}b^4-544\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{7}{2}}b\sqrt{x}+880\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{5}{2}}b^2\right)}{640\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^(1/2)+a*x)^(1/2),x)
```

```
[Out] 1/640*(b*x^(1/2)+a*x)^(1/2)*(256*x*(b*x^(1/2)+a*x)^(3/2)*a^(9/2)-544*a^(7/2
)*x^(1/2)*(b*x^(1/2)+a*x)^(3/2)*b-1300*a^(5/2)*x^(1/2)*(b*x^(1/2)+a*x)^(1/2
)*b^3+880*a^(5/2)*(b*x^(1/2)+a*x)^(3/2)*b^2+1280*a^(3/2)*(x^(1/2)*(b+a*x^(1
/2)))^(1/2)*b^4-650*a^(3/2)*(b*x^(1/2)+a*x)^(1/2)*b^4+325*ln(1/2*(2*(b*x^(1
/2)+a*x)^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*a*b^5-640*a*ln(1/2*(2*(x^(1/
2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*b^5)/(x^(1/2)*(b+a*
x^(1/2)))^(1/2)/a^(13/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(a*x + b*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^(1/2))^(1/2),x)
```

```
[Out] int(x^2/(a*x + b*x^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a*x + b*sqrt(x)), x)
```

3.61 $\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=116

$$-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{4a^{7/2}}+\frac{5b^2\sqrt{ax+b\sqrt{x}}}{4a^3}-\frac{5b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a^2}+\frac{2x\sqrt{ax+b\sqrt{x}}}{3a}$$

Rubi [A] time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2018, 670, 640, 620, 206}

$$\frac{5b^2\sqrt{ax+b\sqrt{x}}}{4a^3}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{4a^{7/2}}-\frac{5b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a^2}+\frac{2x\sqrt{ax+b\sqrt{x}}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (5*b^2*Sqrt[b*Sqrt[x] + a*x])/(4*a^3) - (5*b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])
/(6*a^2) + (2*x*Sqrt[b*Sqrt[x] + a*x])/(3*a) - (5*b^3*ArcTanh[(Sqrt[a]*Sqrt
[x])/Sqrt[b*Sqrt[x] + a*x]])/(4*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\,\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{(5b)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{3a}$$
$$= -\frac{5b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} + \frac{(5b^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{4a^2}$$
$$= \frac{5b^2\sqrt{b\sqrt{x}+ax}}{4a^3} - \frac{5b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{(5b^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{\cdot}\right.}{8a^3}$$
$$= \frac{5b^2\sqrt{b\sqrt{x}+ax}}{4a^3} - \frac{5b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{(5b^3)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{\cdot}}{\sqrt{b\sqrt{\cdot}}}\right.}{4a^3}$$
$$= \frac{5b^2\sqrt{b\sqrt{x}+ax}}{4a^3} - \frac{5b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{4a^{7/2}}$$

Mathematica [A] time = 0.12, size = 129, normalized size = 1.11

$$\frac{5b^4\left(\frac{a\sqrt{x}}{b}+1\right)\left(\frac{16a^3x^{3/2}}{15b^3}-\frac{4a^2x}{3b^2}+\frac{2a\sqrt{x}}{b}-\frac{2\sqrt{a}\sqrt[4]{x}\sinh^{-1}\left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{\frac{a\sqrt{x}}{b}+1}}\right)}{8a^4\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (5*b^4*(1 + (a*Sqrt[x])/b)*((2*a*Sqrt[x])/b - (4*a^2*x)/(3*b^2) + (16*a^3*x
^(3/2))/(15*b^3) - (2*Sqrt[a]*x^(1/4)*ArcSinh[(Sqrt[a]*x^(1/4))/Sqrt[b]])/(
Sqrt[b]*Sqrt[1 + (a*Sqrt[x])/b])))/(8*a^4*Sqrt[(b + a*Sqrt[x])*Sqrt[x]])
```

IntegrateAlgebraic [A] time = 0.25, size = 95, normalized size = 0.82

$$\frac{\sqrt{ax+b\sqrt{x}}\left(8a^2x-10ab\sqrt{x}+15b^2\right)}{12a^3}+\frac{5b^3\log\left(-2a^{7/2}\sqrt{ax+b\sqrt{x}}+2a^4\sqrt{x}+a^3b\right)}{8a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (Sqrt[b*Sqrt[x] + a*x]*(15*b^2 - 10*a*b*Sqrt[x] + 8*a^2*x))/(12*a^3) + (5*b
^3*Log[a^3*b + 2*a^4*Sqrt[x] - 2*a^(7/2)*Sqrt[b*Sqrt[x] + a*x]])/(8*a^(7/2)
)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

[Out] Timed out

giac [A] time = 0.31, size = 83, normalized size = 0.72

$$\frac{1}{12}\sqrt{ax+b\sqrt{x}}\left(2\sqrt{x}\left(\frac{4\sqrt{x}}{a}-\frac{5b}{a^2}\right)+\frac{15b^2}{a^3}\right)+\frac{5b^3\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{8a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(a*x + b*sqrt(x))*(2*sqrt(x)*(4*sqrt(x)/a - 5*b/a^2) + 15*b^2/a^3)
 + 5/8*b^3*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b
))/a^(7/2)
```

maple [B] time = 0.05, size = 181, normalized size = 1.56

$$-\frac{\sqrt{ax+b\sqrt{x}}\left(24a\,b^3\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-9a\,b^3\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+36\sqrt{ax+b\sqrt{x}}\,a^{\frac{5}{2}}b\sqrt{x}-48\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{3}{2}}b^2+18\sqrt{ax+b\sqrt{x}}\,a^{\frac{3}{2}}b^2-16\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{5}{2}}\right)}{24\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x+b*x^(1/2))^(1/2),x)
```

```
[Out] -1/24*(a*x+b*x^(1/2))^(1/2)/a^(9/2)*(36*x^(1/2)*(a*x+b*x^(1/2))^(1/2)*a^(5/
2)*b-16*(a*x+b*x^(1/2))^(3/2)*a^(5/2)-48*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(3
/2)*b^2+18*(a*x+b*x^(1/2))^(1/2)*a^(3/2)*b^2+24*a*ln(1/2*(2*a*x^(1/2)+b+2*(
(a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*b^3-9*ln(1/2*(2*a*x^(1/2)+b+
2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a*b^3)/((a*x^(1/2)+b)*x^(1/2))^(1
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(a*x + b*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^(1/2))^(1/2),x)
```

```
[Out] int(x/(a*x + b*x^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a*x + b*sqrt(x)), x)
```

3.62 $\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=56

$$\frac{2\sqrt{ax+b\sqrt{x}}}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2010, 2013, 620, 206}

$$\frac{2\sqrt{ax+b\sqrt{x}}}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (2*Sqrt[b*Sqrt[x] + a*x])/a - (2*b*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x]
 + a*x]])/a^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2010

```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[(-2*Sqrt
[a*x^j + b*x^n])/(b*(n - 2)*x^(n - 1)), x] - Dist[(a*(2*n - j - 2))/(b*(n -
 2)), Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] &&
LtQ[2*(n - 1), j, n]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx &= \frac{2\sqrt{b\sqrt{x}+ax}}{a}-\frac{b\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx}{2a}\\
&= \frac{2\sqrt{b\sqrt{x}+ax}}{a}-\frac{b\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}\\
&= \frac{2\sqrt{b\sqrt{x}+ax}}{a}-\frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a}\\
&= \frac{2\sqrt{b\sqrt{x}+ax}}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 88, normalized size = 1.57

$$
\frac{2\sqrt{a}\sqrt{x}\left(a\sqrt{x}+b\right)-2b^{3/2}\sqrt[4]{x}\sqrt{\frac{a\sqrt{x}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}}\right)}{a^{3/2}\sqrt{ax+b\sqrt{x}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (2*Sqrt[a]*(b + a*Sqrt[x])*Sqrt[x] - 2*b^(3/2)*Sqrt[1 + (a*Sqrt[x])/b]*x^(1
/4)*ArcSinh[(Sqrt[a]*x^(1/4))/Sqrt[b]])/(a^(3/2)*Sqrt[b*Sqrt[x] + a*x])
```

IntegrateAlgebraic [A] time = 0.20, size = 65, normalized size = 1.16

$$
\frac{b\log\left(-2a^{3/2}\sqrt{ax+b\sqrt{x}}+2a^2\sqrt{x}+ab\right)}{a^{3/2}}+\frac{2\sqrt{ax+b\sqrt{x}}}{a}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (2*Sqrt[b*Sqrt[x] + a*x])/a + (b*Log[a*b + 2*a^2*Sqrt[x] - 2*a^(3/2)*Sqrt[b
*Sqrt[x] + a*x]])/a^(3/2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.28, size = 54, normalized size = 0.96

$$
\frac{b\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{a^{\frac{3}{2}}}+\frac{2\sqrt{ax+b\sqrt{x}}}{a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] b*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(3/2
) + 2*sqrt(a*x + b*sqrt(x))/a
```

maple [A] time = 0.05, size = 83, normalized size = 1.48

$$\frac{\sqrt{ax+b\sqrt{x}}\,\left(-b\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}\right)}{\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x^(1/2))^(1/2),x)
```

```
[Out] (a*x+b*x^(1/2))^(1/2)*(2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2)-b*ln(1/2*(2*
a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2)))/((a*x^(1/2)+
b)*x^(1/2))^(1/2)/a^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*x + b*sqrt(x)), x)
```

mupad [B] time = 5.24, size = 72, normalized size = 1.29

$$\frac{4\,x\,\left(\frac{3\,\sqrt{b}\,\sqrt{b+a\,\sqrt{x}}}{2\,a\,\sqrt{x}}+\frac{b^{3/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,x^{1/4}\,1{}\mathrm{i}}{\sqrt{b}}\right)\,3{}\mathrm{i}}{2\,a^{3/2}\,x^{3/4}}\right)\,\sqrt{\frac{a\,\sqrt{x}}{b}+1}}{3\,\sqrt{a\,x+b\,\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^(1/2))^(1/2),x)
```

```
[Out] (4*x*((3*b^(1/2)*(b + a*x^(1/2))^(1/2))/(2*a*x^(1/2)) + (b^(3/2)*asin((a^(1
/2)*x^(1/4)*1i)/b^(1/2))*3i)/(2*a^(3/2)*x^(3/4)))*((a*x^(1/2))/b + 1)^(1/2)
)/(3*(a*x + b*x^(1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*sqrt(x)), x)
```

3.63 $\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=25

$$-\frac{4\sqrt{ax+b\sqrt{x}}}{b\sqrt{x}}$$

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$-\frac{4\sqrt{ax+b\sqrt{x}}}{b\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[b*Sqrt[x] + a*x]),x]
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(b*Sqrt[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{b\sqrt{x}}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{4\sqrt{ax+b\sqrt{x}}}{b\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[b*Sqrt[x] + a*x]),x]
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(b*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.14, size = 25, normalized size = 1.00

$$-\frac{4\sqrt{ax+b\sqrt{x}}}{b\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[b*Sqrt[x] + a*x]),x]
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(b*Sqrt[x])
```

fricas [A] time = 0.85, size = 19, normalized size = 0.76

$$-\frac{4 \sqrt{ax + b\sqrt{x}}}{b\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -4*sqrt(a*x + b*sqrt(x))/(b*sqrt(x))
```

giac [A] time = 0.18, size = 25, normalized size = 1.00

$$\frac{4}{\sqrt{a}\,\sqrt{x} - \sqrt{ax + b\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))
```

maple [C] time = 0.06, size = 159, normalized size = 6.36

$$\frac{\sqrt{ax + b\sqrt{x}}\,\left(-abx \ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right) + abx \ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right) + 2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{3}{2}}x + 2\sqrt{ax+b\sqrt{x}}\,a^{\frac{3}{2}}x - 4\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}\sqrt{a}\right)}{\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}\,b^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x+b*x^(1/2))^(1/2),x)
```

```
[Out] (a*x+b*x^(1/2))^(1/2)*(2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(3/2)*x-4*(a*x+b*x
^(1/2))^(3/2)*a^(1/2)+2*(a*x+b*x^(1/2))^(1/2)*a^(3/2)*x-ln(1/2*(2*a*x^(1/2)
+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x*a*b+ln(1/2*(2*a*x^(1
/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x*a*b)/((a*x^(1/2)+b)*x^(1/
2))^(1/2)/b^2/x/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + b\sqrt{x}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\,\sqrt{a\,x + b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x*(a*x + b*x^(1/2))^(1/2)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a*x + b*sqrt(x))), x)
```

3.64 $\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=84

$$-\frac{32a^2\sqrt{ax+b\sqrt{x}}}{15b^3\sqrt{x}}+\frac{16a\sqrt{ax+b\sqrt{x}}}{15b^2x}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}$$

Rubi [A] time = 0.12, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$-\frac{32a^2\sqrt{ax+b\sqrt{x}}}{15b^3\sqrt{x}}+\frac{16a\sqrt{ax+b\sqrt{x}}}{15b^2x}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(5*b*x^(3/2)) + (16*a*Sqrt[b*Sqrt[x] + a*x])/(15
*b^2*x) - (32*a^2*Sqrt[b*Sqrt[x] + a*x])/(15*b^3*Sqrt[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx &= -\frac{4\sqrt{b\sqrt{x}+ax}}{5bx^{3/2}}-\frac{(4a)\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{5b}\\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{5bx^{3/2}}+\frac{16a\sqrt{b\sqrt{x}+ax}}{15b^2x}+\frac{\left(8a^2\right)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{15b^2}\\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{5bx^{3/2}}+\frac{16a\sqrt{b\sqrt{x}+ax}}{15b^2x}-\frac{32a^2\sqrt{b\sqrt{x}+ax}}{15b^3\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 48, normalized size = 0.57

$$-\frac{4\sqrt{ax+b\sqrt{x}}\left(8a^2x-4ab\sqrt{x}+3b^2\right)}{15b^3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(3*b^2 - 4*a*b*Sqrt[x] + 8*a^2*x))/(15*b^3*x^(3/2
))
```

IntegrateAlgebraic [A] time = 0.17, size = 48, normalized size = 0.57

$$-\frac{4\sqrt{ax+b\sqrt{x}}\,\left(8a^2x-4ab\sqrt{x}+3b^2\right)}{15b^3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(3*b^2 - 4*a*b*Sqrt[x] + 8*a^2*x))/(15*b^3*x^(3/2
))
```

fricas [A] time = 0.90, size = 42, normalized size = 0.50

$$\frac{4\left(4\,abx-\left(8\,a^2x+3\,b^2\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{15\,b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 4/15*(4*a*b*x - (8*a^2*x + 3*b^2)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(b^3*x^2)
```

giac [A] time = 0.23, size = 84, normalized size = 1.00

$$\frac{4\left(20\,a\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^2+15\,\sqrt{a}\,b\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+3\,b^2\right)}{15\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(20*a*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^2 + 15*sqrt(a)*b*(sqrt
(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + 3*b^2)/(sqrt(a)*sqrt(x) - sqrt(a*x +
 b*sqrt(x)))^5
```

maple [C] time = 0.07, size = 218, normalized size = 2.60

$$\frac{\sqrt{ax+b\sqrt{x}}\,\left(-15a^3b\,x^{\frac{7}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+15a^3b\,x^{\frac{7}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+30\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{7}{2}}x^{\frac{7}{2}}+30\sqrt{ax+b\sqrt{x}}\,a^{\frac{7}{2}}x^{\frac{7}{2}}-60\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{5}{2}}x^{\frac{5}{2}}+28\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{3}{2}}b\,x^2-12\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}\sqrt{a}\,b^2x^{\frac{3}{2}}\right)}{15\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}\,b^4x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a*x+b*x^(1/2))^(1/2),x)
```

```
[Out] 1/15*(a*x+b*x^(1/2))^(1/2)*(30*x^(7/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(7/2
)-60*x^(5/2)*(a*x+b*x^(1/2))^(3/2)*a^(5/2)+30*x^(7/2)*(a*x+b*x^(1/2))^(1/2)
*a^(7/2)-15*x^(7/2)*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a
^(1/2))/a^(1/2))*a^3*b+15*x^(7/2)*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(
1/2)*a^(1/2))/a^(1/2))*a^3*b-12*x^(3/2)*(a*x+b*x^(1/2))^(3/2)*a^(1/2)*b^2+2
8*a^(3/2)*(a*x+b*x^(1/2))^(3/2)*b*x^2)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/b^4/x^
(7/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + b\sqrt{x}}\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\, \sqrt{a\, x + b\, \sqrt{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^2*(a*x + b*x^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{ax + b\sqrt{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a*x + b*sqrt(x))), x)
```

3.65 $\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=142

$$-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{315b^5\sqrt{x}}+\frac{256a^3\sqrt{ax+b\sqrt{x}}}{315b^4x}-\frac{64a^2\sqrt{ax+b\sqrt{x}}}{105b^3x^{3/2}}+\frac{32a\sqrt{ax+b\sqrt{x}}}{63b^2x^2}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}$$

Rubi [A] time = 0.20, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$-\frac{64a^2\sqrt{ax+b\sqrt{x}}}{105b^3x^{3/2}}-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{315b^5\sqrt{x}}+\frac{256a^3\sqrt{ax+b\sqrt{x}}}{315b^4x}+\frac{32a\sqrt{ax+b\sqrt{x}}}{63b^2x^2}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(9*b*x^(5/2)) + (32*a*Sqrt[b*Sqrt[x] + a*x])/(63
*b^2*x^2) - (64*a^2*Sqrt[b*Sqrt[x] + a*x])/(105*b^3*x^(3/2)) + (256*a^3*Sqr
t[b*Sqrt[x] + a*x])/(315*b^4*x) - (512*a^4*Sqrt[b*Sqrt[x] + a*x])/(315*b^5*
Sqrt[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}} - \frac{(8a)\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{9b}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}} + \frac{32a\sqrt{b\sqrt{x}+ax}}{63b^2x^2} + \frac{\left(16a^2\right)\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{21b^2}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}} + \frac{32a\sqrt{b\sqrt{x}+ax}}{63b^2x^2} - \frac{64a^2\sqrt{b\sqrt{x}+ax}}{105b^3x^{3/2}} - \frac{\left(64a^3\right)\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{105b^3}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}} + \frac{32a\sqrt{b\sqrt{x}+ax}}{63b^2x^2} - \frac{64a^2\sqrt{b\sqrt{x}+ax}}{105b^3x^{3/2}} + \frac{256a^3\sqrt{b\sqrt{x}+ax}}{315b^4x} + \frac{\left(128a^4\right)}{}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}} + \frac{32a\sqrt{b\sqrt{x}+ax}}{63b^2x^2} - \frac{64a^2\sqrt{b\sqrt{x}+ax}}{105b^3x^{3/2}} + \frac{256a^3\sqrt{b\sqrt{x}+ax}}{315b^4x} - \frac{512a^4\sqrt{}}{315}$$

Mathematica [A] time = 0.05, size = 72, normalized size = 0.51

$$-\frac{4\sqrt{ax+b\sqrt{x}}\left(128a^4x^2-64a^3bx^{3/2}+48a^2b^2x-40ab^3\sqrt{x}+35b^4\right)}{315b^5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(35*b^4 - 40*a*b^3*Sqrt[x] + 48*a^2*b^2*x - 64*a^
3*b*x^(3/2) + 128*a^4*x^2))/(315*b^5*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.20, size = 72, normalized size = 0.51

$$-\frac{4\sqrt{ax+b\sqrt{x}}\left(128a^4x^2-64a^3bx^{3/2}+48a^2b^2x-40ab^3\sqrt{x}+35b^4\right)}{315b^5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(35*b^4 - 40*a*b^3*Sqrt[x] + 48*a^2*b^2*x - 64*a^
3*b*x^(3/2) + 128*a^4*x^2))/(315*b^5*x^(5/2))
```

fricas [A] time = 0.70, size = 64, normalized size = 0.45

$$\frac{4\left(64\,a^3bx^2+40\,ab^3x-\left(128\,a^4x^2+48\,a^2b^2x+35\,b^4\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{315\,b^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 4/315*(64*a^3*b*x^2 + 40*a*b^3*x - (128*a^4*x^2 + 48*a^2*b^2*x + 35*b^4)*sq
rt(x))*sqrt(a*x + b*sqrt(x))/(b^5*x^3)
```

giac [A] time = 0.19, size = 146, normalized size = 1.03

$$\frac{4\left(1008\,a^2\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^4+1680\,a^{\frac{3}{2}}b\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^3+1080\,ab^2\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^2+315\,\sqrt{a}\,b^3\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+35\,b^4\right)}{315\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/315*(1008*a^2*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^4 + 1680*a^(3/2)*
b*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^3 + 1080*a*b^2*(sqrt(a)*sqrt(x)
 - sqrt(a*x + b*sqrt(x)))^2 + 315*sqrt(a)*b^3*(sqrt(a)*sqrt(x) - sqrt(a*x +
 b*sqrt(x))) + 35*b^4)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^9
```

maple [C] time = 0.06, size = 262, normalized size = 1.85

$$-\frac{\sqrt{ax+b\sqrt{x}}\left(315a^5bx^{\frac{11}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-315a^5bx^{\frac{11}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-630\sqrt{ax+b\sqrt{x}}\,a^{\frac{11}{2}}x^{\frac{11}{2}}-630\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{11}{2}}x^{\frac{11}{2}}+1260\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{9}{2}}x^{\frac{9}{2}}-748\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{7}{2}}bx^4+492\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{5}{2}}b^2x^{\frac{7}{2}}-300\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{3}{2}}b^3x^3+140\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}\sqrt{a}\,b^4x^{\frac{5}{2}}\right)}{315\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}\,b^6x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*x+b*x^(1/2))^(1/2),x)
```

```
[Out] -1/315*(a*x+b*x^(1/2))^(1/2)*(1260*(a*x+b*x^(1/2))^(3/2)*a^(9/2)*x^(9/2)-63
0*(a*x+b*x^(1/2))^(1/2)*a^(11/2)*x^(11/2)+315*ln(1/2*(2*a*x^(1/2)+b+2*((a*x
^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^(11/2)*a^5*b-630*a^(11/2)*x^(1
1/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)-315*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/
2))^(1/2)*a^(1/2))/a^(1/2))*x^(11/2)*a^5*b+492*(a*x+b*x^(1/2))^(3/2)*a^(5/2
)*x^(7/2)*b^2+140*(a*x+b*x^(1/2))^(3/2)*a^(1/2)*x^(5/2)*b^4-748*a^(7/2)*(a*
x+b*x^(1/2))^(3/2)*b*x^4-300*(a*x+b*x^(1/2))^(3/2)*a^(3/2)*x^3*b^3)/((a*x^(
1/2)+b)*x^(1/2))^(1/2)/b^6/x^(11/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+b\sqrt{x}}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x + b*x^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^3*(a*x + b*x^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a*x + b*sqrt(x))), x)
```

3.66 $\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=200

$$-\frac{4096a^6\sqrt{ax+b\sqrt{x}}}{3003b^7\sqrt{x}}+\frac{2048a^5\sqrt{ax+b\sqrt{x}}}{3003b^6x}-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{1001b^5x^{3/2}}+\frac{1280a^3\sqrt{ax+b\sqrt{x}}}{3003b^4x^2}-\frac{160a^2\sqrt{ax+b\sqrt{x}}}{429b^3x^{5/2}}+\frac{48a\sqrt{}}{1}$$

Rubi [A] time = 0.30, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{1001b^5x^{3/2}}+\frac{1280a^3\sqrt{ax+b\sqrt{x}}}{3003b^4x^2}-\frac{160a^2\sqrt{ax+b\sqrt{x}}}{429b^3x^{5/2}}-\frac{4096a^6\sqrt{ax+b\sqrt{x}}}{3003b^7\sqrt{x}}+\frac{2048a^5\sqrt{ax+b\sqrt{x}}}{3003b^6x}+\frac{48a\sqrt{ax+b\sqrt{x}}}{143b^2x^3}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(13*b*x^(7/2)) + (48*a*Sqrt[b*Sqrt[x] + a*x])/(1
43*b^2*x^3) - (160*a^2*Sqrt[b*Sqrt[x] + a*x])/(429*b^3*x^(5/2)) + (1280*a^3
*Sqrt[b*Sqrt[x] + a*x])/(3003*b^4*x^2) - (512*a^4*Sqrt[b*Sqrt[x] + a*x])/(1
001*b^5*x^(3/2)) + (2048*a^5*Sqrt[b*Sqrt[x] + a*x])/(3003*b^6*x) - (4096*a^
6*Sqrt[b*Sqrt[x] + a*x])/(3003*b^7*Sqrt[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} - \frac{(12a)\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx}{13b}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3} + \frac{\left(120a^2\right)\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx}{143b^2}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}} - \frac{\left(320a^3\right)\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{429b^3}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}} + \frac{1280a^3\sqrt{b\sqrt{x}+ax}}{3003b^4x^2} + \frac{(64}{}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}} + \frac{1280a^3\sqrt{b\sqrt{x}+ax}}{3003b^4x^2} - \frac{512}{}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}} + \frac{1280a^3\sqrt{b\sqrt{x}+ax}}{3003b^4x^2} - \frac{512}{}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}} + \frac{1280a^3\sqrt{b\sqrt{x}+ax}}{3003b^4x^2} - \frac{512}{}$$

Mathematica [A] time = 0.06, size = 96, normalized size = 0.48

$$-\frac{4\sqrt{ax+b\sqrt{x}}\left(1024a^6x^3-512a^5bx^{5/2}+384a^4b^2x^2-320a^3b^3x^{3/2}+280a^2b^4x-252ab^5\sqrt{x}+231b^6\right)}{3003b^7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(231*b^6 - 252*a*b^5*Sqrt[x] + 280*a^2*b^4*x - 32
0*a^3*b^3*x^(3/2) + 384*a^4*b^2*x^2 - 512*a^5*b*x^(5/2) + 1024*a^6*x^3))/(3
003*b^7*x^(7/2))
```

IntegrateAlgebraic [A] time = 0.23, size = 96, normalized size = 0.48

$$-\frac{4\sqrt{ax+b\sqrt{x}}\left(1024a^6x^3-512a^5bx^{5/2}+384a^4b^2x^2-320a^3b^3x^{3/2}+280a^2b^4x-252ab^5\sqrt{x}+231b^6\right)}{3003b^7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(231*b^6 - 252*a*b^5*Sqrt[x] + 280*a^2*b^4*x - 32
0*a^3*b^3*x^(3/2) + 384*a^4*b^2*x^2 - 512*a^5*b*x^(5/2) + 1024*a^6*x^3))/(3
003*b^7*x^(7/2))
```

fricas [A] time = 0.77, size = 86, normalized size = 0.43

$$\frac{4\left(512\,a^5bx^3+320\,a^3b^3x^2+252\,ab^5x-\left(1024\,a^6x^3+384\,a^4b^2x^2+280\,a^2b^4x+231\,b^6\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{3003\,b^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 4/3003*(512*a^5*b*x^3 + 320*a^3*b^3*x^2 + 252*a*b^5*x - (1024*a^6*x^3 + 384
*a^4*b^2*x^2 + 280*a^2*b^4*x + 231*b^6)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(b^7
*x^4)
```

giac [A] time = 0.20, size = 208, normalized size = 1.04

$$\frac{4\left(27456\,a^3\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^6+72072\,a^{\frac{5}{2}}b\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^5+80080\,a^2b^2\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^4+48048\,a^{\frac{3}{2}}b^3\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^3+16380\,ab^4\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^2+3003\,\sqrt{a}b^5\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+231\,b^6\right)}{3003\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/3003*(27456*a^3*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^6 + 72072*a^(5/
2)*b*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^5 + 80080*a^2*b^2*(sqrt(a)*s
qrt(x) - sqrt(a*x + b*sqrt(x)))^4 + 48048*a^(3/2)*b^3*(sqrt(a)*sqrt(x) - sq
rt(a*x + b*sqrt(x)))^3 + 16380*a*b^4*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x
)))^2 + 3003*sqrt(a)*b^5*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + 231*b^
6)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^13
```

maple [C] time = 0.07, size = 306, normalized size = 1.53

$$\frac{\sqrt{ax+b\sqrt{x}}\left(-3003a^7bx^{\frac{15}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+3003a^7bx^{\frac{15}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+6006\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{15}{2}}x^{\frac{15}{2}}+6006\sqrt{ax+b\sqrt{x}}\,a^{\frac{15}{2}}x^{\frac{15}{2}}-12012\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{13}{2}}x^{\frac{13}{2}}+7916\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{11}{2}}bx^6-5868\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{9}{2}}b^2x^{\frac{11}{2}}+4332\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{7}{2}}b^3x^5-3052\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{5}{2}}b^4x^{\frac{9}{2}}+1932\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{3}{2}}b^5x^4-924\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}\sqrt{a}\,b^6x^{\frac{7}{2}}\right)}{3003\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}\,b^8x^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a*x+b*x^(1/2))^(1/2),x)
```

```
[Out] 1/3003*(a*x+b*x^(1/2))^(1/2)*(6006*x^(15/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a
^(15/2)-12012*x^(13/2)*(a*x+b*x^(1/2))^(3/2)*a^(13/2)+6006*x^(15/2)*(a*x+b*
x^(1/2))^(1/2)*a^(15/2)-3003*x^(15/2)*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b
)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^7*b+3003*x^(15/2)*ln(1/2*(2*a*x^(1/2)+
b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^7*b-5868*x^(11/2)*(a*x+b*x^(1
/2))^(3/2)*a^(9/2)*b^2-3052*x^(9/2)*(a*x+b*x^(1/2))^(3/2)*a^(5/2)*b^4+7916*
a^(11/2)*(a*x+b*x^(1/2))^(3/2)*b*x^6-924*x^(7/2)*(a*x+b*x^(1/2))^(3/2)*a^(1
/2)*b^6+4332*x^5*(a*x+b*x^(1/2))^(3/2)*a^(7/2)*b^3+1932*(a*x+b*x^(1/2))^(3/
2)*a^(3/2)*x^4*b^5)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/b^8/x^(15/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+b\sqrt{x}}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a*x + b*x^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^4*(a*x + b*x^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**(1/2)+a*x)**(1/2),x)
[Out] Integral(1/(x**4*sqrt(a*x + b*sqrt(x))), x)
```

3.67 $\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=197

$$-\frac{693b^5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{64a^{13/2}}+\frac{693b^4\sqrt{ax+b\sqrt{x}}}{64a^6}-\frac{231b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}}{32a^5}+\frac{231b^2x\sqrt{ax+b\sqrt{x}}}{40a^4}-\frac{99bx^{3/2}\sqrt{ax+b\sqrt{x}}}{20a^3}$$

Rubi [A] time = 0.18, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 668, 670, 640, 620, 206}

$$\frac{693b^4\sqrt{ax+b\sqrt{x}}}{64a^6}-\frac{231b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}}{32a^5}+\frac{231b^2x\sqrt{ax+b\sqrt{x}}}{40a^4}-\frac{693b^5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{64a^{13/2}}-\frac{99bx^{3/2}\sqrt{ax+b\sqrt{x}}}{20a^3}+\frac{22x^2\sqrt{ax+b\sqrt{x}}}{5a^2}-\frac{4x^3}{a\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*x^3)/(a*Sqrt[b*Sqrt[x] + a*x]) + (693*b^4*Sqrt[b*Sqrt[x] + a*x])/(64*a^
6) - (231*b^3*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(32*a^5) + (231*b^2*x*Sqrt[b*S
qrt[x] + a*x])/(40*a^4) - (99*b*x^(3/2)*Sqrt[b*Sqrt[x] + a*x])/(20*a^3) + (
22*x^2*Sqrt[b*Sqrt[x] + a*x])/(5*a^2) - (693*b^5*ArcTanh[(Sqrt[a]*Sqrt[x])/
Sqrt[b*Sqrt[x] + a*x]])/(64*a^(13/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
```

```
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = 2\,\text{Subst}\left(\int \frac{x^7}{\left(bx+ax^2\right)^{3/2}}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} + \frac{22\,\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} + \frac{22x^2\sqrt{b\sqrt{x}+ax}}{5a^2} - \frac{(99b)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{5a^2}$$

$$= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} - \frac{99bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^3} + \frac{22x^2\sqrt{b\sqrt{x}+ax}}{5a^2} + \frac{\left(693b^2\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+a\ldots}}\right.}{40a^3}$$

$$= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} + \frac{231b^2x\sqrt{b\sqrt{x}+ax}}{40a^4} - \frac{99bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^3} + \frac{22x^2\sqrt{b\sqrt{x}+ax}}{5a^2} - \ldots$$

$$= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} - \frac{231b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{231b^2x\sqrt{b\sqrt{x}+ax}}{40a^4} - \frac{99bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^3} \ldots$$

$$= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} + \frac{693b^4\sqrt{b\sqrt{x}+ax}}{64a^6} - \frac{231b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{231b^2x\sqrt{b\sqrt{x}+ax}}{40a^4} \ldots$$

$$= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} + \frac{693b^4\sqrt{b\sqrt{x}+ax}}{64a^6} - \frac{231b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{231b^2x\sqrt{b\sqrt{x}+ax}}{40a^4} \ldots$$

$$= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} + \frac{693b^4\sqrt{b\sqrt{x}+ax}}{64a^6} - \frac{231b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{231b^2x\sqrt{b\sqrt{x}+ax}}{40a^4} \ldots$$

Mathematica [C] time = 0.06, size = 64, normalized size = 0.32

$$\frac{4x^{7/2}\sqrt{\frac{a\sqrt{x}}{b}+1}\;{}_2F_1\left(\frac{3}{2},\frac{13}{2};\frac{15}{2};-\frac{a\sqrt{x}}{b}\right)}{13b\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (4*Sqrt[1 + (a*Sqrt[x])/b]*x^(7/2)*Hypergeometric2F1[3/2, 13/2, 15/2, -((a*
Sqrt[x])/b)])/(13*b*Sqrt[b*Sqrt[x] + a*x])
```

IntegrateAlgebraic [A] time = 0.50, size = 137, normalized size = 0.70

$$\frac{693b^5\log\left(-2\sqrt{a}\sqrt{ax+b\sqrt{x}}+2a\sqrt{x}+b\right)}{128a^{13/2}}+\frac{\sqrt{ax+b\sqrt{x}}\left(128a^5x^{5/2}-176a^4bx^2+264a^3b^2x^{3/2}-462a^2b^3x+1155ab^4\sqrt{x}+3465b^5\right)}{320a^6\left(a\sqrt{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (Sqrt[b*Sqrt[x] + a*x]*(3465*b^5 + 1155*a*b^4*Sqrt[x] - 462*a^2*b^3*x + 264
*a^3*b^2*x^(3/2) - 176*a^4*b*x^2 + 128*a^5*x^(5/2)))/(320*a^6*(b + a*Sqrt[x
])) + (693*b^5*Log[b + 2*a*Sqrt[x] - 2*Sqrt[a]*Sqrt[b*Sqrt[x] + a*x]])/(128
*a^(13/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueDone
```

maple [B] time = 0.06, size = 549, normalized size = 2.79

$$\frac{\sqrt{ax+b\sqrt{x}}\left(-4480a^3b^5x\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+1015a^3b^5x\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-8960a^2b^6\sqrt{x}\,\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+2030a^2b^6\sqrt{x}\,\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-4060\sqrt{ax+b\sqrt{x}}\,a^{\frac{9}{2}}b^3x^{\frac{3}{2}}-4480ab^7\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+1015ab^7\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+256\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{13}{2}}x^2-10150\sqrt{ax+b\sqrt{x}}\,a^{\frac{7}{2}}b^4x+8960\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{7}{2}}b^4x-352\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{11}{2}}bx^{\frac{3}{2}}-8120\sqrt{ax+b\sqrt{x}}\,a^{\frac{5}{2}}b^5\sqrt{x}+17920\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{5}{2}}b^5\sqrt{x}+528\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{9}{2}}b^2x-2030\sqrt{ax+b\sqrt{x}}\,a^{\frac{3}{2}}b^6+8960\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{3}{2}}b^6+3136\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{7}{2}}b^3\sqrt{x}+2000\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{5}{2}}b^4-2560\left(\left(a\sqrt{x}+b\right)\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{5}{2}}b^4\right)}{640\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\left(a\sqrt{x}+b\right)^2a^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] 1/640*(a*x+b*x^(1/2))^(1/2)*(256*(a*x+b*x^(1/2))^(3/2)*a^(13/2)*x^2-352*(a*
x+b*x^(1/2))^(3/2)*a^(11/2)*x^(3/2)*b-4060*(a*x+b*x^(1/2))^(1/2)*a^(9/2)*x^
(3/2)*b^3+528*(a*x+b*x^(1/2))^(3/2)*a^(9/2)*x*b^2+3136*(a*x+b*x^(1/2))^(3/2
)*a^(7/2)*x^(1/2)*b^3-10150*(a*x+b*x^(1/2))^(1/2)*a^(7/2)*x*b^4+8960*a^(7/2
)*x*((a*x^(1/2)+b)*x^(1/2))^(1/2)*b^4+2000*(a*x+b*x^(1/2))^(3/2)*a^(5/2)*b^
4-8120*(a*x+b*x^(1/2))^(1/2)*a^(5/2)*x^(1/2)*b^5+17920*a^(5/2)*x^(1/2)*((a*
x^(1/2)+b)*x^(1/2))^(1/2)*b^5-2560*a^(5/2)*((a*x^(1/2)+b)*x^(1/2))^(3/2)*b^
4-2030*(a*x+b*x^(1/2))^(1/2)*a^(3/2)*b^6+8960*a^(3/2)*((a*x^(1/2)+b)*x^(1/2
))^(1/2)*b^6+2030*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^
(1/2))*x^(1/2)*a^2*b^6+1015*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a
^(1/2))/a^(1/2))*x*a^3*b^5-8960*a^2*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*
x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^(1/2)*b^6-4480*a^3*ln(1/2*(2*a*x^(1/2)+b
+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x*b^5+1015*ln(1/2*(2*a*x
^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a*b^7-4480*a*ln(1/2*(2*a
```

```
*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*b^7)/a^(15/2)/
((a*x^(1/2)+b)*x^(1/2))^(1/2)/(a*x^(1/2)+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x^3/(a*x + b*sqrt(x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(a\,x + b\,\sqrt{x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^(1/2))^(3/2),x)
[Out] int(x^3/(a*x + b*x^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**(1/2)+a*x)**(3/2),x)
[Out] Integral(x**3/(a*x + b*sqrt(x))**(3/2), x)
```

3.68 $\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=139

$$-\frac{35b^3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{4a^{9/2}}+\frac{35b^2\sqrt{ax+b\sqrt{x}}}{4a^4}-\frac{35b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a^3}+\frac{14x\sqrt{ax+b\sqrt{x}}}{3a^2}-\frac{4x^2}{a\sqrt{ax+b\sqrt{x}}}$$

Rubi [A] time = 0.13, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2018, 668, 670, 640, 620, 206}

$$\frac{35b^2\sqrt{ax+b\sqrt{x}}}{4a^4}-\frac{35b^3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{4a^{9/2}}-\frac{35b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a^3}+\frac{14x\sqrt{ax+b\sqrt{x}}}{3a^2}-\frac{4x^2}{a\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*x^2)/(a*Sqrt[b*Sqrt[x] + a*x]) + (35*b^2*Sqrt[b*Sqrt[x] + a*x])/(4*a^4)
 - (35*b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(6*a^3) + (14*x*Sqrt[b*Sqrt[x] + a*
x])/(3*a^2) - (35*b^3*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(4*
a^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
```

```
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = 2\,\text{Subst}\left(\int \frac{x^5}{\left(bx+ax^2\right)^{3/2}}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} + \frac{14\,\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} + \frac{14x\sqrt{b\sqrt{x}+ax}}{3a^2} - \frac{(35b)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{3a^2}$$

$$= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} - \frac{35b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^3} + \frac{14x\sqrt{b\sqrt{x}+ax}}{3a^2} + \frac{\left(35b^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\right.}{4a^3}$$

$$= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} + \frac{35b^2\sqrt{b\sqrt{x}+ax}}{4a^4} - \frac{35b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^3} + \frac{14x\sqrt{b\sqrt{x}+ax}}{3a^2} - \frac{(35b}{}$$

$$= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} + \frac{35b^2\sqrt{b\sqrt{x}+ax}}{4a^4} - \frac{35b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^3} + \frac{14x\sqrt{b\sqrt{x}+ax}}{3a^2} - \frac{(35b}{}$$

$$= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} + \frac{35b^2\sqrt{b\sqrt{x}+ax}}{4a^4} - \frac{35b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^3} + \frac{14x\sqrt{b\sqrt{x}+ax}}{3a^2} - \frac{35b^3}{}$$

Mathematica [C] time = 0.08, size = 64, normalized size = 0.46

$$\frac{4x^{5/2}\sqrt{\frac{a\sqrt{x}}{b}+1}\;{}_2F_1\left(\frac{3}{2},\frac{9}{2};\frac{11}{2};-\frac{a\sqrt{x}}{b}\right)}{9b\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (4*Sqrt[1 + (a*Sqrt[x])/b]*x^(5/2)*Hypergeometric2F1[3/2, 9/2, 11/2, -((a*S
qrt[x])/b)])/(9*b*Sqrt[b*Sqrt[x] + a*x])
```

IntegrateAlgebraic [A] time = 0.52, size = 119, normalized size = 0.86

$$\frac{35b^3\log\left(-2a^{9/2}\sqrt{ax+b\sqrt{x}}+2a^5\sqrt{x}+a^4b\right)}{8a^{9/2}}+\frac{\sqrt{ax+b\sqrt{x}}\left(8a^3x^{3/2}-14a^2bx+35ab^2\sqrt{x}+105b^3\right)}{12a^4\left(a\sqrt{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (Sqrt[b*Sqrt[x] + a*x]*(105*b^3 + 35*a*b^2*Sqrt[x] - 14*a^2*b*x + 8*a^3*x^(
3/2)))/(12*a^4*(b + a*Sqrt[x])) + (35*b^3*Log[a^4*b + 2*a^5*Sqrt[x] - 2*a^(
9/2)*Sqrt[b*Sqrt[x] + a*x]])/(8*a^(9/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueDone
```

maple [B] time = 0.06, size = 503, normalized size = 3.62

$$\frac{\sqrt{ax+b\sqrt{x}}\left(-120a^3b^3x\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+15a^3b^3x\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-240a^2b^4\sqrt{x}\,\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+30a^2b^4\sqrt{x}\,\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-60\sqrt{ax+b\sqrt{x}}\,a^{\frac{9}{2}}bx^{\frac{3}{2}}-120ab^5\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+15ab^5\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-150\sqrt{ax+b\sqrt{x}}\,a^{\frac{7}{2}}b^2x+240\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{7}{2}}b^2x-120\sqrt{ax+b\sqrt{x}}\,a^{\frac{5}{2}}b^3\sqrt{x}+480\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{5}{2}}b^3\sqrt{x}+16\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{9}{2}}x-30\sqrt{ax+b\sqrt{x}}\,a^{\frac{3}{2}}b^4+240\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{3}{2}}b^4+32\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{7}{2}}b\sqrt{x}+16\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{5}{2}}b^2-96\left(\left(a\sqrt{x}+b\right)\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{5}{2}}b^2\right)}{24\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\left(a\sqrt{x}+b\right)^2a^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] 1/24*(a*x+b*x^(1/2))^(1/2)/a^(11/2)*(16*(a*x+b*x^(1/2))^(3/2)*a^(9/2)*x-60*
(a*x+b*x^(1/2))^(1/2)*a^(9/2)*x^(3/2)*b-120*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(
1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^3*x*b^3+32*(a*x+b*x^(1/2))^(3/2)
*a^(7/2)*b*x^(1/2)-150*(a*x+b*x^(1/2))^(1/2)*a^(7/2)*x*b^2+240*a^(7/2)*x*((
a*x^(1/2)+b)*x^(1/2))^(1/2)*b^2+15*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^
(1/2)*a^(1/2))/a^(1/2))*x*a^3*b^3-240*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b
)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^2*x^(1/2)*b^4+16*(a*x+b*x^(1/2))^(3/2)
*a^(5/2)*b^2-120*(a*x+b*x^(1/2))^(1/2)*a^(5/2)*b^3*x^(1/2)+480*a^(5/2)*x^(1
/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*b^3-96*a^(5/2)*((a*x^(1/2)+b)*x^(1/2))^(3
/2)*b^2+30*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*
x^(1/2)*a^2*b^4-120*a*b^5*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(
1/2)*a^(1/2))/a^(1/2))-30*(a*x+b*x^(1/2))^(1/2)*a^(3/2)*b^4+240*((a*x^(1/2)
+b)*x^(1/2))^(1/2)*a^(3/2)*b^4+15*a*b^5*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1
/2))^(1/2)*a^(1/2))/a^(1/2)))/((a*x^(1/2)+b)*x^(1/2))^(1/2)/(a*x^(1/2)+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x^2/(a*x + b*sqrt(x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^(1/2))^(3/2),x)
[Out] int(x^2/(a*x + b*x^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**(1/2)+a*x)**(3/2),x)
[Out] Integral(x**2/(a*x + b*sqrt(x))**(3/2), x)
```

3.69 $\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{6b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{5/2}}+\frac{6\sqrt{ax+b\sqrt{x}}}{a^2}-\frac{4x}{a\sqrt{ax+b\sqrt{x}}}$$

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2018, 668, 640, 620, 206}

$$\frac{6\sqrt{ax+b\sqrt{x}}}{a^2}-\frac{6b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{5/2}}-\frac{4x}{a\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*x)/(a*Sqrt[b*Sqrt[x] + a*x]) + (6*Sqrt[b*Sqrt[x] + a*x])/a^2 - (6*b*Arc
Tanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/a^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^3}{\left(bx+ax^2\right)^{3/2}}\,dx,x,\sqrt{x}\right)\\
&= -\frac{4x}{a\sqrt{b\sqrt{x}+ax}}+\frac{6\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{a}\\
&= -\frac{4x}{a\sqrt{b\sqrt{x}+ax}}+\frac{6\sqrt{b\sqrt{x}+ax}}{a^2}-\frac{(3b)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{a^2}\\
&= -\frac{4x}{a\sqrt{b\sqrt{x}+ax}}+\frac{6\sqrt{b\sqrt{x}+ax}}{a^2}-\frac{(6b)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^2}\\
&= -\frac{4x}{a\sqrt{b\sqrt{x}+ax}}+\frac{6\sqrt{b\sqrt{x}+ax}}{a^2}-\frac{6b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 64, normalized size = 0.83

$$
\frac{4x^{3/2}\sqrt{\frac{a\sqrt{x}}{b}+1}\;{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};-\frac{a\sqrt{x}}{b}\right)}{5b\sqrt{ax+b\sqrt{x}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (4*Sqrt[1 + (a*Sqrt[x])/b]*x^(3/2)*Hypergeometric2F1[3/2, 5/2, 7/2, -((a*Sq
rt[x])/b)])/(5*b*Sqrt[b*Sqrt[x] + a*x])
```

IntegrateAlgebraic [A] time = 0.39, size = 90, normalized size = 1.17

$$
\frac{2\sqrt{ax+b\sqrt{x}}\left(a\sqrt{x}+3b\right)}{a^2\left(a\sqrt{x}+b\right)}+\frac{3b\log\left(-2a^{5/2}\sqrt{ax+b\sqrt{x}}+2a^3\sqrt{x}+a^2b\right)}{a^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (2*(3*b + a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(a^2*(b + a*Sqrt[x])) + (3*b*Lo
g[a^2*b + 2*a^3*Sqrt[x] - 2*a^(5/2)*Sqrt[b*Sqrt[x] + a*x]])/a^(5/2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.32, size = 94, normalized size = 1.22

$$\frac{3\,b\log\left(\left|-2\,\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{a^{\frac{5}{2}}}+\frac{4\,b^2}{\left(a\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+\sqrt{a}\,b\right)a^2}+\frac{2\,\sqrt{ax+b\sqrt{x}}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 3*b*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(5
/2) + 4*b^2/((a*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + sqrt(a)*b)*a^2)
 + 2*sqrt(a*x + b*sqrt(x))/a^2
```

maple [B] time = 0.05, size = 236, normalized size = 3.06

$$\frac{\sqrt{ax+b\sqrt{x}}\,\left(-3a^2bx\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-6a\,b^2\sqrt{x}\,\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-3b^3\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+6\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{5}{2}}x+12\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{3}{2}}b\sqrt{x}+6\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}\,b^2-4\left(\left(a\sqrt{x}+b\right)\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{3}{2}}\right)}{\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\left(a\sqrt{x}+b\right)^2a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] (a*x+b*x^(1/2))^(1/2)/a^(5/2)*(6*x*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(5/2)-3*
x*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a
^2*b+12*x^(1/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(3/2)*b-6*x^(1/2)*ln(1/2*(2
*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a*b^2-4*a^(3
/2)*((a*x^(1/2)+b)*x^(1/2))^(3/2)+6*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2)*b
^2-3*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2)
)*b^3)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/(a*x^(1/2)+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(a*x + b*sqrt(x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x}{{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^(1/2))^(3/2),x)
```

```
[Out] int(x/(a*x + b*x^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(x/(a*x + b*sqrt(x))**(3/2), x)
```

3.70 $\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\frac{4\sqrt{x}}{b\sqrt{ax+b\sqrt{x}}}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2000}

$$\frac{4\sqrt{x}}{b\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sqrt[x] + a*x)^(-3/2),x]
```

```
[Out] (4*Sqrt[x])/(b*Sqrt[b*Sqrt[x] + a*x])
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{4\sqrt{x}}{b\sqrt{b\sqrt{x}+ax}}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{4\sqrt{x}}{b\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sqrt[x] + a*x)^(-3/2),x]
```

```
[Out] (4*Sqrt[x])/(b*Sqrt[b*Sqrt[x] + a*x])
```

IntegrateAlgebraic [A] time = 0.27, size = 31, normalized size = 1.24

$$\frac{4\sqrt{ax+b\sqrt{x}}}{b\left(a\sqrt{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*Sqrt[x] + a*x)^(-3/2),x]
```

```
[Out] (4*Sqrt[b*Sqrt[x] + a*x])/(b*(b + a*Sqrt[x]))
```

fricas [A] time = 0.77, size = 36, normalized size = 1.44

$$\frac{4\sqrt{ax+b\sqrt{x}}\left(a\sqrt{x}-b\right)}{a^{2}bx-b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 4*sqrt(a*x + b*sqrt(x))*(a*sqrt(x) - b)/(a^2*b*x - b^3)
```

giac [A] time = 0.19, size = 34, normalized size = 1.36

$$\frac{4}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 4/((sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)*sqrt(a))
```

maple [C] time = 0.06, size = 404, normalized size = 16.16

$$\frac{\sqrt{ax+b\sqrt{x}}\left(-a^{2}bx\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\sqrt{a}}{2\sqrt{a}}\right)+a^{2}bx\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\sqrt{a}}{2\sqrt{a}}\right)-2ab^{2}\sqrt{x}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\sqrt{a}}{2\sqrt{a}}\right)+2ab^{2}\sqrt{x}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\sqrt{a}}{2\sqrt{a}}\right)-b^{3}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\sqrt{a}}{2\sqrt{a}}\right)+b^{3}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\sqrt{a}}{2\sqrt{a}}\right)+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{5}{2}}x+2\sqrt{ax+b\sqrt{x}}\,a^{\frac{5}{2}}x+4\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{3}{2}}b\sqrt{x}+4\sqrt{ax+b\sqrt{x}}\,a^{\frac{3}{2}}b\sqrt{x}+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\sqrt{a}\,b^{2}+2\sqrt{ax+b\sqrt{x}}\sqrt{a}\,b^{2}-4\left(\left(a\sqrt{x}+b\right)\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{3}{2}}\right)}{\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\left(a\sqrt{x}+b\right)^{2}\sqrt{a}\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] (a*x+b*x^(1/2))^(1/2)*(2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(5/2)*x+2*x*(a*x+b
*x^(1/2))^(1/2)*a^(5/2)-a^2*b*x*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1
/2))^(1/2)*a^(1/2))/a^(1/2))+x*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2
)*a^(1/2))/a^(1/2))*a^2*b+4*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(3/2)*b*x^(1/2)
+4*x^(1/2)*(a*x+b*x^(1/2))^(1/2)*a^(3/2)*b-2*a*b^2*x^(1/2)*ln(1/2*(2*a*x^(1
/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))+2*x^(1/2)*ln(1/2*(2
*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a*b^2-4*((a*x^(1/2)+
b)*x^(1/2))^(3/2)*a^(3/2)+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2)*b^2+2*(a*
x+b*x^(1/2))^(1/2)*a^(1/2)*b^2-b^3*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x
^(1/2))^(1/2)*a^(1/2))/a^(1/2))+ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/
2)*a^(1/2))/a^(1/2))*b^3)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/b^2/(a*x^(1/2)+b)^2
/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*sqrt(x))^(-3/2), x)
```

mupad [B] time = 5.43, size = 40, normalized size = 1.60

$$-\frac{4\,x\,\left(\frac{b}{a\,\sqrt{x}}+1\right)}{{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}\,\left(\sqrt{\frac{b}{a\,\sqrt{x}}+1}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^(1/2))^(3/2),x)
```

```
[Out] -(4*x*(b/(a*x^(1/2)) + 1))/((a*x + b*x^(1/2))^(3/2)*((b/(a*x^(1/2)) + 1)^(1
/2) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral((a*x + b*sqrt(x))**(-3/2), x)
```

3.71 $\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{32a\sqrt{ax+b\sqrt{x}}}{3b^3\sqrt{x}}-\frac{16\sqrt{ax+b\sqrt{x}}}{3b^2x}+\frac{4}{b\sqrt{x}\sqrt{ax+b\sqrt{x}}}$$

Rubi [A] time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 2014}

$$\frac{32a\sqrt{ax+b\sqrt{x}}}{3b^3\sqrt{x}}-\frac{16\sqrt{ax+b\sqrt{x}}}{3b^2x}+\frac{4}{b\sqrt{x}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]) - (16*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*x)
+ (32*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^3*Sqrt[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{b\sqrt{x}\sqrt{b\sqrt{x}+ax}} + \frac{4\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{b}$$

$$= \frac{4}{b\sqrt{x}\sqrt{b\sqrt{x}+ax}} - \frac{16\sqrt{b\sqrt{x}+ax}}{3b^2x} - \frac{(8a)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{3b^2}$$

$$= \frac{4}{b\sqrt{x}\sqrt{b\sqrt{x}+ax}} - \frac{16\sqrt{b\sqrt{x}+ax}}{3b^2x} + \frac{32a\sqrt{b\sqrt{x}+ax}}{3b^3\sqrt{x}}$$

Mathematica [A] time = 0.08, size = 48, normalized size = 0.61

$$\frac{4\left(8a^2x+4ab\sqrt{x}-b^2\right)}{3b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (4*(-b^2 + 4*a*b*Sqrt[x] + 8*a^2*x))/(3*b^3*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])
```

IntegrateAlgebraic [A] time = 0.30, size = 57, normalized size = 0.72

$$\frac{4\sqrt{ax+b\sqrt{x}}\left(8a^2x+4ab\sqrt{x}-b^2\right)}{3b^3x\left(a\sqrt{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (4*Sqrt[b*Sqrt[x] + a*x]*(-b^2 + 4*a*b*Sqrt[x] + 8*a^2*x))/(3*b^3*(b + a*Sq
rt[x])*x)
```

fricas [A] time = 0.82, size = 63, normalized size = 0.80

$$-\frac{4\left(4\,a^2bx-b^3-\left(8\,a^3x-5\,ab^2\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{3\left(a^2b^3x^2-b^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -4/3*(4*a^2*b*x - b^3 - (8*a^3*x - 5*a*b^2)*sqrt(x))*sqrt(a*x + b*sqrt(x))/
(a^2*b^3*x^2 - b^5*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x), x)
```

maple [C] time = 0.07, size = 524, normalized size = 6.63

$$\frac{\sqrt{ax+b\sqrt{x}}\left(3a^4bx^{\frac{7}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-3a^4bx^{\frac{7}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+6a^3b^2x^3\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-6a^3b^2x^3\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+3a^2b^3x^{\frac{5}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-3a^2b^3x^{\frac{5}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-6\sqrt{ax+b\sqrt{x}}\,a^{\frac{9}{2}}x^{\frac{7}{2}}-6\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{9}{2}}x^{\frac{7}{2}}-12\sqrt{ax+b\sqrt{x}}\,a^{\frac{7}{2}}bx^3-12\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{7}{2}}bx^3-6\sqrt{ax+b\sqrt{x}}\,a^{\frac{5}{2}}b^2x^{\frac{5}{2}}-6\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{5}{2}}b^2x^{\frac{5}{2}}+24\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{7}{2}}x^{\frac{5}{2}}-12\left(\left(a\sqrt{x}+b\right)\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{7}{2}}x^{\frac{5}{2}}+44\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{5}{2}}bx^2+16\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{3}{2}}b^2x^{\frac{3}{2}}-4\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}\sqrt{a}\,b^3x\right)}{3\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\left(a\sqrt{x}+b\right)^2\sqrt{a}\,b^4x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] 1/3*(a*x+b*x^(1/2))^(1/2)*(24*(a*x+b*x^(1/2))^(3/2)*a^(7/2)*x^(5/2)-6*(a*x+
b*x^(1/2))^(1/2)*a^(9/2)*x^(7/2)+3*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x
^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^(7/2)*a^4*b-6*a^(9/2)*x^(7/2)*((a*x^(1/2)
+b)*x^(1/2))^(1/2)-3*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))
/a^(1/2))*x^(7/2)*a^4*b+44*(a*x+b*x^(1/2))^(3/2)*a^(5/2)*x^2*b-12*(a*x+b*x^
(1/2))^(1/2)*a^(7/2)*x^3*b+6*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2)
)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^3*b^2-12*a^(7/2)*x^3*((a*x^(1/2)+b)*x^(1/2)
)^(1/2)*b-12*a^(7/2)*x^(5/2)*((a*x^(1/2)+b)*x^(1/2))^(3/2)-6*ln(1/2*(2*a*x^
(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^3*a^3*b^2+16*(a*x+b*x^(
1/2))^(3/2)*a^(3/2)*x^(3/2)*b^2-6*(a*x+b*x^(1/2))^(1/2)*a^(5/2)*x^(5/2)*b^2
+3*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*
x^(5/2)*a^2*b^3-6*a^(5/2)*x^(5/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*b^2-3*ln(1/
2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^(5/2)*a^2*b^3-
4*(a*x+b*x^(1/2))^(3/2)*a^(1/2)*x*b^3)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/b^4/a^
(1/2)/x^(5/2)/(a*x^(1/2)+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^(1/2))^(3/2)),x)
```

```
[Out] int(1/(x*(a*x + b*x^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x*(a*x + b*sqrt(x))**(3/2)), x)
```

3.72 $\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{512a^3\sqrt{ax+b\sqrt{x}}}{35b^5\sqrt{x}}-\frac{256a^2\sqrt{ax+b\sqrt{x}}}{35b^4x}+\frac{192a\sqrt{ax+b\sqrt{x}}}{35b^3x^{3/2}}-\frac{32\sqrt{ax+b\sqrt{x}}}{7b^2x^2}+\frac{4}{bx^{3/2}\sqrt{ax+b\sqrt{x}}}$$

Rubi [A] time = 0.20, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 2014}

$$\frac{512a^3\sqrt{ax+b\sqrt{x}}}{35b^5\sqrt{x}}-\frac{256a^2\sqrt{ax+b\sqrt{x}}}{35b^4x}+\frac{192a\sqrt{ax+b\sqrt{x}}}{35b^3x^{3/2}}-\frac{32\sqrt{ax+b\sqrt{x}}}{7b^2x^2}+\frac{4}{bx^{3/2}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*x^(3/2)*Sqrt[b*Sqrt[x] + a*x]) - (32*Sqrt[b*Sqrt[x] + a*x])/(7*b^2*x^2
) + (192*a*Sqrt[b*Sqrt[x] + a*x])/(35*b^3*x^(3/2)) - (256*a^2*Sqrt[b*Sqrt[x
] + a*x])/(35*b^4*x) + (512*a^3*Sqrt[b*Sqrt[x] + a*x])/(35*b^5*Sqrt[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}} + \frac{8\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{b}$$

$$= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}} - \frac{32\sqrt{b\sqrt{x}+ax}}{7b^2x^2} - \frac{(48a)\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{7b^2}$$

$$= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}} - \frac{32\sqrt{b\sqrt{x}+ax}}{7b^2x^2} + \frac{192a\sqrt{b\sqrt{x}+ax}}{35b^3x^{3/2}} + \frac{\left(192a^2\right)\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{35b^3}$$

$$= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}} - \frac{32\sqrt{b\sqrt{x}+ax}}{7b^2x^2} + \frac{192a\sqrt{b\sqrt{x}+ax}}{35b^3x^{3/2}} - \frac{256a^2\sqrt{b\sqrt{x}+ax}}{35b^4x} - \frac{(128a}{}$$

$$= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}} - \frac{32\sqrt{b\sqrt{x}+ax}}{7b^2x^2} + \frac{192a\sqrt{b\sqrt{x}+ax}}{35b^3x^{3/2}} - \frac{256a^2\sqrt{b\sqrt{x}+ax}}{35b^4x} + \frac{512a^3}{}$$

Mathematica [A] time = 0.08, size = 72, normalized size = 0.53

$$\frac{4\left(128a^4x^2+64a^3bx^{3/2}-16a^2b^2x+8ab^3\sqrt{x}-5b^4\right)}{35b^5x^{3/2}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (4*(-5*b^4 + 8*a*b^3*Sqrt[x] - 16*a^2*b^2*x + 64*a^3*b*x^(3/2) + 128*a^4*x^
2))/(35*b^5*x^(3/2)*Sqrt[b*Sqrt[x] + a*x])
```

IntegrateAlgebraic [A] time = 0.33, size = 81, normalized size = 0.59

$$\frac{4\sqrt{ax+b\sqrt{x}}\,\left(128a^4x^2+64a^3bx^{3/2}-16a^2b^2x+8ab^3\sqrt{x}-5b^4\right)}{35b^5x^2\left(a\sqrt{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (4*Sqrt[b*Sqrt[x] + a*x]*(-5*b^4 + 8*a*b^3*Sqrt[x] - 16*a^2*b^2*x + 64*a^3*
b*x^(3/2) + 128*a^4*x^2))/(35*b^5*(b + a*Sqrt[x])*x^2)
```

fricas [A] time = 1.16, size = 87, normalized size = 0.64

$$-\frac{4\left(64\,a^4bx^2-24\,a^2b^3x-5\,b^5-\left(128\,a^5x^2-80\,a^3b^2x-13\,ab^4\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{35\left(a^2b^5x^3-b^7x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -4/35*(64*a^4*b*x^2 - 24*a^2*b^3*x - 5*b^5 - (128*a^5*x^2 - 80*a^3*b^2*x -
13*a*b^4)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^5*x^3 - b^7*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^2), x)
```

maple [C] time = 0.06, size = 570, normalized size = 4.16

$$-\frac{\sqrt{ax+b\sqrt{x}}\left(-105 a^{6} b x^{\frac{11}{2}} \ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+105 a^{6} b x^{\frac{11}{2}} \ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-210 a^{5} b^{2} x^{5} \ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+210 a^{5} b^{2} x^{5} \ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-105 a^{4} b^{3} x^{\frac{9}{2}} \ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+105 a^{4} b^{3} x^{\frac{9}{2}} \ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+210\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\, a^{\frac{13}{2}} x^{\frac{11}{2}}+210\sqrt{ax+b\sqrt{x}}\, a^{\frac{13}{2}} x^{\frac{11}{2}}+420\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\, a^{\frac{11}{2}} b x^{5}+420\sqrt{ax+b\sqrt{x}}\, a^{\frac{11}{2}} b x^{5}+210\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\, a^{\frac{9}{2}} b^{2} x^{\frac{9}{2}}+210\sqrt{ax+b\sqrt{x}}\, a^{\frac{9}{2}} b^{2} x^{\frac{9}{2}}-560\left(ax+b\sqrt{x}\right)^{\frac{3}{2}} a^{\frac{11}{2}} x^{\frac{9}{2}}+140\left(\left(a\sqrt{x}+b\right)\sqrt{x}\right)^{\frac{3}{2}} a^{\frac{11}{2}} x^{\frac{9}{2}}-932\left(ax+b\sqrt{x}\right)^{\frac{3}{2}} a^{\frac{9}{2}} b x^{4}-256\left(ax+b\sqrt{x}\right)^{\frac{3}{2}} a^{\frac{7}{2}} b^{2} x^{\frac{7}{2}}+64\left(ax+b\sqrt{x}\right)^{\frac{3}{2}} a^{\frac{5}{2}} b^{3} x^{3}-32\left(ax+b\sqrt{x}\right)^{\frac{3}{2}} a^{\frac{3}{2}} b^{4} x^{\frac{5}{2}}+20\left(ax+b\sqrt{x}\right)^{\frac{3}{2}} \sqrt{a}\, b^{5} x^{2}\right)}{35\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\left(a\sqrt{x}+b\right)^{2}\sqrt{a}\, b^{6} x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] -1/35*(a*x+b*x^(1/2))^(1/2)*(210*x^(11/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(
13/2)-560*x^(9/2)*(a*x+b*x^(1/2))^(3/2)*a^(11/2)+210*x^(11/2)*(a*x+b*x^(1/2
))^(1/2)*a^(13/2)-105*x^(11/2)*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/
2))^(1/2)*a^(1/2))/a^(1/2))*a^6*b+105*x^(11/2)*ln(1/2*(2*a*x^(1/2)+b+2*(a*x
+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^6*b-256*x^(7/2)*(a*x+b*x^(1/2))^(3/2)
*a^(7/2)*b^2+420*x^5*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(11/2)*b-932*x^4*(a*x+
b*x^(1/2))^(3/2)*a^(9/2)*b+420*x^5*(a*x+b*x^(1/2))^(1/2)*a^(11/2)*b-210*x^5
*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^
5*b^2+210*x^5*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2
))*a^5*b^2+140*x^(9/2)*((a*x^(1/2)+b)*x^(1/2))^(3/2)*a^(11/2)+64*x^3*(a*x+b
*x^(1/2))^(3/2)*a^(5/2)*b^3+210*x^(9/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(9/
2)*b^2+210*x^(9/2)*(a*x+b*x^(1/2))^(1/2)*a^(9/2)*b^2-105*x^(9/2)*ln(1/2*(2*
a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^4*b^3+105*x
^(9/2)*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^4*
b^3-32*x^(5/2)*(a*x+b*x^(1/2))^(3/2)*a^(3/2)*b^4+20*(a*x+b*x^(1/2))^(3/2)*a
^(1/2)*x^2*b^5)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/b^6/x^(9/2)/a^(1/2)/(a*x^(1/2
)+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^(1/2))^(3/2)),x)
```

```
[Out] int(1/(x^2*(a*x + b*x^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(ax + b\sqrt{x}\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a*x + b*sqrt(x))**(3/2)), x)
```

3.73 $\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=195

$$\frac{4096a^5\sqrt{ax+b\sqrt{x}}}{231b^7\sqrt{x}}-\frac{2048a^4\sqrt{ax+b\sqrt{x}}}{231b^6x}+\frac{512a^3\sqrt{ax+b\sqrt{x}}}{77b^5x^{3/2}}-\frac{1280a^2\sqrt{ax+b\sqrt{x}}}{231b^4x^2}+\frac{160a\sqrt{ax+b\sqrt{x}}}{33b^3x^{5/2}}-48\sqrt{}$$

Rubi [A] time = 0.30, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 2014}

$$\frac{512a^3\sqrt{ax+b\sqrt{x}}}{77b^5x^{3/2}}-\frac{1280a^2\sqrt{ax+b\sqrt{x}}}{231b^4x^2}+\frac{4096a^5\sqrt{ax+b\sqrt{x}}}{231b^7\sqrt{x}}-\frac{2048a^4\sqrt{ax+b\sqrt{x}}}{231b^6x}+\frac{160a\sqrt{ax+b\sqrt{x}}}{33b^3x^{5/2}}-\frac{48\sqrt{ax+b\sqrt{x}}}{11b^2x^3}+\frac{4}{bx^{5/2}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*x^(5/2)*Sqrt[b*Sqrt[x] + a*x]) - (48*Sqrt[b*Sqrt[x] + a*x])/(11*b^2*x^
3) + (160*a*Sqrt[b*Sqrt[x] + a*x])/(33*b^3*x^(5/2)) - (1280*a^2*Sqrt[b*Sqrt
[x] + a*x])/(231*b^4*x^2) + (512*a^3*Sqrt[b*Sqrt[x] + a*x])/(77*b^5*x^(3/2)
) - (2048*a^4*Sqrt[b*Sqrt[x] + a*x])/(231*b^6*x) + (4096*a^5*Sqrt[b*Sqrt[x]
 + a*x])/(231*b^7*Sqrt[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}} + \frac{12\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx}{b}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}} - \frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3} - \frac{(120a)\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx}{11b^2}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}} - \frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3} + \frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}} + \frac{\left(320a^2\right)\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{33b^3}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}} - \frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3} + \frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}} - \frac{1280a^2\sqrt{b\sqrt{x}+ax}}{231b^4x^2} - \frac{(640}{}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}} - \frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3} + \frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}} - \frac{1280a^2\sqrt{b\sqrt{x}+ax}}{231b^4x^2} + \frac{512a}{}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}} - \frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3} + \frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}} - \frac{1280a^2\sqrt{b\sqrt{x}+ax}}{231b^4x^2} + \frac{512a}{}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}} - \frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3} + \frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}} - \frac{1280a^2\sqrt{b\sqrt{x}+ax}}{231b^4x^2} + \frac{512a}{}$$

Mathematica [A] time = 0.09, size = 96, normalized size = 0.49

$$\frac{4\left(1024a^6x^3+512a^5bx^{5/2}-128a^4b^2x^2+64a^3b^3x^{3/2}-40a^2b^4x+28ab^5\sqrt{x}-21b^6\right)}{231b^7x^{5/2}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (4*(-21*b^6 + 28*a*b^5*Sqrt[x] - 40*a^2*b^4*x + 64*a^3*b^3*x^(3/2) - 128*a^
4*b^2*x^2 + 512*a^5*b*x^(5/2) + 1024*a^6*x^3))/(231*b^7*x^(5/2)*Sqrt[b*Sqrt
[x] + a*x])
```

IntegrateAlgebraic [A] time = 0.35, size = 105, normalized size = 0.54

$$\frac{4\sqrt{ax+b\sqrt{x}}\left(1024a^6x^3+512a^5bx^{5/2}-128a^4b^2x^2+64a^3b^3x^{3/2}-40a^2b^4x+28ab^5\sqrt{x}-21b^6\right)}{231b^7x^3\left(a\sqrt{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (4*Sqrt[b*Sqrt[x] + a*x]*(-21*b^6 + 28*a*b^5*Sqrt[x] - 40*a^2*b^4*x + 64*a^
3*b^3*x^(3/2) - 128*a^4*b^2*x^2 + 512*a^5*b*x^(5/2) + 1024*a^6*x^3))/(231*b
^7*(b + a*Sqrt[x])*x^3)
```

fricas [A]   time = 0.87, size = 109, normalized size = 0.56

$$-\frac{4 \left(512 \, a^{6} b x^{3} - 192 \, a^{4} b^{3} x^{2} - 68 \, a^{2} b^{5} x - 21 \, b^{7} - \left(1024 \, a^{7} x^{3} - 640 \, a^{5} b^{2} x^{2} - 104 \, a^{3} b^{4} x - 49 \, a b^{6}\right) \sqrt{x}\right) \sqrt{a x + b \sqrt{x}}}{231 \left(a^{2} b^{7} x^{4} - b^{9} x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -4/231*(512*a^6*b*x^3 - 192*a^4*b^3*x^2 - 68*a^2*b^5*x - 21*b^7 - (1024*a^7
*x^3 - 640*a^5*b^2*x^2 - 104*a^3*b^4*x - 49*a*b^6)*sqrt(x))*sqrt(a*x + b*sq
rt(x))/(a^2*b^7*x^4 - b^9*x^3)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a x + b \sqrt{x}\right)^{\frac{3}{2}} x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^3), x)
```

maple [C]   time = 0.07, size = 614, normalized size = 3.15

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] 1/231*(a*x+b*x^(1/2))^(1/2)*(-2310*a^(13/2)*x^(13/2)*((a*x^(1/2)+b)*x^(1/2)
)^(1/2)*b^2-2310*(a*x+b*x^(1/2))^(1/2)*a^(13/2)*x^(13/2)*b^2-512*(a*x+b*x^(
1/2))^(3/2)*a^(9/2)*x^5*b^3-4620*a^(15/2)*x^7*((a*x^(1/2)+b)*x^(1/2))^(1/2)
*b-4620*(a*x+b*x^(1/2))^(1/2)*a^(15/2)*x^7*b+2048*(a*x+b*x^(1/2))^(3/2)*a^(
11/2)*x^(11/2)*b^2+8716*(a*x+b*x^(1/2))^(3/2)*a^(13/2)*x^6*b-84*(a*x+b*x^(1
/2))^(3/2)*a^(1/2)*x^3*b^7+5544*(a*x+b*x^(1/2))^(3/2)*a^(15/2)*x^(13/2)-231
0*(a*x+b*x^(1/2))^(1/2)*a^(17/2)*x^(15/2)+1155*ln(1/2*(2*a*x^(1/2)+b+2*((a*
x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^(15/2)*a^8*b-2310*a^(17/2)*x^
(15/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)-1155*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^
(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^(15/2)*a^8*b+2310*ln(1/2*(2*a*x^(1/2)+b+2*
((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^7*a^7*b^2-924*a^(15/2)*x^
(13/2)*((a*x^(1/2)+b)*x^(1/2))^(3/2)-2310*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^
(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^7*a^7*b^2+256*(a*x+b*x^(1/2))^(3/2)*a^(7/2
)*x^(9/2)*b^4+1155*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^
(1/2))/a^(1/2))*x^(13/2)*a^6*b^3-1155*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2
))^(1/2)*a^(1/2))/a^(1/2))*x^(13/2)*a^6*b^3-160*(a*x+b*x^(1/2))^(3/2)*a^(5/
2)*x^4*b^5+112*(a*x+b*x^(1/2))^(3/2)*a^(3/2)*x^(7/2)*b^6)/((a*x^(1/2)+b)*x^
(1/2))^(1/2)/b^8/x^(13/2)/a^(1/2)/(a*x^(1/2)+b)^2
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a x + b \sqrt{x}\right)^{\frac{3}{2}} x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x + b*x^(1/2))^(3/2)),x)
```

```
[Out] int(1/(x^3*(a*x + b*x^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a*x + b*sqrt(x))**(3/2)), x)
```

3.74 $\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=204

$$\frac{231b^6\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{256a^{13/2}}-\frac{231b^5\sqrt{ax+b\sqrt{x}}}{256a^6}+\frac{77b^4\sqrt{x}\sqrt{ax+b\sqrt{x}}}{128a^5}-\frac{77b^3x\sqrt{ax+b\sqrt{x}}}{160a^4}+\frac{33b^2x^{3/2}\sqrt{ax+b\sqrt{x}}}{80a^3}$$

Rubi [A] time = 0.17, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2018, 670, 640, 620, 206}

$$\frac{33b^2x^{3/2}\sqrt{ax+b\sqrt{x}}}{80a^3}-\frac{231b^5\sqrt{ax+b\sqrt{x}}}{256a^6}+\frac{77b^4\sqrt{x}\sqrt{ax+b\sqrt{x}}}{128a^5}-\frac{77b^3x\sqrt{ax+b\sqrt{x}}}{160a^4}+\frac{231b^6\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{256a^{13/2}}-\frac{11bx^2\sqrt{ax+b\sqrt{x}}}{30a^2}+\frac{x^{5/2}\sqrt{ax+b\sqrt{x}}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (-231*b^5*Sqrt[b*Sqrt[x] + a*x])/(256*a^6) + (77*b^4*Sqrt[x]*Sqrt[b*Sqrt[x]
 + a*x])/(128*a^5) - (77*b^3*x*Sqrt[b*Sqrt[x] + a*x])/(160*a^4) + (33*b^2*x
^(3/2)*Sqrt[b*Sqrt[x] + a*x])/(80*a^3) - (11*b*x^2*Sqrt[b*Sqrt[x] + a*x])/(
30*a^2) + (x^(5/2)*Sqrt[b*Sqrt[x] + a*x])/(3*a) + (231*b^6*ArcTanh[(Sqrt[a]
*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(256*a^(13/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\,\text{Subst}\left(\int \frac{x^6}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)$$
$$= \frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a} - \frac{(11b)\,\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{6a}$$
$$= -\frac{11bx^2\sqrt{b\sqrt{x}+ax}}{30a^2} + \frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a} + \frac{\left(33b^2\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{20a^2}$$
$$= \frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3} - \frac{11bx^2\sqrt{b\sqrt{x}+ax}}{30a^2} + \frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a} - \frac{\left(231b^3\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\right.}{160a^3}$$
$$= -\frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4} + \frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3} - \frac{11bx^2\sqrt{b\sqrt{x}+ax}}{30a^2} + \frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a} + \frac{(7}{}$$
$$= \frac{77b^4\sqrt{x}\sqrt{b\sqrt{x}+ax}}{128a^5} - \frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4} + \frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3} - \frac{11bx^2\sqrt{b\sqrt{x}+ax}}{30a^2} +$$
$$= -\frac{231b^5\sqrt{b\sqrt{x}+ax}}{256a^6} + \frac{77b^4\sqrt{x}\sqrt{b\sqrt{x}+ax}}{128a^5} - \frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4} + \frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3}$$
$$= -\frac{231b^5\sqrt{b\sqrt{x}+ax}}{256a^6} + \frac{77b^4\sqrt{x}\sqrt{b\sqrt{x}+ax}}{128a^5} - \frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4} + \frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3}$$
$$= -\frac{231b^5\sqrt{b\sqrt{x}+ax}}{256a^6} + \frac{77b^4\sqrt{x}\sqrt{b\sqrt{x}+ax}}{128a^5} - \frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4} + \frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3}$$

Mathematica [A] time = 0.26, size = 164, normalized size = 0.80

$$\frac{\left(a\sqrt{x}+b\right)\left(\sqrt{a}\sqrt{x}\sqrt{\frac{a\sqrt{x}}{b}+1}\left(1280a^5x^{5/2}-1408a^4bx^2+1584a^3b^2x^{3/2}-1848a^2b^3x+2310ab^4\sqrt{x}-3465b^5\right)+3465b^{11/2}\sqrt[4]{x}\sinh^{-1}\left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}}\right)\right)}{3840a^{13/2}\sqrt{\frac{a\sqrt{x}}{b}+1}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] ((b + a*Sqrt[x])*(Sqrt[a]*Sqrt[1 + (a*Sqrt[x])/b]*Sqrt[x]*(-3465*b^5 + 2310
*a*b^4*Sqrt[x] - 1848*a^2*b^3*x + 1584*a^3*b^2*x^(3/2) - 1408*a^4*b*x^2 + 1
280*a^5*x^(5/2)) + 3465*b^(11/2)*x^(1/4)*ArcSinh[(Sqrt[a]*x^(1/4))/Sqrt[b]]
))/(3840*a^(13/2)*Sqrt[1 + (a*Sqrt[x])/b]*Sqrt[b*Sqrt[x] + a*x])
```

IntegrateAlgebraic [A] time = 0.35, size = 126, normalized size = 0.62

$$\frac{\sqrt{ax+b\sqrt{x}}\left(1280a^5x^{5/2}-1408a^4bx^2+1584a^3b^2x^{3/2}-1848a^2b^3x+2310ab^4\sqrt{x}-3465b^5\right)}{3840a^6} - \frac{231b^6\log\left(-2\sqrt{a}\sqrt{ax+b\sqrt{x}}+2a\sqrt{x}+b\right)}{512a^{13/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (Sqrt[b*Sqrt[x] + a*x]*(-3465*b^5 + 2310*a*b^4*Sqrt[x] - 1848*a^2*b^3*x + 1
584*a^3*b^2*x^(3/2) - 1408*a^4*b*x^2 + 1280*a^5*x^(5/2)))/(3840*a^6) - (231
*b^6*Log[b + 2*a*Sqrt[x] - 2*Sqrt[a]*Sqrt[b*Sqrt[x] + a*x]])/(512*a^(13/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.28, size = 125, normalized size = 0.61

$$\frac{1}{3840}\,\sqrt{ax+b\sqrt{x}}\left(2\left(4\left(2\left(8\,\sqrt{x}\left(\frac{10\,\sqrt{x}}{a}-\frac{11\,b}{a^{2}}\right)+\frac{99\,b^{2}}{a^{3}}\right)\sqrt{x}-\frac{231\,b^{3}}{a^{4}}\right)\sqrt{x}+\frac{1155\,b^{4}}{a^{5}}\right)\sqrt{x}-\frac{3465\,b^{5}}{a^{6}}\right)-\frac{231\,b^{6}\log\left(\left|-2\,\sqrt{a}\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{512\,a^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*sqrt(a*x + b*sqrt(x))*(2*(4*(2*(8*sqrt(x)*(10*sqrt(x)/a - 11*b/a^2)
+ 99*b^2/a^3)*sqrt(x) - 231*b^3/a^4)*sqrt(x) + 1155*b^4/a^5)*sqrt(x) - 3465
*b^5/a^6) - 231/512*b^6*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*
sqrt(x))) - b))/a^(13/2)
```

maple [A] time = 0.06, size = 245, normalized size = 1.20

$$\frac{\sqrt{ax+b\sqrt{x}}\left(7680a\,b^{6}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\;\sqrt{a}}{2\sqrt{a}}\right)-4215a\,b^{6}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\;\sqrt{a}}{2\sqrt{a}}\right)+2560\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{11}{2}}x^{\frac{3}{2}}+16860\sqrt{ax+b\sqrt{x}}\;a^{\frac{5}{2}}b^{4}\sqrt{x}-5376\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{9}{2}}bx-15360\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\;a^{\frac{3}{2}}b^{5}+8430\sqrt{ax+b\sqrt{x}}\;a^{\frac{3}{2}}b^{5}+8544\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{7}{2}}b^{2}\sqrt{x}-12240\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{5}{2}}b^{3}\right)}{7680\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\;a^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a*x+b*x^(1/2))^(1/2),x)
```

```
[Out] 1/7680*(a*x+b*x^(1/2))^(1/2)*(2560*x^(3/2)*(a*x+b*x^(1/2))^(3/2)*a^(11/2)+8
544*x^(1/2)*(a*x+b*x^(1/2))^(3/2)*a^(7/2)*b^2-5376*(a*x+b*x^(1/2))^(3/2)*a^
(9/2)*x*b+16860*x^(1/2)*(a*x+b*x^(1/2))^(1/2)*a^(5/2)*b^4-12240*(a*x+b*x^(1
/2))^(3/2)*a^(5/2)*b^3-15360*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(3/2)*b^5+8430
*(a*x+b*x^(1/2))^(1/2)*a^(3/2)*b^5+7680*a*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/
2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*b^6-4215*ln(1/2*(2*a*x^(1/2)+b+2*(a*
x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a*b^6)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/a
^(15/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/sqrt(a*x + b*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{5/2}}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a*x + b*x^(1/2))^(1/2),x)
```

```
[Out] int(x^(5/2)/(a*x + b*x^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{ax + b\sqrt{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(x**(5/2)/sqrt(a*x + b*sqrt(x)), x)
```

3.75 $\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=146

$$\frac{35b^4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{32a^{9/2}}-\frac{35b^3\sqrt{ax+b\sqrt{x}}}{32a^4}+\frac{35b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{48a^3}-\frac{7bx\sqrt{ax+b\sqrt{x}}}{12a^2}+\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{2a}$$

Rubi [A] time = 0.12, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2018, 670, 640, 620, 206}

$$-\frac{35b^3\sqrt{ax+b\sqrt{x}}}{32a^4}+\frac{35b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{48a^3}+\frac{35b^4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{32a^{9/2}}-\frac{7bx\sqrt{ax+b\sqrt{x}}}{12a^2}+\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (-35*b^3*Sqrt[b*Sqrt[x] + a*x])/(32*a^4) + (35*b^2*Sqrt[x]*Sqrt[b*Sqrt[x] +
 a*x])/(48*a^3) - (7*b*x*Sqrt[b*Sqrt[x] + a*x])/(12*a^2) + (x^(3/2)*Sqrt[b*
Sqrt[x] + a*x])/(2*a) + (35*b^4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] +
a*x]])/(32*a^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\,\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a} - \frac{(7b)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{4a}$$
$$= -\frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2} + \frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a} + \frac{\left(35b^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{24a^2}$$
$$= \frac{35b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{48a^3} - \frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2} + \frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a} - \frac{\left(35b^3\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx,\right.}{32a^3}$$
$$= -\frac{35b^3\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{35b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{48a^3} - \frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2} + \frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a} + \frac{\left(35b^4\right.}{}$$
$$= -\frac{35b^3\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{35b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{48a^3} - \frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2} + \frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a} + \frac{\left(35b^4\right.}{}$$
$$= -\frac{35b^3\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{35b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{48a^3} - \frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2} + \frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a} + \frac{35b^4\,\text{t}}{}$$

Mathematica [A] time = 0.21, size = 142, normalized size = 0.97

$$-\frac{35b^5\left(\frac{a\sqrt{x}}{b}+1\right)\left(-\frac{32a^4x^2}{35b^4}+\frac{16a^3x^{3/2}}{15b^3}-\frac{4a^2x}{3b^2}+\frac{2a\sqrt{x}}{b}-\frac{2\sqrt{a}\sqrt[4]{x}\sinh^{-1}\left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{\frac{a\sqrt{x}}{b}+1}}\right)}{64a^5\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (-35*b^5*(1 + (a*Sqrt[x])/b)*((2*a*Sqrt[x])/b - (4*a^2*x)/(3*b^2) + (16*a^3
*x^(3/2))/(15*b^3) - (32*a^4*x^2)/(35*b^4) - (2*Sqrt[a]*x^(1/4)*ArcSinh[(Sq
rt[a]*x^(1/4))/Sqrt[b]])/(Sqrt[b]*Sqrt[1 + (a*Sqrt[x])/b])))/(64*a^5*Sqrt[(
b + a*Sqrt[x])*Sqrt[x]])
```

IntegrateAlgebraic [A] time = 0.30, size = 108, normalized size = 0.74

$$\frac{\sqrt{ax+b\sqrt{x}}\left(48a^3x^{3/2}-56a^2bx+70ab^2\sqrt{x}-105b^3\right)}{96a^4}-\frac{35b^4\log\left(-2a^{9/2}\sqrt{ax+b\sqrt{x}}+2a^5\sqrt{x}+a^4b\right)}{64a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (Sqrt[b*Sqrt[x] + a*x]*(-105*b^3 + 70*a*b^2*Sqrt[x] - 56*a^2*b*x + 48*a^3*x
^(3/2)))/(96*a^4) - (35*b^4*Log[a^4*b + 2*a^5*Sqrt[x] - 2*a^(9/2)*Sqrt[b*Sq
rt[x] + a*x]])/(64*a^(9/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.41, size = 97, normalized size = 0.66

$$\frac{1}{96}\sqrt{ax+b\sqrt{x}}\left(2\left(4\sqrt{x}\left(\frac{6\sqrt{x}}{a}-\frac{7b}{a^2}\right)+\frac{35b^2}{a^3}\right)\sqrt{x}-\frac{105b^3}{a^4}\right)-\frac{35b^4\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{64a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*sqrt(a*x + b*sqrt(x))*(2*(4*sqrt(x)*(6*sqrt(x)/a - 7*b/a^2) + 35*b^2/a
^3)*sqrt(x) - 105*b^3/a^4) - 35/64*b^4*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x)
- sqrt(a*x + b*sqrt(x))) - b))/a^(9/2)
```

maple [A] time = 0.05, size = 203, normalized size = 1.39

$$\frac{\sqrt{ax+b\sqrt{x}}\left(192ab^4\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\sqrt{a}}{2\sqrt{a}}\right)-87ab^4\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\sqrt{a}}{2\sqrt{a}}\right)+348\sqrt{ax+b\sqrt{x}}\,a^{\frac{5}{2}}b^2\sqrt{x}-384\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{3}{2}}b^3+174\sqrt{ax+b\sqrt{x}}\,a^{\frac{3}{2}}b^3+96\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{7}{2}}\sqrt{x}-208\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{5}{2}}b\right)}{192\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a*x+b*x^(1/2))^(1/2),x)
```

```
[Out] 1/192*(a*x+b*x^(1/2))^(1/2)*(96*x^(1/2)*(a*x+b*x^(1/2))^(3/2)*a^(7/2)+348*x
^(1/2)*(a*x+b*x^(1/2))^(1/2)*a^(5/2)*b^2-208*(a*x+b*x^(1/2))^(3/2)*a^(5/2)*
b-384*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(3/2)*b^3+174*(a*x+b*x^(1/2))^(1/2)*a
^(3/2)*b^3+192*a*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1
/2))/a^(1/2))*b^4-87*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))
/a^(1/2))*a*b^4)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/a^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{3}{2}}}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/sqrt(a*x + b*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^{3/2}}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a*x + b*x^(1/2))^(1/2),x)
```

```
[Out] int(x^(3/2)/(a*x + b*x^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{3}{2}}}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(x**(3/2)/sqrt(a*x + b*sqrt(x)), x)
```

3.76 $\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=87

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{2a^{5/2}}-\frac{3b\sqrt{ax+b\sqrt{x}}}{2a^2}+\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a}$$

Rubi [A] time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2018, 670, 640, 620, 206}

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{2a^{5/2}}-\frac{3b\sqrt{ax+b\sqrt{x}}}{2a^2}+\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (-3*b*Sqrt[b*Sqrt[x] + a*x])/(2*a^2) + (Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/a +
(3*b^2*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(2*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a} - \frac{(3b)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{2a}$$
$$= -\frac{3b\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a} + \frac{\left(3b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{4a^2}$$
$$= -\frac{3b\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a} + \frac{\left(3b^2\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{2a^2}$$
$$= -\frac{3b\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a} + \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{2a^{5/2}}$$

Mathematica [A] time = 0.12, size = 102, normalized size = 1.17

$$\frac{\sqrt{a}\sqrt{x}\left(2a^2x-ab\sqrt{x}-3b^2\right)+3b^{5/2}\sqrt[4]{x}\sqrt{\frac{a\sqrt{x}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}}\right)}{2a^{5/2}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (Sqrt[a]*Sqrt[x]*(-3*b^2 - a*b*Sqrt[x] + 2*a^2*x) + 3*b^(5/2)*Sqrt[1 + (a*S
qrt[x])/b]*x^(1/4)*ArcSinh[(Sqrt[a]*x^(1/4))/Sqrt[b]])/(2*a^(5/2)*Sqrt[b*Sq
rt[x] + a*x])
```

IntegrateAlgebraic [A] time = 0.27, size = 86, normalized size = 0.99

$$\frac{\left(2a\sqrt{x}-3b\right)\sqrt{ax+b\sqrt{x}}}{2a^2} - \frac{3b^2\log\left(-2a^{5/2}\sqrt{ax+b\sqrt{x}}+2a^3\sqrt{x}+a^2b\right)}{4a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] ((-3*b + 2*a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(2*a^2) - (3*b^2*Log[a^2*b + 2
*a^3*Sqrt[x] - 2*a^(5/2)*Sqrt[b*Sqrt[x] + a*x]])/(4*a^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.27, size = 69, normalized size = 0.79

$$\frac{1}{2}\sqrt{ax+b\sqrt{x}}\left(\frac{2\sqrt{x}}{a}-\frac{3b}{a^2}\right) - \frac{3b^2\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{4a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a*x + b*sqrt(x))*(2*sqrt(x)/a - 3*b/a^2) - 3/4*b^2*log(abs(-2*sqrt
(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(5/2)
```

maple [B] time = 0.05, size = 160, normalized size = 1.84

$$\frac{\sqrt{ax+b\sqrt{x}}\,\left(4a\,b^{2}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-a\,b^{2}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+4\sqrt{ax+b\sqrt{x}}\,a^{\frac{5}{2}}\sqrt{x}-8\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{3}{2}}b+2\sqrt{ax+b\sqrt{x}}\,a^{\frac{3}{2}}b\right)}{4\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a*x+b*x^(1/2))^(1/2),x)
```

```
[Out] 1/4*(a*x+b*x^(1/2))^(1/2)*(4*x^(1/2)*(a*x+b*x^(1/2))^(1/2)*a^(5/2)-8*((a*x^
(1/2)+b)*x^(1/2))^(1/2)*a^(3/2)*b+2*(a*x+b*x^(1/2))^(1/2)*a^(3/2)*b+4*a*ln(
1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*b^2-b^
2*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a)/((a*x^
(1/2)+b)*x^(1/2))^(1/2)/a^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt(a*x + b*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x}}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a*x + b*x^(1/2))^(1/2),x)
```

```
[Out] int(x^(1/2)/(a*x + b*x^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x)/sqrt(a*x + b*sqrt(x)), x)
```

3.77 $\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=34

$$\frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{\sqrt{a}}$$

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2013, 620, 206}

$$\frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/Sqrt[a]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)\\
&= 4\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)\\
&= \frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 65, normalized size = 1.91

$$\frac{4\sqrt{b}\sqrt[4]{x}\sqrt{\frac{a\sqrt{x}}{b}+1}\sinh^{-1}\left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (4*Sqrt[b]*Sqrt[1 + (a*Sqrt[x])/b]*x^(1/4)*ArcSinh[(Sqrt[a]*x^(1/4))/Sqrt[b
]])/(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])
```

IntegrateAlgebraic [A] time = 0.14, size = 40, normalized size = 1.18

$$-\frac{2\log\left(-2\sqrt{a}\sqrt{ax+b\sqrt{x}}+2a\sqrt{x}+b\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-2*Log[b + 2*a*Sqrt[x] - 2*Sqrt[a]*Sqrt[b*Sqrt[x] + a*x]])/Sqrt[a]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.26, size = 37, normalized size = 1.09

$$-\frac{2\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/sqrt(
a)
```

maple [B] time = 0.05, size = 136, normalized size = 4.00

$$-\frac{\sqrt{ax+b\sqrt{x}}\left(-b\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\sqrt{a}}{2\sqrt{a}}\right)-b\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\sqrt{a}}{2\sqrt{a}}\right)+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\sqrt{a}-2\sqrt{ax+b\sqrt{x}}\sqrt{a}\right)}{\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\sqrt{a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(a*x+b*x^(1/2))^(1/2),x)
```

```
[Out] -(a*x+b*x^(1/2))^(1/2)*(2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2)-2*(a*x+b*x^
(1/2))^(1/2)*a^(1/2)-b*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2
)*a^(1/2))/a^(1/2))-b*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2)
)/a^(1/2)))/((a*x^(1/2)+b)*x^(1/2))^(1/2)/b/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + b\sqrt{x}}\,\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{x}\,\sqrt{a\,x + b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a*x + b*x^(1/2))^(1/2)),x)
[Out] int(1/(x^(1/2)*(a*x + b*x^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\sqrt{ax + b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x**(1/2)+a*x)**(1/2),x)
[Out] Integral(1/(sqrt(x)*sqrt(a*x + b*sqrt(x))), x)
```

3.78 $\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=54

$$\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}$$

Rubi [A] time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(3*b*x) + (8*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*Sqr
t[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx &= -\frac{4\sqrt{b\sqrt{x}+ax}}{3bx}-\frac{(2a)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{3b}\\ &= -\frac{4\sqrt{b\sqrt{x}+ax}}{3bx}+\frac{8a\sqrt{b\sqrt{x}+ax}}{3b^2\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 37, normalized size = 0.69

$$\frac{4\left(2a\sqrt{x}-b\right)\sqrt{ax+b\sqrt{x}}}{3b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (4*(-b + 2*a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*x)
```

IntegrateAlgebraic [A] time = 0.16, size = 37, normalized size = 0.69

$$\frac{4\left(2a\sqrt{x}-b\right)\sqrt{ax+b\sqrt{x}}}{3b^2x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*Sqrt[b*Sqrt[x] + a*x]),x]
[Out] (4*(-b + 2*a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*x)
```

fricas [A] time = 0.70, size = 29, normalized size = 0.54

$$\frac{4\sqrt{ax+b\sqrt{x}}\left(2\,a\sqrt{x}-b\right)}{3\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
[Out] 4/3*sqrt(a*x + b*sqrt(x))*(2*a*sqrt(x) - b)/(b^2*x)
```

giac [A] time = 0.19, size = 53, normalized size = 0.98

$$\frac{4\left(3\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)}{3\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
[Out] 4/3*(3*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)/(sqrt(a)*sqrt
(x) - sqrt(a*x + b*sqrt(x)))^3
```

maple [C] time = 0.06, size = 194, normalized size = 3.59

$$-\frac{\sqrt{ax+b\sqrt{x}}\left(-3a^2b\,x^{\frac{5}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\;\sqrt{a}}{2\sqrt{a}}\right)+3a^2b\,x^{\frac{5}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\;\sqrt{a}}{2\sqrt{a}}\right)+6\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\;a^{\frac{5}{2}}x^{\frac{5}{2}}+6\sqrt{ax+b\sqrt{x}}\;a^{\frac{5}{2}}x^{\frac{5}{2}}-12\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{3}{2}}x^{\frac{3}{2}}+4\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}\sqrt{a}\;bx\right)}{3\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\;\sqrt{a}\;b^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(a*x+b*x^(1/2))^(1/2),x)
[Out] -1/3*(a*x+b*x^(1/2))^(1/2)*(6*x^(5/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(5/2)
-12*x^(3/2)*(a*x+b*x^(1/2))^(3/2)*a^(3/2)+6*x^(5/2)*(a*x+b*x^(1/2))^(1/2)*a
^(5/2)-3*x^(5/2)*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1
/2))/a^(1/2))*a^2*b+3*x^(5/2)*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)
*a^(1/2))/a^(1/2))*a^2*b+4*(a*x+b*x^(1/2))^(3/2)*b*a^(1/2)*x)/((a*x^(1/2)+b
)*x^(1/2))^(1/2)/b^3/a^(1/2)/x^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+b\sqrt{x}}\,x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^{3/2}\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a*x + b*x^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^(3/2)*(a*x + b*x^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**(3/2)*sqrt(a*x + b*sqrt(x))), x)
```

3.79 $\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=112

$$\frac{64a^3\sqrt{ax+b\sqrt{x}}}{35b^4\sqrt{x}}-\frac{32a^2\sqrt{ax+b\sqrt{x}}}{35b^3x}+\frac{24a\sqrt{ax+b\sqrt{x}}}{35b^2x^{3/2}}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}$$

Rubi [A] time = 0.15, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$\frac{64a^3\sqrt{ax+b\sqrt{x}}}{35b^4\sqrt{x}}-\frac{32a^2\sqrt{ax+b\sqrt{x}}}{35b^3x}+\frac{24a\sqrt{ax+b\sqrt{x}}}{35b^2x^{3/2}}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(7*b*x^2) + (24*a*Sqrt[b*Sqrt[x] + a*x])/(35*b^2
*x^(3/2)) - (32*a^2*Sqrt[b*Sqrt[x] + a*x])/(35*b^3*x) + (64*a^3*Sqrt[b*Sqrt
[x] + a*x])/(35*b^4*Sqrt[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx &= -\frac{4\sqrt{b\sqrt{x}+ax}}{7bx^2}-\frac{(6a)\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{7b}\\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{7bx^2}+\frac{24a\sqrt{b\sqrt{x}+ax}}{35b^2x^{3/2}}+\frac{\left(24a^2\right)\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{35b^2}\\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{7bx^2}+\frac{24a\sqrt{b\sqrt{x}+ax}}{35b^2x^{3/2}}-\frac{32a^2\sqrt{b\sqrt{x}+ax}}{35b^3x}-\frac{\left(16a^3\right)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{35b^3}\\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{7bx^2}+\frac{24a\sqrt{b\sqrt{x}+ax}}{35b^2x^{3/2}}-\frac{32a^2\sqrt{b\sqrt{x}+ax}}{35b^3x}+\frac{64a^3\sqrt{b\sqrt{x}+ax}}{35b^4\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 59, normalized size = 0.53

$$\frac{4\sqrt{ax+b\sqrt{x}}\left(16a^3x^{3/2}-8a^2bx+6ab^2\sqrt{x}-5b^3\right)}{35b^4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (4*Sqrt[b*Sqrt[x] + a*x]*(-5*b^3 + 6*a*b^2*Sqrt[x] - 8*a^2*b*x + 16*a^3*x^(
3/2)))/(35*b^4*x^2)
```

IntegrateAlgebraic [A] time = 0.18, size = 59, normalized size = 0.53

$$\frac{4\sqrt{ax+b\sqrt{x}}\left(16a^3x^{3/2}-8a^2bx+6ab^2\sqrt{x}-5b^3\right)}{35b^4x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (4*Sqrt[b*Sqrt[x] + a*x]*(-5*b^3 + 6*a*b^2*Sqrt[x] - 8*a^2*b*x + 16*a^3*x^(
3/2)))/(35*b^4*x^2)
```

fricas [A] time = 0.85, size = 50, normalized size = 0.45

$$-\frac{4\left(8\,a^2bx+5\,b^3-2\left(8\,a^3x+3\,ab^2\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{35\,b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -4/35*(8*a^2*b*x + 5*b^3 - 2*(8*a^3*x + 3*a*b^2)*sqrt(x))*sqrt(a*x + b*sqrt
(x))/(b^4*x^2)
```

giac [A] time = 0.21, size = 115, normalized size = 1.03

$$\frac{4\left(70\,a^{\frac{3}{2}}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^3+84\,ab\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^2+35\,\sqrt{a}\,b^2\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+5\,b^3\right)}{35\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/35*(70*a^(3/2)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^3 + 84*a*b*(sqrt
(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^2 + 35*sqrt(a)*b^2*(sqrt(a)*sqrt(x) -
sqrt(a*x + b*sqrt(x))) + 5*b^3)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^7
```

maple [C] time = 0.06, size = 240, normalized size = 2.14

$$\frac{\sqrt{ax+b\sqrt{x}}\left(35a^4b\,x^{\frac{9}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-35a^4b\,x^{\frac{9}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-70\sqrt{ax+b\sqrt{x}}\,a^{\frac{9}{2}}x^{\frac{9}{2}}-70\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{9}{2}}x^{\frac{9}{2}}+140\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{7}{2}}x^{\frac{7}{2}}-76\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{5}{2}}b\,x^3+44\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{3}{2}}b^2x^{\frac{5}{2}}-20\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}\sqrt{a}\,b^3x^2\right)}{35\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}\,b^5x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(a*x+b*x^(1/2))^(1/2),x)
```

```
[Out] 1/35*(a*x+b*x^(1/2))^(1/2)*(140*(a*x+b*x^(1/2))^(3/2)*a^(7/2)*x^(7/2)-70*(a
*x+b*x^(1/2))^(1/2)*a^(9/2)*x^(9/2)+35*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+
```

```
b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^(9/2)*a^4*b-70*a^(9/2)*x^(9/2)*((a*x^
(1/2)+b)*x^(1/2))^(1/2)-35*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^
(1/2))/a^(1/2))*x^(9/2)*a^4*b+44*(a*x+b*x^(1/2))^(3/2)*a^(3/2)*x^(5/2)*b^2-
76*a^(5/2)*(a*x+b*x^(1/2))^(3/2)*b*x^3-20*(a*x+b*x^(1/2))^(3/2)*a^(1/2)*x^2
*b^3)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/b^5/x^(9/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + b\sqrt{x}}\, x^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{5/2}\,\sqrt{a\,x + b\,\sqrt{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a*x + b*x^(1/2))^(1/2)),x)
[Out] int(1/(x^(5/2)*(a*x + b*x^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}}\sqrt{ax + b\sqrt{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**(1/2)+a*x)**(1/2),x)
[Out] Integral(1/(x**(5/2)*sqrt(a*x + b*sqrt(x))), x)
```

3.80 $\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=170

$$\frac{1024a^5\sqrt{ax+b\sqrt{x}}}{693b^6\sqrt{x}}-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{693b^5x}+\frac{128a^3\sqrt{ax+b\sqrt{x}}}{231b^4x^{3/2}}-\frac{320a^2\sqrt{ax+b\sqrt{x}}}{693b^3x^2}+\frac{40a\sqrt{ax+b\sqrt{x}}}{99b^2x^{5/2}}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}$$

Rubi [A] time = 0.24, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$\frac{128a^3\sqrt{ax+b\sqrt{x}}}{231b^4x^{3/2}}-\frac{320a^2\sqrt{ax+b\sqrt{x}}}{693b^3x^2}+\frac{1024a^5\sqrt{ax+b\sqrt{x}}}{693b^6\sqrt{x}}-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{693b^5x}+\frac{40a\sqrt{ax+b\sqrt{x}}}{99b^2x^{5/2}}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(11*b*x^3) + (40*a*Sqrt[b*Sqrt[x] + a*x])/(99*b^
2*x^(5/2)) - (320*a^2*Sqrt[b*Sqrt[x] + a*x])/(693*b^3*x^2) + (128*a^3*Sqrt[
b*Sqrt[x] + a*x])/(231*b^4*x^(3/2)) - (512*a^4*Sqrt[b*Sqrt[x] + a*x])/(693*
b^5*x) + (1024*a^5*Sqrt[b*Sqrt[x] + a*x])/(693*b^6*Sqrt[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} - \frac{(10a)\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx}{11b}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} + \frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}} + \frac{\left(80a^2\right)\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{99b^2}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} + \frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}} - \frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2} - \frac{\left(160a^3\right)\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{231b^3}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} + \frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}} - \frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2} + \frac{128a^3\sqrt{b\sqrt{x}+ax}}{231b^4x^{3/2}} + \frac{(128a}{}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} + \frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}} - \frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2} + \frac{128a^3\sqrt{b\sqrt{x}+ax}}{231b^4x^{3/2}} - \frac{512a^4}{6}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} + \frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}} - \frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2} + \frac{128a^3\sqrt{b\sqrt{x}+ax}}{231b^4x^{3/2}} - \frac{512a^4}{6}$$

Mathematica [A] time = 0.09, size = 83, normalized size = 0.49

$$\frac{4\sqrt{ax+b\sqrt{x}}\left(256a^5x^{5/2}-128a^4bx^2+96a^3b^2x^{3/2}-80a^2b^3x+70ab^4\sqrt{x}-63b^5\right)}{693b^6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (4*Sqrt[b*Sqrt[x] + a*x]*(-63*b^5 + 70*a*b^4*Sqrt[x] - 80*a^2*b^3*x + 96*a^
3*b^2*x^(3/2) - 128*a^4*b*x^2 + 256*a^5*x^(5/2)))/(693*b^6*x^3)
```

IntegrateAlgebraic [A] time = 0.20, size = 83, normalized size = 0.49

$$\frac{4\sqrt{ax+b\sqrt{x}}\left(256a^5x^{5/2}-128a^4bx^2+96a^3b^2x^{3/2}-80a^2b^3x+70ab^4\sqrt{x}-63b^5\right)}{693b^6x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (4*Sqrt[b*Sqrt[x] + a*x]*(-63*b^5 + 70*a*b^4*Sqrt[x] - 80*a^2*b^3*x + 96*a^
3*b^2*x^(3/2) - 128*a^4*b*x^2 + 256*a^5*x^(5/2)))/(693*b^6*x^3)
```

fricas [A] time = 0.71, size = 72, normalized size = 0.42

$$-\frac{4\left(128\,a^4bx^2+80\,a^2b^3x+63\,b^5-2\left(128\,a^5x^2+48\,a^3b^2x+35\,ab^4\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{693\,b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -4/693*(128*a^4*b*x^2 + 80*a^2*b^3*x + 63*b^5 - 2*(128*a^5*x^2 + 48*a^3*b^2
*x + 35*a*b^4)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(b^6*x^3)
```

giac [A] time = 0.24, size = 177, normalized size = 1.04

$$\frac{4\left(3696\,a^{\frac{5}{2}}\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{5}+7920\,a^{2}b\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{4}+6930\,a^{\frac{3}{2}}b^{2}\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{3}+3080\,ab^{3}\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{2}+693\,\sqrt{a}\,b^{4}\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+63\,b^{5}\right)}{693\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/693*(3696*a^(5/2)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^5 + 7920*a^2*
b*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^4 + 6930*a^(3/2)*b^2*(sqrt(a)*s
qrt(x) - sqrt(a*x + b*sqrt(x)))^3 + 3080*a*b^3*(sqrt(a)*sqrt(x) - sqrt(a*x
+ b*sqrt(x)))^2 + 693*sqrt(a)*b^4*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))
 + 63*b^5)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^11
```

maple [C] time = 0.06, size = 284, normalized size = 1.67

$$\frac{\sqrt{ax+b\sqrt{x}}\,\left(693a^{6}b\,x^{\frac{13}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-693a^{6}b\,x^{\frac{13}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-1386\sqrt{ax+b\sqrt{x}}\,a^{\frac{13}{2}}x^{\frac{13}{2}}-1386\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{13}{2}}x^{\frac{13}{2}}+2772\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{11}{2}}x^{\frac{11}{2}}-1748\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{9}{2}}b\,x^{5}+1236\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{7}{2}}b^{2}x^{\frac{9}{2}}-852\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{5}{2}}b^{3}x^{4}+532\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{3}{2}}b^{4}x^{\frac{7}{2}}-252\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}\sqrt{a}\,b^{5}x^{3}\right)}{693\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}\,b^{7}x^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(a*x+b*x^(1/2))^(1/2),x)
```

```
[Out] 1/693*(a*x+b*x^(1/2))^(1/2)*(2772*(a*x+b*x^(1/2))^(3/2)*a^(11/2)*x^(11/2)-1
386*(a*x+b*x^(1/2))^(1/2)*a^(13/2)*x^(13/2)+693*ln(1/2*(2*a*x^(1/2)+b+2*((a
*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^(13/2)*a^6*b-1386*a^(13/2)*x
^(13/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)-693*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^
(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^(13/2)*a^6*b+1236*(a*x+b*x^(1/2))^(3/2)*a^
(7/2)*x^(9/2)*b^2+532*(a*x+b*x^(1/2))^(3/2)*a^(3/2)*x^(7/2)*b^4-1748*a^(9/2
)*(a*x+b*x^(1/2))^(3/2)*b*x^5-852*(a*x+b*x^(1/2))^(3/2)*a^(5/2)*x^4*b^3-252
*(a*x+b*x^(1/2))^(3/2)*a^(1/2)*x^3*b^5)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/b^7/x
^(13/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+b\sqrt{x}}\,x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{7/2}\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a*x + b*x^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^(7/2)*(a*x + b*x^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{7}{2}}\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**(7/2)*sqrt(a*x + b*sqrt(x))), x)
```

3.81 $\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=171

$$\frac{315b^4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{32a^{11/2}}-\frac{315b^3\sqrt{ax+b\sqrt{x}}}{32a^5}+\frac{105b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{16a^4}-\frac{21bx\sqrt{ax+b\sqrt{x}}}{4a^3}+\frac{9x^{3/2}\sqrt{ax+b\sqrt{x}}}{2a^2}-$$

Rubi [A] time = 0.15, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2018, 668, 670, 640, 620, 206}

$$-\frac{315b^3\sqrt{ax+b\sqrt{x}}}{32a^5}+\frac{105b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{16a^4}+\frac{315b^4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{32a^{11/2}}+\frac{9x^{3/2}\sqrt{ax+b\sqrt{x}}}{2a^2}-\frac{21bx\sqrt{ax+b\sqrt{x}}}{4a^3}-\frac{4x^{5/2}}{a\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*x^(5/2))/(a*Sqrt[b*Sqrt[x] + a*x]) - (315*b^3*Sqrt[b*Sqrt[x] + a*x])/(3
2*a^5) + (105*b^2*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(16*a^4) - (21*b*x*Sqrt[b*
Sqrt[x] + a*x])/(4*a^3) + (9*x^(3/2)*Sqrt[b*Sqrt[x] + a*x])/(2*a^2) + (315*
b^4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(32*a^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
```

```
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = 2\,\text{Subst}\left(\int \frac{x^6}{\left(bx+ax^2\right)^{3/2}}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} + \frac{18\,\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} + \frac{9x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a^2} - \frac{(63b)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{4a^2}$$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3} + \frac{9x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{\left(105b^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx,\right.}{8a^3}$$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{16a^4} - \frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3} + \frac{9x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a^2} - \frac{(315}{}$$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{315b^3\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{16a^4} - \frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3} + \frac{9x^3}{}$$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{315b^3\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{16a^4} - \frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3} + \frac{9x^3}{}$$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{315b^3\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{16a^4} - \frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3} + \frac{9x^3}{}$$

Mathematica [C] time = 0.08, size = 62, normalized size = 0.36

$$\frac{4x^3\sqrt{\frac{a\sqrt{x}}{b}+1}\;{}_2F_1\left(\frac{3}{2},\frac{11}{2};\frac{13}{2};-\frac{a\sqrt{x}}{b}\right)}{11b\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (4*Sqrt[1 + (a*Sqrt[x])/b]*x^3*Hypergeometric2F1[3/2, 11/2, 13/2, -((a*Sqrt
[x])/b)])/(11*b*Sqrt[b*Sqrt[x] + a*x])
```

IntegrateAlgebraic [A] time = 0.45, size = 124, normalized size = 0.73

$$\frac{\sqrt{ax+b\sqrt{x}}\,\left(16a^4x^2-24a^3bx^{3/2}+42a^2b^2x-105ab^3\sqrt{x}-315b^4\right)}{32a^5\left(a\sqrt{x}+b\right)}-\frac{315b^4\log\left(-2\sqrt{a}\,\sqrt{ax+b\sqrt{x}}\,+2a\sqrt{x}\,+b\right)}{64a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (Sqrt[b*Sqrt[x] + a*x]*(-315*b^4 - 105*a*b^3*Sqrt[x] + 42*a^2*b^2*x - 24*a^
3*b*x^(3/2) + 16*a^4*x^2))/(32*a^5*(b + a*Sqrt[x])) - (315*b^4*Log[b + 2*a*
Sqrt[x] - 2*Sqrt[a]*Sqrt[b*Sqrt[x] + a*x]])/(64*a^(11/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueDone
```

maple [B] time = 0.06, size = 527, normalized size = 3.08

$$\frac{\sqrt{ax+b\sqrt{x}}\,\left(384a^3b^4x\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-69a^3b^4x\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+768a^2b^5\sqrt{x}\,\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-138a^2b^5\sqrt{x}\,\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+276\sqrt{ax+b\sqrt{x}}\,a^{\frac{9}{2}}b^2x^{\frac{3}{2}}+384ab^6\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-69ab^6\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-768\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{7}{2}}b^3x+690\sqrt{ax+b\sqrt{x}}\,a^{\frac{7}{2}}b^3x+32\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{11}{2}}x^{\frac{3}{2}}-1536\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{5}{2}}b^4\sqrt{x}+552\sqrt{ax+b\sqrt{x}}\,a^{\frac{5}{2}}b^4\sqrt{x}-48\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{9}{2}}bx-768\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{3}{2}}b^5+138\sqrt{ax+b\sqrt{x}}\,a^{\frac{3}{2}}b^5-192\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{7}{2}}b^2\sqrt{x}+256\left(\left(a\sqrt{x}+b\right)\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{5}{2}}b^3-112\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{5}{2}}b^3\right)}{64\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\left(a\sqrt{x}+b\right)^2a^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] 1/64*(a*x+b*x^(1/2))^(1/2)/a^(13/2)*(32*(a*x+b*x^(1/2))^(3/2)*a^(11/2)*x^(3
/2)+276*x^(3/2)*(a*x+b*x^(1/2))^(1/2)*a^(9/2)*b^2-48*(a*x+b*x^(1/2))^(3/2)*
a^(9/2)*b*x-768*x*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(7/2)*b^3+690*x*(a*x+b*x^
(1/2))^(1/2)*a^(7/2)*b^3+384*x*a^3*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x
^(1/2))^(1/2)*a^(1/2))/a^(1/2))*b^4-192*(a*x+b*x^(1/2))^(3/2)*a^(7/2)*b^2*x
^(1/2)-69*x*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))
*a^3*b^4-1536*x^(1/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(5/2)*b^4+552*(a*x+b*
x^(1/2))^(1/2)*a^(5/2)*b^4*x^(1/2)+768*x^(1/2)*a^2*ln(1/2*(2*a*x^(1/2)+b+2*
((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*b^5+256*((a*x^(1/2)+b)*x^(1
/2))^(3/2)*a^(5/2)*b^3-112*(a*x+b*x^(1/2))^(3/2)*a^(5/2)*b^3-138*x^(1/2)*ln
(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^2*b^5-768*(
(a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(3/2)*b^5+138*(a*x+b*x^(1/2))^(1/2)*a^(3/2)*
b^5+384*a*b^6*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2)
)/a^(1/2))-69*a*b^6*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/
a^(1/2)))/((a*x^(1/2)+b)*x^(1/2))^(1/2)/(a*x^(1/2)+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/(a*x + b*sqrt(x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a*x + b*x^(1/2))^(3/2),x)
```

```
[Out] int(x^(5/2)/(a*x + b*x^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(x**(5/2)/(a*x + b*sqrt(x))**(3/2), x)
```

3.82 $\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=113

$$\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{2a^{7/2}}-\frac{15b\sqrt{ax+b\sqrt{x}}}{2a^3}+\frac{5\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a^2}-\frac{4x^{3/2}}{a\sqrt{ax+b\sqrt{x}}}$$

Rubi [A] time = 0.10, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2018, 668, 670, 640, 620, 206}

$$\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{2a^{7/2}}-\frac{15b\sqrt{ax+b\sqrt{x}}}{2a^3}+\frac{5\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a^2}-\frac{4x^{3/2}}{a\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*x^(3/2))/(a*Sqrt[b*Sqrt[x] + a*x]) - (15*b*Sqrt[b*Sqrt[x] + a*x])/(2*a^
3) + (5*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/a^2 + (15*b^2*ArcTanh[(Sqrt[a]*Sqrt[
x])/Sqrt[b*Sqrt[x] + a*x]])/(2*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
```

```
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = 2\,\text{Subst}\left(\int \frac{x^4}{\left(bx+ax^2\right)^{3/2}}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}} + \frac{10\,\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}} + \frac{5\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a^2} - \frac{(15b)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{2a^2}$$

$$= -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{15b\sqrt{b\sqrt{x}+ax}}{2a^3} + \frac{5\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a^2} + \frac{\left(15b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x,\right.}{4a^3}$$

$$= -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{15b\sqrt{b\sqrt{x}+ax}}{2a^3} + \frac{5\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a^2} + \frac{\left(15b^2\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \right.}{2a^3}$$

$$= -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{15b\sqrt{b\sqrt{x}+ax}}{2a^3} + \frac{5\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a^2} + \frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{2a^{7/2}}$$

Mathematica [C] time = 0.07, size = 62, normalized size = 0.55

$$\frac{4x^2\sqrt{\frac{a\sqrt{x}}{b}+1}\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};-\frac{a\sqrt{x}}{b}\right)}{7b\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (4*Sqrt[1 + (a*Sqrt[x])/b]*x^2*Hypergeometric2F1[3/2, 7/2, 9/2, -((a*Sqrt[x
])/b)])/(7*b*Sqrt[b*Sqrt[x] + a*x])
```

IntegrateAlgebraic [A] time = 0.47, size = 106, normalized size = 0.94

$$\frac{\sqrt{ax+b\sqrt{x}}\left(2a^2x-5ab\sqrt{x}-15b^2\right)}{2a^3\left(a\sqrt{x}+b\right)} - \frac{15b^2\log\left(-2a^{7/2}\sqrt{ax+b\sqrt{x}}+2a^4\sqrt{x}+a^3b\right)}{4a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (Sqrt[b*Sqrt[x] + a*x]*(-15*b^2 - 5*a*b*Sqrt[x] + 2*a^2*x))/(2*a^3*(b + a*S
qrt[x])) - (15*b^2*Log[a^3*b + 2*a^4*Sqrt[x] - 2*a^(7/2)*Sqrt[b*Sqrt[x] + a
*x]])/(4*a^(7/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueDone
```

maple [B] time = 0.06, size = 440, normalized size = 3.89

$$\frac{\sqrt{ax+b\sqrt{x}}\,\left(16a^{3}b^{2}x\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-a^{3}b^{2}x\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+32a^{2}b^{3}\sqrt{x}\,\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-2a^{2}b^{3}\sqrt{x}\,\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+4\sqrt{ax+b\sqrt{x}}\,a^{\frac{9}{2}}x^{\frac{3}{2}}+16a\,b^{4}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-a\,b^{4}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-32\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{7}{2}}bx+10\sqrt{ax+b\sqrt{x}}\,a^{\frac{7}{2}}bx-64\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{5}{2}}b^{2}\sqrt{x}+8\sqrt{ax+b\sqrt{x}}\,a^{\frac{5}{2}}b^{2}\sqrt{x}-32\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{3}{2}}b^{3}+2\sqrt{ax+b\sqrt{x}}\,a^{\frac{3}{2}}b^{3}+16\left(\left(a\sqrt{x}+b\right)\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{5}{2}}b\right)}{4\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\left(a\sqrt{x}+b\right)^{2}a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] 1/4*(a*x+b*x^(1/2))^(1/2)/a^(9/2)*(4*x^(3/2)*(a*x+b*x^(1/2))^(1/2)*a^(9/2)-
32*x*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(7/2)*b+10*x*(a*x+b*x^(1/2))^(1/2)*a^(
7/2)*b+16*x*a^3*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/
2))/a^(1/2))*b^2-x*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a
^(1/2))*a^3*b^2-64*x^(1/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(5/2)*b^2+8*(a*x
+b*x^(1/2))^(1/2)*a^(5/2)*b^2*x^(1/2)+32*x^(1/2)*a^2*ln(1/2*(2*a*x^(1/2)+b+
2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*b^3+16*((a*x^(1/2)+b)*x^(
1/2))^(3/2)*a^(5/2)*b-2*x^(1/2)*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/
2)*a^(1/2))/a^(1/2))*a^2*b^3-32*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(3/2)*b^3+2
*(a*x+b*x^(1/2))^(1/2)*a^(3/2)*b^3+16*a*b^4*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(
1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))-a*b^4*ln(1/2*(2*a*x^(1/2)+b+2*(a*x
+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2)))/((a*x^(1/2)+b)*x^(1/2))^(1/2)/(a*x^(1/
2)+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/(a*x + b*sqrt(x))^(3/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a*x + b*x^(1/2))^(3/2),x)
```

```
[Out] int(x^(3/2)/(a*x + b*x^(1/2))^(3/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(x**(3/2)/(a*x + b*sqrt(x))**(3/2), x)
```

3.83 $\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=60

$$\frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}-\frac{4\sqrt{x}}{a\sqrt{ax+b\sqrt{x}}}$$

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2018, 652, 620, 206}

$$\frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}-\frac{4\sqrt{x}}{a\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*Sqrt[x])/(a*Sqrt[b*Sqrt[x] + a*x]) + (4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[
b*Sqrt[x] + a*x]])/a^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))^2*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[(e*(d + e*x)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[
(e^2*(p + 2))/(c*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{\left(bx+ax^2\right)^{3/2}}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{4\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}$$
$$= -\frac{4\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}} + \frac{4\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a}$$
$$= -\frac{4\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}} + \frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{3/2}}$$

Mathematica [A] time = 0.11, size = 79, normalized size = 1.32

$$\frac{4\sqrt[4]{x}\left(\sqrt{b}\sqrt{\frac{a\sqrt{x}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}}\right)-\sqrt{a}\sqrt[4]{x}\right)}{a^{3/2}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (4*x^(1/4)*(-(Sqrt[a]*x^(1/4)) + Sqrt[b]*Sqrt[1 + (a*Sqrt[x])/b]*ArcSinh[(S
qrt[a]*x^(1/4))/Sqrt[b]]))/(a^(3/2)*Sqrt[b*Sqrt[x] + a*x])
```

IntegrateAlgebraic [A] time = 0.30, size = 76, normalized size = 1.27

$$-\frac{2\log\left(-2a^{3/2}\sqrt{ax+b\sqrt{x}}+2a^2\sqrt{x}+ab\right)}{a^{3/2}}-\frac{4\sqrt{ax+b\sqrt{x}}}{a\left(a\sqrt{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(a*(b + a*Sqrt[x])) - (2*Log[a*b + 2*a^2*Sqrt[x]
 - 2*a^(3/2)*Sqrt[b*Sqrt[x] + a*x]])/a^(3/2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%%{1,[1]%%%},[2,2]%%%}+%%%{%%{[-2,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,3]%
%%}+%%%{1,[0,4]%%%} / %%%{%%%{1,[2]%%%},[2,0]%%%}+%%%{%%{[%%%{-2,[1]%%%},0]
:[1,0,%%%{-1,[1]%%%}]%%},[1,1]%%%}+%%%{%%%{1,[1]%%%},[0,2]%%%} Error: Bad A
rgument Value
```

maple [B] time = 0.05, size = 240, normalized size = 4.00

$$-\frac{2\sqrt{ax+b\sqrt{x}}\,\left(-a^{2}bx\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-2ab^{2}\sqrt{x}\,\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-b^{3}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{5}{2}}x+4\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{3}{2}}b\sqrt{x}+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}\,b^{2}-2\left(\left(a\sqrt{x}+b\right)\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{3}{2}}\right)}{\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\left(a\sqrt{x}+b\right)^{2}a^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] -2*(a*x+b*x^(1/2))^(1/2)/a^(3/2)*(2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(5/2)*x
-a^2*b*x*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(
1/2))+4*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(3/2)*b*x^(1/2)-2*a*b^2*x^(1/2)*ln(
1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))-2*((a*
x^(1/2)+b)*x^(1/2))^(3/2)*a^(3/2)+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2)*b
^2-b^3*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/
2)))/((a*x^(1/2)+b)*x^(1/2))^(1/2)/b/(a*x^(1/2)+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/(a*x + b*sqrt(x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a*x + b*x^(1/2))^(3/2),x)
```

```
[Out] int(x^(1/2)/(a*x + b*x^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(sqrt(x)/(a*x + b*sqrt(x))**(3/2), x)
```

3.84 $\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=30

$$-\frac{4\left(2a\sqrt{x}+b\right)}{b^2\sqrt{ax+b\sqrt{x}}}$$

Rubi [A] time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2013, 613}

$$-\frac{4\left(2a\sqrt{x}+b\right)}{b^2\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(b*Sqrt[x] + a*x)^(3/2)),x]
[Out] (-4*(b + 2*a*Sqrt[x]))/(b^2*Sqrt[b*Sqrt[x] + a*x])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\left(bx+ax^2\right)^{3/2}}\,dx, x, \sqrt{x}\right)\\ &= -\frac{4\left(b+2a\sqrt{x}\right)}{b^2\sqrt{b\sqrt{x}+ax}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 45, normalized size = 1.50

$$-\frac{4\left(2a\sqrt{x}+b\right)\sqrt{ax+b\sqrt{x}}}{ab^2x+b^3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(b*Sqrt[x] + a*x)^(3/2)),x]
[Out] (-4*(b + 2*a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(b^3*Sqrt[x] + a*b^2*x)
```

IntegrateAlgebraic [A] time = 0.28, size = 46, normalized size = 1.53

$$-\frac{4\left(2a\sqrt{x}+b\right)\sqrt{ax+b\sqrt{x}}}{b^2\sqrt{x}\left(a\sqrt{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(b*Sqrt[x] + a*x)^(3/2)),x]
[Out] (-4*(b + 2*a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(b^2*(b + a*Sqrt[x])*Sqrt[x])
```

fricas [B] time = 0.63, size = 54, normalized size = 1.80

$$\frac{4\left(abx-\left(2\,a^2x-b^2\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{a^2b^2x^2-b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
[Out] 4*(a*b*x - (2*a^2*x - b^2)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^2*x^2 - b^
4*x)
```

giac [A] time = 0.18, size = 26, normalized size = 0.87

$$-\frac{4\left(\frac{2\,a\sqrt{x}}{b^2}+\frac{1}{b}\right)}{\sqrt{ax+b\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
[Out] -4*(2*a*sqrt(x)/b^2 + 1/b)/sqrt(a*x + b*sqrt(x))
```

maple [B] time = 0.06, size = 111, normalized size = 3.70

$$-\frac{4\sqrt{ax+b\sqrt{x}}\,\left(\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^2x-\left(\left(a\sqrt{x}+b\right)\sqrt{x}\right)^{\frac{3}{2}}a^2x+2\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}ab\sqrt{x}+\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}b^2\right)}{\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\left(a\sqrt{x}+b\right)^2b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(a*x+b*x^(1/2))^(3/2),x)
[Out] -4*(a*x+b*x^(1/2))^(1/2)*((a*x+b*x^(1/2))^(3/2)*x*a^2+2*(a*x+b*x^(1/2))^(3/
2)*x^(1/2)*a*b-((a*x^(1/2)+b)*x^(1/2))^(3/2)*x*a^2+(a*x+b*x^(1/2))^(3/2)*b^
2)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/b^3/(a*x^(1/2)+b)^2/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{x}\,\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a*x + b*x^(1/2))^(3/2)),x)
[Out] int(1/(x^(1/2)*(a*x + b*x^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x**(1/2)+a*x)**(3/2),x)
[Out] Integral(1/(sqrt(x)*(a*x + b*sqrt(x))**(3/2)), x)
```

3.85 $\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{64a^2\sqrt{ax+b\sqrt{x}}}{5b^4\sqrt{x}}+\frac{32a\sqrt{ax+b\sqrt{x}}}{5b^3x}-\frac{24\sqrt{ax+b\sqrt{x}}}{5b^2x^{3/2}}+\frac{4}{bx\sqrt{ax+b\sqrt{x}}}$$

Rubi [A] time = 0.16, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2015, 2016, 2014}

$$-\frac{64a^2\sqrt{ax+b\sqrt{x}}}{5b^4\sqrt{x}}-\frac{24\sqrt{ax+b\sqrt{x}}}{5b^2x^{3/2}}+\frac{32a\sqrt{ax+b\sqrt{x}}}{5b^3x}+\frac{4}{bx\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*x*Sqrt[b*Sqrt[x] + a*x]) - (24*Sqrt[b*Sqrt[x] + a*x])/(5*b^2*x^(3/2))
+ (32*a*Sqrt[b*Sqrt[x] + a*x])/(5*b^3*x) - (64*a^2*Sqrt[b*Sqrt[x] + a*x])/(
5*b^4*Sqrt[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{bx\sqrt{b\sqrt{x}+ax}} + \frac{6\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{b}$$

$$= \frac{4}{bx\sqrt{b\sqrt{x}+ax}} - \frac{24\sqrt{b\sqrt{x}+ax}}{5b^2x^{3/2}} - \frac{(24a)\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{5b^2}$$

$$= \frac{4}{bx\sqrt{b\sqrt{x}+ax}} - \frac{24\sqrt{b\sqrt{x}+ax}}{5b^2x^{3/2}} + \frac{32a\sqrt{b\sqrt{x}+ax}}{5b^3x} + \frac{\left(16a^2\right)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{5b^3}$$

$$= \frac{4}{bx\sqrt{b\sqrt{x}+ax}} - \frac{24\sqrt{b\sqrt{x}+ax}}{5b^2x^{3/2}} + \frac{32a\sqrt{b\sqrt{x}+ax}}{5b^3x} - \frac{64a^2\sqrt{b\sqrt{x}+ax}}{5b^4\sqrt{x}}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.53

$$-\frac{4\left(16a^3x^{3/2}+8a^2bx-2ab^2\sqrt{x}+b^3\right)}{5b^4x\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*(b^3 - 2*a*b^2*Sqrt[x] + 8*a^2*b*x + 16*a^3*x^(3/2)))/(5*b^4*x*Sqrt[b*S
qrt[x] + a*x])
```

IntegrateAlgebraic [A] time = 0.31, size = 70, normalized size = 0.65

$$-\frac{4\sqrt{ax+b\sqrt{x}}\left(16a^3x^{3/2}+8a^2bx-2ab^2\sqrt{x}+b^3\right)}{5b^4x^{3/2}\left(a\sqrt{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(b^3 - 2*a*b^2*Sqrt[x] + 8*a^2*b*x + 16*a^3*x^(3/
2)))/(5*b^4*(b + a*Sqrt[x])*x^(3/2))
```

fricas [A] time = 0.65, size = 79, normalized size = 0.74

$$\frac{4\left(8a^3bx^2-3ab^3x-\left(16a^4x^2-10a^2b^2x-b^4\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{5\left(a^2b^4x^3-b^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 4/5*(8*a^3*b*x^2 - 3*a*b^3*x - (16*a^4*x^2 - 10*a^2*b^2*x - b^4)*sqrt(x))*s
qrt(a*x + b*sqrt(x))/(a^2*b^4*x^3 - b^6*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(3/2)), x)
```

maple [C] time = 0.06, size = 548, normalized size = 5.12

$$\frac{2\sqrt{ax+b\sqrt{x}}\left(-5a^{5}bx^{\frac{9}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+5a^{5}bx^{\frac{9}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-10a^{4}b^{2}x^{4}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+10a^{4}b^{2}x^{4}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-5a^{3}b^{3}x^{\frac{7}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+5a^{3}b^{3}x^{\frac{7}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+10\sqrt{ax+b\sqrt{x}}\,a^{\frac{11}{2}}x^{\frac{9}{2}}+10\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{11}{2}}x^{\frac{9}{2}}+20\sqrt{ax+b\sqrt{x}}\,a^{\frac{9}{2}}bx^{4}+20\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{9}{2}}bx^{4}+10\sqrt{ax+b\sqrt{x}}\,a^{\frac{7}{2}}b^{2}x^{\frac{7}{2}}+10\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{7}{2}}b^{2}x^{\frac{7}{2}}-30\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{9}{2}}x^{\frac{7}{2}}+10\left(\left(a\sqrt{x}+b\right)\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{9}{2}}x^{\frac{7}{2}}-52\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{7}{2}}bx^{3}-16\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{5}{2}}b^{2}x^{\frac{5}{2}}+4\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{3}{2}}b^{3}x^{2}-2\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}\sqrt{a}\,b^{4}x^{\frac{3}{2}}\right)}{5\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\left(a\sqrt{x}+b\right)^{2}\sqrt{a}\,b^{5}x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] 2/5*(a*x+b*x^(1/2))^(1/2)*(5*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*
a^(1/2))/a^(1/2))*x^(9/2)*a^5*b-30*(a*x+b*x^(1/2))^(3/2)*a^(9/2)*x^(7/2)+10
*(a*x+b*x^(1/2))^(1/2)*a^(11/2)*x^(9/2)-5*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/
2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^(9/2)*a^5*b+10*a^(11/2)*x^(9/2)*((
a*x^(1/2)+b)*x^(1/2))^(1/2)+10*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2
)*a^(1/2))/a^(1/2))*x^4*a^4*b^2-16*(a*x+b*x^(1/2))^(3/2)*a^(5/2)*x^(5/2)*b^
2-52*(a*x+b*x^(1/2))^(3/2)*a^(7/2)*x^3*b+20*(a*x+b*x^(1/2))^(1/2)*a^(9/2)*x
^4*b-10*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1
/2))*x^4*a^4*b^2+20*a^(9/2)*x^4*((a*x^(1/2)+b)*x^(1/2))^(1/2)*b+10*a^(9/2)*
x^(7/2)*((a*x^(1/2)+b)*x^(1/2))^(3/2)+5*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1
/2))^(1/2)*a^(1/2))/a^(1/2))*x^(7/2)*a^3*b^3+4*(a*x+b*x^(1/2))^(3/2)*a^(3/2
)*x^2*b^3+10*(a*x+b*x^(1/2))^(1/2)*a^(7/2)*x^(7/2)*b^2-5*ln(1/2*(2*a*x^(1/2
)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^(7/2)*a^3*b^3+10*a^
(7/2)*x^(7/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*b^2-2*(a*x+b*x^(1/2))^(3/2)*a^(
1/2)*x^(3/2)*b^4)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/b^5/x^(7/2)/a^(1/2)/(a*x^(1
/2)+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3/2}\,{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a*x + b*x^(1/2))^(3/2)),x)
```

```
[Out] int(1/(x^(3/2)*(a*x + b*x^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**(3/2)*(a*x + b*sqrt(x))**(3/2)), x)
```

3.86 $\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=165

$$-\frac{1024a^4\sqrt{ax+b\sqrt{x}}}{63b^6\sqrt{x}}+\frac{512a^3\sqrt{ax+b\sqrt{x}}}{63b^5x}-\frac{128a^2\sqrt{ax+b\sqrt{x}}}{21b^4x^{3/2}}+\frac{320a\sqrt{ax+b\sqrt{x}}}{63b^3x^2}-\frac{40\sqrt{ax+b\sqrt{x}}}{9b^2x^{5/2}}+\frac{4}{bx^2\sqrt{ax+b\sqrt{x}}}$$

Rubi [A] time = 0.26, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2015, 2016, 2014}

$$-\frac{128a^2\sqrt{ax+b\sqrt{x}}}{21b^4x^{3/2}}-\frac{1024a^4\sqrt{ax+b\sqrt{x}}}{63b^6\sqrt{x}}+\frac{512a^3\sqrt{ax+b\sqrt{x}}}{63b^5x}+\frac{320a\sqrt{ax+b\sqrt{x}}}{63b^3x^2}-\frac{40\sqrt{ax+b\sqrt{x}}}{9b^2x^{5/2}}+\frac{4}{bx^2\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*x^2*Sqrt[b*Sqrt[x] + a*x]) - (40*Sqrt[b*Sqrt[x] + a*x])/(9*b^2*x^(5/2)
) + (320*a*Sqrt[b*Sqrt[x] + a*x])/(63*b^3*x^2) - (128*a^2*Sqrt[b*Sqrt[x] +
a*x])/(21*b^4*x^(3/2)) + (512*a^3*Sqrt[b*Sqrt[x] + a*x])/(63*b^5*x) - (1024
*a^4*Sqrt[b*Sqrt[x] + a*x])/(63*b^6*Sqrt[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}} + \frac{10\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx}{b}$$

$$= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}} - \frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}} - \frac{(80a)\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{9b^2}$$

$$= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}} - \frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}} + \frac{320a\sqrt{b\sqrt{x}+ax}}{63b^3x^2} + \frac{\left(160a^2\right)\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{21b^3}$$

$$= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}} - \frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}} + \frac{320a\sqrt{b\sqrt{x}+ax}}{63b^3x^2} - \frac{128a^2\sqrt{b\sqrt{x}+ax}}{21b^4x^{3/2}} - \cdots$$

$$= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}} - \frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}} + \frac{320a\sqrt{b\sqrt{x}+ax}}{63b^3x^2} - \frac{128a^2\sqrt{b\sqrt{x}+ax}}{21b^4x^{3/2}} + \cdots$$

$$= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}} - \frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}} + \frac{320a\sqrt{b\sqrt{x}+ax}}{63b^3x^2} - \frac{128a^2\sqrt{b\sqrt{x}+ax}}{21b^4x^{3/2}} + \cdots$$

Mathematica [A] time = 0.07, size = 83, normalized size = 0.50

$$-\frac{4\left(256a^5x^{5/2}+128a^4bx^2-32a^3b^2x^{3/2}+16a^2b^3x-10ab^4\sqrt{x}+7b^5\right)}{63b^6x^2\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*(7*b^5 - 10*a*b^4*Sqrt[x] + 16*a^2*b^3*x - 32*a^3*b^2*x^(3/2) + 128*a^4
*b*x^2 + 256*a^5*x^(5/2)))/(63*b^6*x^2*Sqrt[b*Sqrt[x] + a*x])
```

IntegrateAlgebraic [A] time = 0.35, size = 96, normalized size = 0.58

$$-\frac{4\sqrt{ax+b\sqrt{x}}\left(256a^5x^{5/2}+128a^4bx^2-32a^3b^2x^{3/2}+16a^2b^3x-10ab^4\sqrt{x}+7b^5\right)}{63b^6x^{5/2}\left(a\sqrt{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(7*b^5 - 10*a*b^4*Sqrt[x] + 16*a^2*b^3*x - 32*a^3
*b^2*x^(3/2) + 128*a^4*b*x^2 + 256*a^5*x^(5/2)))/(63*b^6*(b + a*Sqrt[x])*x^
(5/2))
```

fricas [A] time = 0.58, size = 101, normalized size = 0.61

$$\frac{4\left(128\,a^5bx^3-48\,a^3b^3x^2-17\,ab^5x-\left(256\,a^6x^3-160\,a^4b^2x^2-26\,a^2b^4x-7\,b^6\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{63\left(a^2b^6x^4-b^8x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 4/63*(128*a^5*b*x^3 - 48*a^3*b^3*x^2 - 17*a*b^5*x - (256*a^6*x^3 - 160*a^4*
b^2*x^2 - 26*a^2*b^4*x - 7*b^6)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^6*x^4
 - b^8*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}} x^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(5/2)), x)
```

maple [C] time = 0.07, size = 592, normalized size = 3.59

$$\frac{4\sqrt{ax+b\sqrt{x}}\,\left(-63a^{7}bx^{\frac{13}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+63a^{7}bx^{\frac{13}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-126a^{6}b^{2}x^{6}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+126a^{6}b^{2}x^{6}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-63a^{5}b^{3}x^{\frac{11}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+63a^{5}b^{3}x^{\frac{11}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+126\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{15}{2}}x^{\frac{13}{2}}+126\sqrt{ax+b\sqrt{x}}\,a^{\frac{15}{2}}x^{\frac{13}{2}}+252\sqrt{ax+b\sqrt{x}}\,a^{\frac{13}{2}}bx^{6}+252\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{13}{2}}bx^{6}+126\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{11}{2}}b^{2}x^{\frac{11}{2}}+126\sqrt{ax+b\sqrt{x}}\,a^{\frac{11}{2}}b^{2}x^{\frac{11}{2}}+63\left(\left(a\sqrt{x}+b\right)\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{13}{2}}x^{\frac{11}{2}}-315\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{13}{2}}x^{\frac{11}{2}}-508\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{11}{2}}bx^{5}-128\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{9}{2}}b^{2}x^{\frac{9}{2}}+32\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{7}{2}}b^{3}x^{4}-16\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{5}{2}}b^{4}x^{\frac{7}{2}}+10\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{3}{2}}b^{5}x^{3}-7\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}\sqrt{a}\,b^{6}x^{\frac{5}{2}}\right)}{63\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\left(a\sqrt{x}+b\right)^{2}\sqrt{a}\,b^{7}x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] 4/63*(a*x+b*x^(1/2))^(1/2)*(126*x^6*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))
^(1/2)*a^(1/2))/a^(1/2))*a^6*b^2+63*x^(11/2)*((a*x^(1/2)+b)*x^(1/2))^(3/2)*
a^(13/2)-16*x^(7/2)*(a*x+b*x^(1/2))^(3/2)*a^(5/2)*b^4-63*x^(11/2)*ln(1/2*(2
*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^5*b^3+63*x
^(11/2)*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^5
*b^3+10*x^3*(a*x+b*x^(1/2))^(3/2)*a^(3/2)*b^5+126*x^(11/2)*((a*x^(1/2)+b)*x
^(1/2))^(1/2)*a^(11/2)*b^2+126*x^(11/2)*(a*x+b*x^(1/2))^(1/2)*a^(11/2)*b^2+
252*x^6*(a*x+b*x^(1/2))^(1/2)*a^(13/2)*b+32*x^4*(a*x+b*x^(1/2))^(3/2)*a^(7/
2)*b^3-508*x^5*(a*x+b*x^(1/2))^(3/2)*a^(11/2)*b-128*x^(9/2)*(a*x+b*x^(1/2))
^(3/2)*a^(9/2)*b^2+252*x^6*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(13/2)*b-7*x^(5/
2)*(a*x+b*x^(1/2))^(3/2)*a^(1/2)*b^6+126*x^(13/2)*((a*x^(1/2)+b)*x^(1/2))^(
1/2)*a^(15/2)-315*x^(11/2)*(a*x+b*x^(1/2))^(3/2)*a^(13/2)+126*x^(13/2)*(a*x
+b*x^(1/2))^(1/2)*a^(15/2)-63*x^(13/2)*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+
b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^7*b+63*x^(13/2)*ln(1/2*(2*a*x^(1/2)+b
+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^7*b-126*x^6*ln(1/2*(2*a*x^(1/2
)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^6*b^2)/((a*x^(1/2)+
b)*x^(1/2))^(1/2)/b^7/x^(11/2)/a^(1/2)/(a*x^(1/2)+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}} x^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{5/2}\,{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a*x + b*x^(1/2))^(3/2)),x)
[Out] int(1/(x^(5/2)*(a*x + b*x^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}} \left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**(1/2)+a*x)**(3/2),x)
[Out] Integral(1/(x**(5/2)*(a*x + b*sqrt(x))**(3/2)), x)
```

3.87 $\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=223

$$-\frac{8192a^6\sqrt{ax+b\sqrt{x}}}{429b^8\sqrt{x}}+\frac{4096a^5\sqrt{ax+b\sqrt{x}}}{429b^7x}-\frac{1024a^4\sqrt{ax+b\sqrt{x}}}{143b^6x^{3/2}}+\frac{2560a^3\sqrt{ax+b\sqrt{x}}}{429b^5x^2}-\frac{2240a^2\sqrt{ax+b\sqrt{x}}}{429b^4x^{5/2}}+67$$

Rubi [A] time = 0.35, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2015, 2016, 2014}

$$-\frac{1024a^4\sqrt{ax+b\sqrt{x}}}{143b^6x^{3/2}}+\frac{2560a^3\sqrt{ax+b\sqrt{x}}}{429b^5x^2}-\frac{2240a^2\sqrt{ax+b\sqrt{x}}}{429b^4x^{5/2}}-\frac{8192a^6\sqrt{ax+b\sqrt{x}}}{429b^8\sqrt{x}}+\frac{4096a^5\sqrt{ax+b\sqrt{x}}}{429b^7x}+\frac{672a\sqrt{ax+b\sqrt{x}}}{143b^3x^3}-\frac{56\sqrt{ax+b\sqrt{x}}}{13b^2x^{7/2}}+\frac{4}{bx^3\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*x^3*Sqrt[b*Sqrt[x] + a*x]) - (56*Sqrt[b*Sqrt[x] + a*x])/(13*b^2*x^(7/2
)) + (672*a*Sqrt[b*Sqrt[x] + a*x])/(143*b^3*x^3) - (2240*a^2*Sqrt[b*Sqrt[x]
 + a*x])/(429*b^4*x^(5/2)) + (2560*a^3*Sqrt[b*Sqrt[x] + a*x])/(429*b^5*x^2)
 - (1024*a^4*Sqrt[b*Sqrt[x] + a*x])/(143*b^6*x^(3/2)) + (4096*a^5*Sqrt[b*Sq
rt[x] + a*x])/(429*b^7*x) - (8192*a^6*Sqrt[b*Sqrt[x] + a*x])/(429*b^8*Sqrt[
x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}} + \frac{14\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx}{b}$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}} - \frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}} - \frac{(168a)\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx}{13b^2}$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}} - \frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}} + \frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3} + \frac{\left(1680a^2\right)\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,d}{143b^3}$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}} - \frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}} + \frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3} - \frac{2240a^2\sqrt{b\sqrt{x}+ax}}{429b^4x^{5/2}} - (\ldots$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}} - \frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}} + \frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3} - \frac{2240a^2\sqrt{b\sqrt{x}+ax}}{429b^4x^{5/2}} + 2\ldots$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}} - \frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}} + \frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3} - \frac{2240a^2\sqrt{b\sqrt{x}+ax}}{429b^4x^{5/2}} + 2\ldots$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}} - \frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}} + \frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3} - \frac{2240a^2\sqrt{b\sqrt{x}+ax}}{429b^4x^{5/2}} + 2\ldots$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}} - \frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}} + \frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3} - \frac{2240a^2\sqrt{b\sqrt{x}+ax}}{429b^4x^{5/2}} + 2\ldots$$

Mathematica [A] time = 0.07, size = 107, normalized size = 0.48

$$-\frac{4\left(2048a^7x^{7/2}+1024a^6bx^3-256a^5b^2x^{5/2}+128a^4b^3x^2-80a^3b^4x^{3/2}+56a^2b^5x-42ab^6\sqrt{x}+33b^7\right)}{429b^8x^3\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*(33*b^7 - 42*a*b^6*Sqrt[x] + 56*a^2*b^5*x - 80*a^3*b^4*x^(3/2) + 128*a^
4*b^3*x^2 - 256*a^5*b^2*x^(5/2) + 1024*a^6*b*x^3 + 2048*a^7*x^(7/2)))/(429*
b^8*x^3*Sqrt[b*Sqrt[x] + a*x])
```

IntegrateAlgebraic [A] time = 0.36, size = 120, normalized size = 0.54

$$-\frac{4\sqrt{ax+b\sqrt{x}}\left(2048a^7x^{7/2}+1024a^6bx^3-256a^5b^2x^{5/2}+128a^4b^3x^2-80a^3b^4x^{3/2}+56a^2b^5x-42ab^6\sqrt{x}+33b^7\right)}{429b^8x^{7/2}\left(a\sqrt{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(33*b^7 - 42*a*b^6*Sqrt[x] + 56*a^2*b^5*x - 80*a^
3*b^4*x^(3/2) + 128*a^4*b^3*x^2 - 256*a^5*b^2*x^(5/2) + 1024*a^6*b*x^3 + 20
48*a^7*x^(7/2)))/(429*b^8*(b + a*Sqrt[x])*x^(7/2))
```

fricas [A] time = 0.48, size = 123, normalized size = 0.55

$$\frac{4\left(1024\,a^{7}bx^{4}-384\,a^{5}b^{3}x^{3}-136\,a^{3}b^{5}x^{2}-75\,ab^{7}x-\left(2048\,a^{8}x^{4}-1280\,a^{6}b^{2}x^{3}-208\,a^{4}b^{4}x^{2}-98\,a^{2}b^{6}x-33\,b^{8}\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{429\left(a^{2}b^{8}x^{5}-b^{10}x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 4/429*(1024*a^7*b*x^4 - 384*a^5*b^3*x^3 - 136*a^3*b^5*x^2 - 75*a*b^7*x - (2
048*a^8*x^4 - 1280*a^6*b^2*x^3 - 208*a^4*b^4*x^2 - 98*a^2*b^6*x - 33*b^8)*s
qrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^8*x^5 - b^10*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(7/2)), x)
```

maple [C] time = 0.09, size = 636, normalized size = 2.85

$$\frac{2\sqrt{ax+b\sqrt{x}}\left(-1287a^{9}bx^{\frac{17}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+1287a^{9}bx^{\frac{17}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-2574a^{8}b^{2}x^{8}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+2574a^{8}b^{2}x^{8}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-1287a^{7}b^{3}x^{\frac{15}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+1287a^{7}b^{3}x^{\frac{15}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+2574\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{19}{2}}x^{\frac{17}{2}}+2574\sqrt{ax+b\sqrt{x}}\,a^{\frac{19}{2}}x^{\frac{17}{2}}+5148\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{17}{2}}bx^{8}+5148\sqrt{ax+b\sqrt{x}}\,a^{\frac{17}{2}}bx^{8}+2574\sqrt{ax+b\sqrt{x}}\,a^{\frac{15}{2}}b^{2}x^{\frac{15}{2}}+2574\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{15}{2}}b^{2}x^{\frac{15}{2}}-6006\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{17}{2}}x^{\frac{15}{2}}+858\left(\left(a\sqrt{x}+b\right)\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{17}{2}}x^{\frac{15}{2}}-9244\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{15}{2}}bx^{7}-2048\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{13}{2}}b^{2}x^{\frac{13}{2}}+512\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{11}{2}}b^{3}x^{6}-256\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{9}{2}}b^{4}x^{\frac{11}{2}}+160\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{7}{2}}b^{5}x^{5}-112\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{5}{2}}b^{6}x^{\frac{9}{2}}+84\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^{\frac{3}{2}}b^{7}x^{4}-66\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}\sqrt{a}\,b^{8}x^{\frac{7}{2}}\right)}{429\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\left(a\sqrt{x}+b\right)^{2}\sqrt{a}\,b^{9}x^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] 2/429*(a*x+b*x^(1/2))^(1/2)*(2574*x^(15/2)*(a*x+b*x^(1/2))^(1/2)*a^(15/2)*b
^2+160*x^5*(a*x+b*x^(1/2))^(3/2)*a^(7/2)*b^5-2048*x^(13/2)*(a*x+b*x^(1/2))^
(3/2)*a^(13/2)*b^2+5148*x^8*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(17/2)*b-9244*x
^7*(a*x+b*x^(1/2))^(3/2)*a^(15/2)*b+5148*x^8*(a*x+b*x^(1/2))^(1/2)*a^(17/2)
*b-256*x^(11/2)*(a*x+b*x^(1/2))^(3/2)*a^(9/2)*b^4+512*x^6*(a*x+b*x^(1/2))^(
3/2)*a^(11/2)*b^3+2574*x^(15/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(15/2)*b^2+
2574*x^(17/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(19/2)-66*x^(7/2)*(a*x+b*x^(1
/2))^(3/2)*a^(1/2)*b^8-6006*x^(15/2)*(a*x+b*x^(1/2))^(3/2)*a^(17/2)+2574*x^
(17/2)*(a*x+b*x^(1/2))^(1/2)*a^(19/2)-1287*x^(17/2)*ln(1/2*(2*a*x^(1/2)+b+2
*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^9*b+1287*x^(17/2)*ln(1/2
*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^9*b-2574*x^8*ln
(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^8*b
^2+2574*x^8*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))
*a^8*b^2+858*x^(15/2)*((a*x^(1/2)+b)*x^(1/2))^(3/2)*a^(17/2)-1287*x^(15/2)*
ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^7
*b^3+1287*x^(15/2)*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a
^(1/2))*a^7*b^3-112*x^(9/2)*(a*x+b*x^(1/2))^(3/2)*a^(5/2)*b^6+84*x^4*(a*x+b
*x^(1/2))^(3/2)*a^(3/2)*b^7)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/b^9/x^(15/2)/a^(
1/2)/(a*x^(1/2)+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{7/2}\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a*x + b*x^(1/2))^(3/2)),x)
[Out] int(1/(x^(7/2)*(a*x + b*x^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{7}{2}}\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**(1/2)+a*x)**(3/2),x)
[Out] Integral(1/(x**(7/2)*(a*x + b*sqrt(x))**(3/2)), x)
```

3.88 $\int x^3\sqrt{bx^{2/3}+ax}\,dx$

Optimal. Leaf size=371

$$\frac{8388608b^{12}\left(ax+bx^{2/3}\right)^{3/2}}{152108775a^{13}x}-\frac{4194304b^{11}\left(ax+bx^{2/3}\right)^{3/2}}{50702925a^{12}x^{2/3}}+\frac{1048576b^{10}\left(ax+bx^{2/3}\right)^{3/2}}{10140585a^{11}\sqrt[3]{x}}-\frac{524288b^{9}\left(ax+bx^{2/3}\right)^{3/2}}{4345965a^{10}}+$$

Rubi [A] time = 0.63, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$\frac{8388608b^{12}\left(ax+bx^{2/3}\right)^{3/2}}{152108775a^{13}x}-\frac{4194304b^{11}\left(ax+bx^{2/3}\right)^{3/2}}{50702925a^{12}x^{2/3}}+\frac{1048576b^{10}\left(ax+bx^{2/3}\right)^{3/2}}{10140585a^{11}\sqrt[3]{x}}-\frac{524288b^{9}\left(ax+bx^{2/3}\right)^{3/2}}{4345965a^{10}}+\frac{65536b^{8}\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{482885a^{9}}-\frac{360448b^{7}x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{2414425a^{8}}+\frac{90112b^{6}x\left(ax+bx^{2/3}\right)^{3/2}}{557175a^{7}}-\frac{45056b^{5}x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{260015a^{6}}+\frac{2816b^{4}x^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{15295a^{5}}-\frac{1408b^{3}x^{2}\left(ax+bx^{2/3}\right)^{3/2}}{7245a^{4}}+\frac{352b^{2}x^{7/3}\left(ax+bx^{2/3}\right)^{3/2}}{1725a^{3}}-\frac{16bx^{8/3}\left(ax+bx^{2/3}\right)^{3/2}}{75a^{2}}+\frac{2x^{3}\left(ax+bx^{2/3}\right)^{3/2}}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (-524288*b^9*(b*x^(2/3) + a*x)^(3/2))/(4345965*a^10) + (8388608*b^12*(b*x^(
2/3) + a*x)^(3/2))/(152108775*a^13*x) - (4194304*b^11*(b*x^(2/3) + a*x)^(3/
2))/(50702925*a^12*x^(2/3)) + (1048576*b^10*(b*x^(2/3) + a*x)^(3/2))/(10140
585*a^11*x^(1/3)) + (65536*b^8*x^(1/3)*(b*x^(2/3) + a*x)^(3/2))/(482885*a^9
) - (360448*b^7*x^(2/3)*(b*x^(2/3) + a*x)^(3/2))/(2414425*a^8) + (90112*b^6
*x*(b*x^(2/3) + a*x)^(3/2))/(557175*a^7) - (45056*b^5*x^(4/3)*(b*x^(2/3) +
a*x)^(3/2))/(260015*a^6) + (2816*b^4*x^(5/3)*(b*x^(2/3) + a*x)^(3/2))/(1529
5*a^5) - (1408*b^3*x^2*(b*x^(2/3) + a*x)^(3/2))/(7245*a^4) + (352*b^2*x^(7/
3)*(b*x^(2/3) + a*x)^(3/2))/(1725*a^3) - (16*b*x^(8/3)*(b*x^(2/3) + a*x)^(3
/2))/(75*a^2) + (2*x^3*(b*x^(2/3) + a*x)^(3/2))/(9*a)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^3\sqrt{bx^{2/3}+ax}\,dx &= \frac{2x^3\left(bx^{2/3}+ax\right)^{3/2}}{9a}-\frac{(8b)\int x^{8/3}\sqrt{bx^{2/3}+ax}\,dx}{9a}\\
&= -\frac{16bx^{8/3}\left(bx^{2/3}+ax\right)^{3/2}}{75a^2}+\frac{2x^3\left(bx^{2/3}+ax\right)^{3/2}}{9a}+\frac{\left(176b^2\right)\int x^{7/3}\sqrt{bx^{2/3}+ax}\,dx}{225a^2}\\
&= \frac{352b^2x^{7/3}\left(bx^{2/3}+ax\right)^{3/2}}{1725a^3}-\frac{16bx^{8/3}\left(bx^{2/3}+ax\right)^{3/2}}{75a^2}+\frac{2x^3\left(bx^{2/3}+ax\right)^{3/2}}{9a}-\left(704b^3\right)\int x\cdots\\
&= -\frac{1408b^3x^2\left(bx^{2/3}+ax\right)^{3/2}}{7245a^4}+\frac{352b^2x^{7/3}\left(bx^{2/3}+ax\right)^{3/2}}{1725a^3}-\frac{16bx^{8/3}\left(bx^{2/3}+ax\right)^{3/2}}{75a^2}+2x^3\cdots\\
&= \frac{2816b^4x^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{15295a^5}-\frac{1408b^3x^2\left(bx^{2/3}+ax\right)^{3/2}}{7245a^4}+\frac{352b^2x^{7/3}\left(bx^{2/3}+ax\right)^{3/2}}{1725a^3}-16\cdots\\
&= -\frac{45056b^5x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{260015a^6}+\frac{2816b^4x^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{15295a^5}-\frac{1408b^3x^2\left(bx^{2/3}+ax\right)^{3/2}}{7245a^4}\cdots\\
&= \frac{90112b^6x\left(bx^{2/3}+ax\right)^{3/2}}{557175a^7}-\frac{45056b^5x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{260015a^6}+\frac{2816b^4x^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{15295a^5}-\cdots\\
&= -\frac{360448b^7x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{2414425a^8}+\frac{90112b^6x\left(bx^{2/3}+ax\right)^{3/2}}{557175a^7}-\frac{45056b^5x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{260015a^6}\cdots\\
&= \frac{65536b^8\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{482885a^9}-\frac{360448b^7x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{2414425a^8}+\frac{90112b^6x\left(bx^{2/3}+ax\right)^{3/2}}{557175a^7}\cdots\\
&= -\frac{524288b^9\left(bx^{2/3}+ax\right)^{3/2}}{4345965a^{10}}+\frac{65536b^8\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{482885a^9}-\frac{360448b^7x^{2/3}\left(bx^{2/3}+ax\right)^{3}}{2414425a^8}\cdots\\
&= -\frac{524288b^9\left(bx^{2/3}+ax\right)^{3/2}}{4345965a^{10}}+\frac{1048576b^{10}\left(bx^{2/3}+ax\right)^{3/2}}{10140585a^{11}\sqrt[3]{x}}+\frac{65536b^8\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{482885a^9}\cdots\\
&= -\frac{524288b^9\left(bx^{2/3}+ax\right)^{3/2}}{4345965a^{10}}-\frac{4194304b^{11}\left(bx^{2/3}+ax\right)^{3/2}}{50702925a^{12}x^{2/3}}+\frac{1048576b^{10}\left(bx^{2/3}+ax\right)^{3/2}}{10140585a^{11}\sqrt[3]{x}}\cdots\\
&= -\frac{524288b^9\left(bx^{2/3}+ax\right)^{3/2}}{4345965a^{10}}+\frac{8388608b^{12}\left(bx^{2/3}+ax\right)^{3/2}}{152108775a^{13}x}-\frac{4194304b^{11}\left(bx^{2/3}+ax\right)^{3/2}}{50702925a^{12}x^{2/3}}\cdots
\end{aligned}
$$

Mathematica [A] time = 0.18, size = 181, normalized size = 0.49

$$\frac{2\left(a\sqrt[3]{x}+b\right)\sqrt{ax+bx^{2/3}}\left(16900975a^{12}x^4-16224936a^{11}bx^{11/3}+15519504a^{10}b^2x^{10/3}-14780480a^9b^3x^3+14002560a^8b^4x^{8/3}-13178880a^7b^5x^{7/3}+12300288a^6b^6x^2-11354112a^5b^7x^{5/3}+10321920a^4b^8x^{4/3}-9175040a^3b^9x+7864320a^2b^{10}x^{2/3}-6291456ab^{11}\sqrt[3]{x}+4194304b^{12}\right)}{152108775a^{13}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*(4194304*b^12 - 6291456*a*b^11*x^(
1/3) + 7864320*a^2*b^10*x^(2/3) - 9175040*a^3*b^9*x + 10321920*a^4*b^8*x^(4
/3) - 11354112*a^5*b^7*x^(5/3) + 12300288*a^6*b^6*x^2 - 13178880*a^7*b^5*x^
(7/3) + 14002560*a^8*b^4*x^(8/3) - 14780480*a^9*b^3*x^3 + 15519504*a^10*b^2
*x^(10/3) - 16224936*a^11*b*x^(11/3) + 16900975*a^12*x^4))/(152108775*a^13*
x^(1/3))
```

IntegrateAlgebraic [A] time = 0.12, size = 185, normalized size = 0.50

$$\frac{2\sqrt{ax+bx^{2/3}}\left(16900975a^{13}x^{13/3}+676039a^{12}bx^4-705432a^{11}b^2x^{11/3}+739024a^{10}b^3x^{10/3}-777920a^9b^4x^3+823680a^8b^5x^{8/3}-878592a^7b^6x^{7/3}+946176a^6b^7x^2-1032192a^5b^8x^{5/3}+1146880a^4b^9x^{4/3}-1310720a^3b^{10}x+1572864a^2b^{11}x^{2/3}-2097152ab^{12}\sqrt[3]{x}+4194304b^{13}\right)}{152108775a^{13}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(4194304*b^13 - 2097152*a*b^12*x^(1/3) + 1572864*a
^2*b^11*x^(2/3) - 1310720*a^3*b^10*x + 1146880*a^4*b^9*x^(4/3) - 1032192*a^
5*b^8*x^(5/3) + 946176*a^6*b^7*x^2 - 878592*a^7*b^6*x^(7/3) + 823680*a^8*b^
5*x^(8/3) - 777920*a^9*b^4*x^3 + 739024*a^10*b^3*x^(10/3) - 705432*a^11*b^2
*x^(11/3) + 676039*a^12*b*x^4 + 16900975*a^13*x^(13/3)))/(152108775*a^13*x^
(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.22, size = 396, normalized size = 1.07

$$-\frac{8388608\,b^{\frac{27}{2}}}{152108775\,a^{13}}+\frac{2\left(\frac{27\left(676039\left(ax^{\frac{1}{3}}+b\right)^{\frac{25}{2}}-8817900\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}}b+53117350\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}b^{2}-195695500\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}b^{3}+492116625\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b^{4}-892371480\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^{5}+1201269300\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^{6}-1216870200\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^{7}+929553625\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^{8}-531173500\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^{9}+223092870\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{10}-67603900\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{11}+16900975\sqrt{ax^{\frac{1}{3}}+b}\,b^{12}\right)b}{a^{12}}+\frac{13\left(1300075\left(ax^{\frac{1}{3}}+b\right)^{\frac{27}{2}}-18253053\left(ax^{\frac{1}{3}}+b\right)^{\frac{25}{2}}b+119041650\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}}b^{2}-478056150\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}b^{3}+1320944625\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}b^{4}-2657429775\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b^{5}+4015671660\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^{6}-4633467300\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^{7}+4106936925\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^{8}-2788660875\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^{9}+1434168450\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^{10}-547591590\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{11}+152108775\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{12}-35102025\sqrt{ax^{\frac{1}{3}}+b}\,b^{13}\right)}{a^{12}}\right)}{152108775\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -8388608/152108775*b^(27/2)/a^13 + 2/152108775*(27*(676039*(a*x^(1/3) + b)^
(25/2) - 8817900*(a*x^(1/3) + b)^(23/2)*b + 53117350*(a*x^(1/3) + b)^(21/2)
*b^2 - 195695500*(a*x^(1/3) + b)^(19/2)*b^3 + 492116625*(a*x^(1/3) + b)^(17
/2)*b^4 - 892371480*(a*x^(1/3) + b)^(15/2)*b^5 + 1201269300*(a*x^(1/3) + b)
^(13/2)*b^6 - 1216870200*(a*x^(1/3) + b)^(11/2)*b^7 + 929553625*(a*x^(1/3)
+ b)^(9/2)*b^8 - 531173500*(a*x^(1/3) + b)^(7/2)*b^9 + 223092870*(a*x^(1/3)
 + b)^(5/2)*b^10 - 67603900*(a*x^(1/3) + b)^(3/2)*b^11 + 16900975*sqrt(a*x^
(1/3) + b)*b^12)*b/a^12 + 13*(1300075*(a*x^(1/3) + b)^(27/2) - 18253053*(a*
x^(1/3) + b)^(25/2)*b + 119041650*(a*x^(1/3) + b)^(23/2)*b^2 - 478056150*(a
*x^(1/3) + b)^(21/2)*b^3 + 1320944625*(a*x^(1/3) + b)^(19/2)*b^4 - 26574297
75*(a*x^(1/3) + b)^(17/2)*b^5 + 4015671660*(a*x^(1/3) + b)^(15/2)*b^6 - 463
3467300*(a*x^(1/3) + b)^(13/2)*b^7 + 4106936925*(a*x^(1/3) + b)^(11/2)*b^8
- 2788660875*(a*x^(1/3) + b)^(9/2)*b^9 + 1434168450*(a*x^(1/3) + b)^(7/2)*b
^10 - 547591590*(a*x^(1/3) + b)^(5/2)*b^11 + 152108775*(a*x^(1/3) + b)^(3/2
)*b^12 - 35102025*sqrt(a*x^(1/3) + b)*b^13)/a^12)/a
```

maple [A] time = 0.07, size = 156, normalized size = 0.42

$$-\frac{2\sqrt{ax+b\,x^{\frac{2}{3}}}\,\left(a\,x^{\frac{1}{3}}+b\right)\left(-16900975a^{12}x^{4}+16224936a^{11}b\,x^{\frac{11}{3}}-15519504a^{10}b^{2}x^{\frac{10}{3}}+14780480a^{9}b^{3}x^{3}-14002560a^{8}b^{4}x^{\frac{8}{3}}+13178880a^{7}b^{5}x^{\frac{7}{3}}-12300288a^{6}b^{6}x^{2}+11354112a^{5}b^{7}x^{\frac{5}{3}}-10321920a^{4}b^{8}x^{\frac{4}{3}}+9175040a^{3}b^{9}x-7864320a^{2}b^{10}x^{\frac{2}{3}}+6291456a\,b^{11}x^{\frac{1}{3}}-4194304b^{12}\right)}{152108775a^{13}x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^(2/3)+a*x)^(1/2),x)
```

```
[Out] -2/152108775*(b*x^(2/3)+a*x)^(1/2)*(a*x^(1/3)+b)*(16224936*x^(11/3)*a^11*b-
15519504*x^(10/3)*a^10*b^2-14002560*x^(8/3)*a^8*b^4+13178880*x^(7/3)*a^7*b^
5+11354112*x^(5/3)*a^5*b^7-10321920*x^(4/3)*a^4*b^8-16900975*x^4*a^12+14780
480*x^3*a^9*b^3-7864320*x^(2/3)*a^2*b^10-12300288*x^2*a^6*b^6+6291456*x^(1/
3)*a*b^11+9175040*x*a^3*b^9-4194304*b^12)/x^(1/3)/a^13
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax + bx^{\frac{2}{3}}}\, x^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \sqrt{a\,x + b\,x^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] int(x^3*(a*x + b*x^(2/3))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\sqrt{ax + bx^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*x + b*x**(2/3)), x)
```

3.89 $\int x^2\sqrt{bx^{2/3}+ax}\,dx$

Optimal. Leaf size=283

$$-\frac{131072b^9\left(ax+bx^{2/3}\right)^{3/2}}{1616615a^{10}x}+\frac{196608b^8\left(ax+bx^{2/3}\right)^{3/2}}{1616615a^9x^{2/3}}-\frac{49152b^7\left(ax+bx^{2/3}\right)^{3/2}}{323323a^8\sqrt[3]{x}}+\frac{8192b^6\left(ax+bx^{2/3}\right)^{3/2}}{46189a^7}-\frac{9216b^5\sqrt[3]{\cdots}}{\cdots}$$

Rubi [A] time = 0.44, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$-\frac{131072b^9\left(ax+bx^{2/3}\right)^{3/2}}{1616615a^{10}x}+\frac{196608b^8\left(ax+bx^{2/3}\right)^{3/2}}{1616615a^9x^{2/3}}-\frac{49152b^7\left(ax+bx^{2/3}\right)^{3/2}}{323323a^8\sqrt[3]{x}}+\frac{8192b^6\left(ax+bx^{2/3}\right)^{3/2}}{46189a^7}-\frac{9216b^5\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{46189a^6}+\frac{4608b^4x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{20995a^5}-\frac{384b^3x\left(ax+bx^{2/3}\right)^{3/2}}{1615a^4}+\frac{576b^2x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{2261a^3}-\frac{36bx^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{133a^2}+\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (8192*b^6*(b*x^(2/3) + a*x)^(3/2))/(46189*a^7) - (131072*b^9*(b*x^(2/3) + a
*x)^(3/2))/(1616615*a^10*x) + (196608*b^8*(b*x^(2/3) + a*x)^(3/2))/(1616615
*a^9*x^(2/3)) - (49152*b^7*(b*x^(2/3) + a*x)^(3/2))/(323323*a^8*x^(1/3)) -
(9216*b^5*x^(1/3)*(b*x^(2/3) + a*x)^(3/2))/(46189*a^6) + (4608*b^4*x^(2/3)*
(b*x^(2/3) + a*x)^(3/2))/(20995*a^5) - (384*b^3*x*(b*x^(2/3) + a*x)^(3/2))/
(1615*a^4) + (576*b^2*x^(4/3)*(b*x^(2/3) + a*x)^(3/2))/(2261*a^3) - (36*b*x
^(5/3)*(b*x^(2/3) + a*x)^(3/2))/(133*a^2) + (2*x^2*(b*x^(2/3) + a*x)^(3/2))
/(7*a)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^2\sqrt{bx^{2/3}+ax}\,dx &= \frac{2x^2\left(bx^{2/3}+ax\right)^{3/2}}{7a} - \frac{(6b)\int x^{5/3}\sqrt{bx^{2/3}+ax}\,dx}{7a} \\
&= -\frac{36bx^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{133a^2} + \frac{2x^2\left(bx^{2/3}+ax\right)^{3/2}}{7a} + \frac{\left(96b^2\right)\int x^{4/3}\sqrt{bx^{2/3}+ax}\,dx}{133a^2} \\
&= \frac{576b^2x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{2261a^3} - \frac{36bx^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{133a^2} + \frac{2x^2\left(bx^{2/3}+ax\right)^{3/2}}{7a} - \frac{\left(192b^3\right)\int x \cdots}{3\cdots} \\
&= -\frac{384b^3x\left(bx^{2/3}+ax\right)^{3/2}}{1615a^4} + \frac{576b^2x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{2261a^3} - \frac{36bx^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{133a^2} + \frac{2x^2\left(b \cdots\right.}{\cdots} \\
&= \frac{4608b^4x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{20995a^5} - \frac{384b^3x\left(bx^{2/3}+ax\right)^{3/2}}{1615a^4} + \frac{576b^2x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{2261a^3} - \frac{36bx \cdots}{\cdots} \\
&= -\frac{9216b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{46189a^6} + \frac{4608b^4x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{20995a^5} - \frac{384b^3x\left(bx^{2/3}+ax\right)^{3/2}}{1615a^4} + \frac{5\cdots}{\cdots} \\
&= \frac{8192b^6\left(bx^{2/3}+ax\right)^{3/2}}{46189a^7} - \frac{9216b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{46189a^6} + \frac{4608b^4x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{20995a^5} - \frac{384\cdots}{\cdots} \\
&= \frac{8192b^6\left(bx^{2/3}+ax\right)^{3/2}}{46189a^7} - \frac{49152b^7\left(bx^{2/3}+ax\right)^{3/2}}{323323a^8\sqrt[3]{x}} - \frac{9216b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{46189a^6} + \frac{4608b\cdots}{\cdots} \\
&= \frac{8192b^6\left(bx^{2/3}+ax\right)^{3/2}}{46189a^7} + \frac{196608b^8\left(bx^{2/3}+ax\right)^{3/2}}{1616615a^9x^{2/3}} - \frac{49152b^7\left(bx^{2/3}+ax\right)^{3/2}}{323323a^8\sqrt[3]{x}} - \frac{9216b^5\cdots}{\cdots} \\
&= \frac{8192b^6\left(bx^{2/3}+ax\right)^{3/2}}{46189a^7} - \frac{131072b^9\left(bx^{2/3}+ax\right)^{3/2}}{1616615a^{10}x} + \frac{196608b^8\left(bx^{2/3}+ax\right)^{3/2}}{1616615a^9x^{2/3}} - \frac{49152\cdots}{3\cdots}
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 144, normalized size = 0.51

$$\frac{2\left(a\sqrt[3]{x}+b\right)\sqrt{ax+bx^{2/3}}\left(230945a^9x^3-218790a^8bx^{8/3}+205920a^7b^2x^{7/3}-192192a^6b^3x^2+177408a^5b^4x^{5/3}-161280a^4b^5x^{4/3}+143360a^3b^6x-122880a^2b^7x^{2/3}+98304ab^8\sqrt[3]{x}-65536b^9\right)}{1616615a^{10}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*(-65536*b^9 + 98304*a*b^8*x^(1/3)
- 122880*a^2*b^7*x^(2/3) + 143360*a^3*b^6*x - 161280*a^4*b^5*x^(4/3) + 1774
08*a^5*b^4*x^(5/3) - 192192*a^6*b^3*x^2 + 205920*a^7*b^2*x^(7/3) - 218790*a
^8*b*x^(8/3) + 230945*a^9*x^3))/(1616615*a^10*x^(1/3))
```

IntegrateAlgebraic [A] time = 0.10, size = 133, normalized size = 0.47

$$\frac{2\left(ax+bx^{2/3}\right)^{3/2}\left(230945a^9x^3-218790a^8bx^{8/3}+205920a^7b^2x^{7/3}-192192a^6b^3x^2+177408a^5b^4x^{5/3}-161280a^4b^5x^{4/3}+143360a^3b^6x-122880a^2b^7x^{2/3}+98304ab^8\sqrt[3]{x}-65536b^9\right)}{1616615a^{10}x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(b*x^(2/3) + a*x)^(3/2)*(-65536*b^9 + 98304*a*b^8*x^(1/3) - 122880*a^2*b
^7*x^(2/3) + 143360*a^3*b^6*x - 161280*a^4*b^5*x^(4/3) + 177408*a^5*b^4*x^(
5/3) - 192192*a^6*b^3*x^2 + 205920*a^7*b^2*x^(7/3) - 218790*a^8*b*x^(8/3) +
 230945*a^9*x^3))/(1616615*a^10*x)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.22, size = 312, normalized size = 1.10

$$\frac{131072\,b^{\frac{21}{2}}}{1616615\,a^{10}}+\frac{2\left(\frac{21\left(12155\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}-122265\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b+554268\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^{2}-1492260\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^{3}+2645370\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^{4}-3233230\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^{5}+2771340\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^{6}-1662804\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{7}+692835\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{8}-230945\sqrt{ax^{\frac{1}{3}}+b}\,b^{9}\right)b}{a^{9}}+\frac{5\left(46189\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}-510510\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}b+2567565\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b^{2}-7759752\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^{3}+15668730\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^{4}-22221108\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^{5}+22632610\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^{6}-16628040\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^{7}+8729721\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{8}-3233230\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{9}+969969\sqrt{ax^{\frac{1}{3}}+b}\,b^{10}\right)}{a^{9}}\right)}{1616615\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 131072/1616615*b^(21/2)/a^10 + 2/1616615*(21*(12155*(a*x^(1/3) + b)^(19/2)
- 122265*(a*x^(1/3) + b)^(17/2)*b + 554268*(a*x^(1/3) + b)^(15/2)*b^2 - 149
2260*(a*x^(1/3) + b)^(13/2)*b^3 + 2645370*(a*x^(1/3) + b)^(11/2)*b^4 - 3233
230*(a*x^(1/3) + b)^(9/2)*b^5 + 2771340*(a*x^(1/3) + b)^(7/2)*b^6 - 1662804
*(a*x^(1/3) + b)^(5/2)*b^7 + 692835*(a*x^(1/3) + b)^(3/2)*b^8 - 230945*sqrt
(a*x^(1/3) + b)*b^9)*b/a^9 + 5*(46189*(a*x^(1/3) + b)^(21/2) - 510510*(a*x^
(1/3) + b)^(19/2)*b + 2567565*(a*x^(1/3) + b)^(17/2)*b^2 - 7759752*(a*x^(1/
3) + b)^(15/2)*b^3 + 15668730*(a*x^(1/3) + b)^(13/2)*b^4 - 22221108*(a*x^(1
/3) + b)^(11/2)*b^5 + 22632610*(a*x^(1/3) + b)^(9/2)*b^6 - 16628040*(a*x^(1
/3) + b)^(7/2)*b^7 + 8729721*(a*x^(1/3) + b)^(5/2)*b^8 - 3233230*(a*x^(1/3)
 + b)^(3/2)*b^9 + 969969*sqrt(a*x^(1/3) + b)*b^10)/a^9)/a
```

maple [A] time = 0.05, size = 123, normalized size = 0.43

$$-\frac{2\sqrt{ax+bx^{\frac{2}{3}}}\,\left(ax^{\frac{1}{3}}+b\right)\left(-230945a^{9}x^{3}+218790a^{8}bx^{\frac{8}{3}}-205920a^{7}b^{2}x^{\frac{7}{3}}+192192a^{6}b^{3}x^{2}-177408a^{5}b^{4}x^{\frac{5}{3}}+161280a^{4}b^{5}x^{\frac{4}{3}}-143360a^{3}b^{6}x+122880a^{2}b^{7}x^{\frac{2}{3}}-98304a\,b^{8}x^{\frac{1}{3}}+65536b^{9}\right)}{1616615a^{10}x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x+b*x^(2/3))^(1/2),x)
```

```
[Out] -2/1616615*(a*x+b*x^(2/3))^(1/2)*(a*x^(1/3)+b)*(218790*x^(8/3)*a^8*b-205920
*x^(7/3)*a^7*b^2-177408*x^(5/3)*a^5*b^4+161280*x^(4/3)*a^4*b^5-230945*x^3*a
^9+122880*x^(2/3)*a^2*b^7+192192*x^2*a^6*b^3-98304*x^(1/3)*a*b^8-143360*x*a
^3*b^6+65536*b^9)/x^(1/3)/a^10
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax+bx^{\frac{2}{3}}}\,x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{a\,x+b\,x^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] int(x^2*(a*x + b*x^(2/3))^(1/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{ax+bx^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*x + b*x**(2/3)), x)
```

3.90 $\int x\sqrt{bx^{2/3}+ax}\,dx$

Optimal. Leaf size=195

$$\frac{2048b^6\left(ax+bx^{2/3}\right)^{3/2}}{15015a^7x}-\frac{1024b^5\left(ax+bx^{2/3}\right)^{3/2}}{5005a^6x^{2/3}}+\frac{256b^4\left(ax+bx^{2/3}\right)^{3/2}}{1001a^5\sqrt[3]{x}}-\frac{128b^3\left(ax+bx^{2/3}\right)^{3/2}}{429a^4}+\frac{48b^2\sqrt[3]{x}\left(ax+bx^{2/}\right.}{143a^3}$$

Rubi [A] time = 0.27, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2016, 2002, 2014}

$$\frac{2048b^6\left(ax+bx^{2/3}\right)^{3/2}}{15015a^7x}-\frac{1024b^5\left(ax+bx^{2/3}\right)^{3/2}}{5005a^6x^{2/3}}+\frac{256b^4\left(ax+bx^{2/3}\right)^{3/2}}{1001a^5\sqrt[3]{x}}-\frac{128b^3\left(ax+bx^{2/3}\right)^{3/2}}{429a^4}+\frac{48b^2\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{143a^3}-\frac{24bx^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{65a^2}+\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (-128*b^3*(b*x^(2/3) + a*x)^(3/2))/(429*a^4) + (2048*b^6*(b*x^(2/3) + a*x)^
(3/2))/(15015*a^7*x) - (1024*b^5*(b*x^(2/3) + a*x)^(3/2))/(5005*a^6*x^(2/3)
) + (256*b^4*(b*x^(2/3) + a*x)^(3/2))/(1001*a^5*x^(1/3)) + (48*b^2*x^(1/3)*
(b*x^(2/3) + a*x)^(3/2))/(143*a^3) - (24*b*x^(2/3)*(b*x^(2/3) + a*x)^(3/2))
/(65*a^2) + (2*x*(b*x^(2/3) + a*x)^(3/2))/(5*a)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int x\sqrt{bx^{2/3}+ax}\,dx &= \frac{2x\left(bx^{2/3}+ax\right)^{3/2}}{5a} - \frac{(4b)\int x^{2/3}\sqrt{bx^{2/3}+ax}\,dx}{5a} \\
&= -\frac{24bx^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{65a^2} + \frac{2x\left(bx^{2/3}+ax\right)^{3/2}}{5a} + \frac{\left(8b^2\right)\int \sqrt[3]{x}\,\sqrt{bx^{2/3}+ax}\,dx}{13a^2} \\
&= \frac{48b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{143a^3} - \frac{24bx^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{65a^2} + \frac{2x\left(bx^{2/3}+ax\right)^{3/2}}{5a} - \frac{\left(64b^3\right)\int \sqrt{bx^{2/}}}{143a^{3}} \\
&= -\frac{128b^3\left(bx^{2/3}+ax\right)^{3/2}}{429a^4} + \frac{48b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{143a^3} - \frac{24bx^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{65a^2} + \frac{2x\left(bx^{2/3}\right.}{5a} \\
&= -\frac{128b^3\left(bx^{2/3}+ax\right)^{3/2}}{429a^4} + \frac{256b^4\left(bx^{2/3}+ax\right)^{3/2}}{1001a^5\sqrt[3]{x}} + \frac{48b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{143a^3} - \frac{24bx^{2/3}\left(bx\right.}{65} \\
&= -\frac{128b^3\left(bx^{2/3}+ax\right)^{3/2}}{429a^4} - \frac{1024b^5\left(bx^{2/3}+ax\right)^{3/2}}{5005a^6x^{2/3}} + \frac{256b^4\left(bx^{2/3}+ax\right)^{3/2}}{1001a^5\sqrt[3]{x}} + \frac{48b^2\sqrt[3]{x}\left(bx^{2}\right.}{143} \\
&= -\frac{128b^3\left(bx^{2/3}+ax\right)^{3/2}}{429a^4} + \frac{2048b^6\left(bx^{2/3}+ax\right)^{3/2}}{15015a^7x} - \frac{1024b^5\left(bx^{2/3}+ax\right)^{3/2}}{5005a^6x^{2/3}} + \frac{256b^4\left(bx^{2/}\right.}{1001a}
\end{aligned}
$$

Mathematica [**A**] time = 0.09, size = 107, normalized size = 0.55

$$\frac{2\left(a\sqrt[3]{x}+b\right)\sqrt{ax+bx^{2/3}}\left(3003a^6x^2-2772a^5bx^{5/3}+2520a^4b^2x^{4/3}-2240a^3b^3x+1920a^2b^4x^{2/3}-1536ab^5\sqrt[3]{x}+1024b^6\right)}{15015a^7\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*(1024*b^6 - 1536*a*b^5*x^(1/3) + 1
920*a^2*b^4*x^(2/3) - 2240*a^3*b^3*x + 2520*a^4*b^2*x^(4/3) - 2772*a^5*b*x^
(5/3) + 3003*a^6*x^2))/(15015*a^7*x^(1/3))
```

IntegrateAlgebraic [**A**] time = 0.07, size = 96, normalized size = 0.49

$$\frac{2\left(ax+bx^{2/3}\right)^{3/2}\left(3003a^6x^2-2772a^5bx^{5/3}+2520a^4b^2x^{4/3}-2240a^3b^3x+1920a^2b^4x^{2/3}-1536ab^5\sqrt[3]{x}+1024b^6\right)}{15015a^7x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(b*x^(2/3) + a*x)^(3/2)*(1024*b^6 - 1536*a*b^5*x^(1/3) + 1920*a^2*b^4*x^
(2/3) - 2240*a^3*b^3*x + 2520*a^4*b^2*x^(4/3) - 2772*a^5*b*x^(5/3) + 3003*a
^6*x^2))/(15015*a^7*x)
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**A**] time = 0.19, size = 228, normalized size = 1.17

$$-\frac{2048\,b^{\frac{15}{2}}}{15015\,a^{7}} + \frac{2\left(\frac{15\left(231\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}-1638\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b+5005\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^{2}-8580\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^{3}+9009\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{4}-6006\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{5}+3003\sqrt{ax^{\frac{1}{3}}+b}\,b^{6}\right)b}{a^{6}} + \frac{7\left(429\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}-3465\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b+12285\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^{2}-25025\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^{3}+32175\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^{4}-27027\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{5}+15015\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{6}-6435\sqrt{ax^{\frac{1}{3}}+b}\,b^{7}\right)}{a^{6}}\right)}{15015\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")

[Out] -2048/15015*b^(15/2)/a^7 + 2/15015*(15*(231*(a*x^(1/3) + b)^(13/2) - 1638*(a*x^(1/3) + b)^(11/2)*b + 5005*(a*x^(1/3) + b)^(9/2)*b^2 - 8580*(a*x^(1/3) + b)^(7/2)*b^3 + 9009*(a*x^(1/3) + b)^(5/2)*b^4 - 6006*(a*x^(1/3) + b)^(3/2)*b^5 + 3003*sqrt(a*x^(1/3) + b)*b^6)*b/a^6 + 7*(429*(a*x^(1/3) + b)^(15/2) - 3465*(a*x^(1/3) + b)^(13/2)*b + 12285*(a*x^(1/3) + b)^(11/2)*b^2 - 25025*(a*x^(1/3) + b)^(9/2)*b^3 + 32175*(a*x^(1/3) + b)^(7/2)*b^4 - 27027*(a*x^(1/3) + b)^(5/2)*b^5 + 15015*(a*x^(1/3) + b)^(3/2)*b^6 - 6435*sqrt(a*x^(1/3) + b)*b^7)/a^6)/a

maple [A] time = 0.05, size = 90, normalized size = 0.46

$$-\frac{2\sqrt{ax+b\,x^{\frac{2}{3}}}\,\left(a\,x^{\frac{1}{3}}+b\right)\left(-3003a^{6}x^{2}+2772a^{5}b\,x^{\frac{5}{3}}-2520a^{4}b^{2}x^{\frac{4}{3}}+2240a^{3}b^{3}x-1920a^{2}b^{4}x^{\frac{2}{3}}+1536a\,b^{5}x^{\frac{1}{3}}-1024b^{6}\right)}{15015a^{7}x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a*x+b*x^(2/3))^(1/2),x)

[Out] -2/15015*(a*x+b*x^(2/3))^(1/2)*(a*x^(1/3)+b)*(2772*x^(5/3)*a^5*b-2520*x^(4/3)*a^4*b^2-1920*x^(2/3)*a^2*b^4-3003*x^2*a^6+1536*x^(1/3)*a*b^5+2240*x*a^3*b^3-1024*b^6)/x^(1/3)/a^7

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax+bx^{\frac{2}{3}}}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*x + b*x^(2/3))*x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{a\,x+b\,x^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a*x + b*x^(2/3))^(1/2),x)

[Out] int(x*(a*x + b*x^(2/3))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{ax+bx^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x**(2/3)+a*x)**(1/2),x)

[Out] Integral(x*sqrt(a*x + b*x**(2/3)), x)

3.91 $\int \sqrt{bx^{2/3}+ax}\,dx$

Optimal. Leaf size=109

$$-\frac{32b^3\left(ax+bx^{2/3}\right)^{3/2}}{105a^4x}+\frac{16b^2\left(ax+bx^{2/3}\right)^{3/2}}{35a^3x^{2/3}}-\frac{4b\left(ax+bx^{2/3}\right)^{3/2}}{7a^2\sqrt[3]{x}}+\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{3a}$$

Rubi [A] time = 0.14, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2002, 2016, 2014}

$$-\frac{32b^3\left(ax+bx^{2/3}\right)^{3/2}}{105a^4x}+\frac{16b^2\left(ax+bx^{2/3}\right)^{3/2}}{35a^3x^{2/3}}-\frac{4b\left(ax+bx^{2/3}\right)^{3/2}}{7a^2\sqrt[3]{x}}+\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(b*x^(2/3) + a*x)^(3/2))/(3*a) - (32*b^3*(b*x^(2/3) + a*x)^(3/2))/(105*a
^4*x) + (16*b^2*(b*x^(2/3) + a*x)^(3/2))/(35*a^3*x^(2/3)) - (4*b*(b*x^(2/3)
 + a*x)^(3/2))/(7*a^2*x^(1/3))
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{bx^{2/3}+ax}\,dx &= \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{3a}-\frac{(2b)\int \frac{\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}\,dx}{3a}\\
&= \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{3a}-\frac{4b\left(bx^{2/3}+ax\right)^{3/2}}{7a^2\sqrt[3]{x}}+\frac{(8b^2)\int \frac{\sqrt{bx^{2/3}+ax}}{x^{2/3}}\,dx}{21a^2}\\
&= \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{3a}+\frac{16b^2\left(bx^{2/3}+ax\right)^{3/2}}{35a^3x^{2/3}}-\frac{4b\left(bx^{2/3}+ax\right)^{3/2}}{7a^2\sqrt[3]{x}}-\frac{(16b^3)\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx}{105a^3}\\
&= \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{3a}-\frac{32b^3\left(bx^{2/3}+ax\right)^{3/2}}{105a^4x}+\frac{16b^2\left(bx^{2/3}+ax\right)^{3/2}}{35a^3x^{2/3}}-\frac{4b\left(bx^{2/3}+ax\right)^{3/2}}{7a^2\sqrt[3]{x}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 70, normalized size = 0.64

$$\frac{2\left(a\sqrt[3]{x}+b\right)\sqrt{ax+bx^{2/3}}\left(35a^3x-30a^2bx^{2/3}+24ab^2\sqrt[3]{x}-16b^3\right)}{105a^4\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*(-16*b^3 + 24*a*b^2*x^(1/3) - 30*a
^2*b*x^(2/3) + 35*a^3*x))/(105*a^4*x^(1/3))
```

IntegrateAlgebraic [A] time = 0.06, size = 74, normalized size = 0.68

$$\frac{2\sqrt{ax+bx^{2/3}}\left(35a^4x^{4/3}+5a^3bx-6a^2b^2x^{2/3}+8ab^3\sqrt[3]{x}-16b^4\right)}{105a^4\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(-16*b^4 + 8*a*b^3*x^(1/3) - 6*a^2*b^2*x^(2/3) + 5
*a^3*b*x + 35*a^4*x^(4/3)))/(105*a^4*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.18, size = 143, normalized size = 1.31

$$\frac{32\,b^{\frac{9}{2}}}{105\,a^4}+\frac{2\left(\frac{9\left(5\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}-21\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b+35\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^2-35\sqrt{ax^{\frac{1}{3}}+b}\,b^3\right)b}{a^3}+\frac{35\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}-180\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b+378\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^2-420\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^3+315\sqrt{ax^{\frac{1}{3}}+b}\,b^4}{a^3}\right)}{105\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 32/105*b^(9/2)/a^4 + 2/105*(9*(5*(a*x^(1/3) + b)^(7/2) - 21*(a*x^(1/3) + b)
^(5/2)*b + 35*(a*x^(1/3) + b)^(3/2)*b^2 - 35*sqrt(a*x^(1/3) + b)*b^3)*b/a^3
 + (35*(a*x^(1/3) + b)^(9/2) - 180*(a*x^(1/3) + b)^(7/2)*b + 378*(a*x^(1/3)
 + b)^(5/2)*b^2 - 420*(a*x^(1/3) + b)^(3/2)*b^3 + 315*sqrt(a*x^(1/3) + b)*b
^4)/a^3)/a
```

maple [A] time = 0.04, size = 57, normalized size = 0.52

$$-\frac{2\sqrt{ax+b\,x^{\frac{2}{3}}}\,\left(a\,x^{\frac{1}{3}}+b\right)\left(-35a^{3}x+30a^{2}b\,x^{\frac{2}{3}}-24a\,b^{2}x^{\frac{1}{3}}+16b^{3}\right)}{105a^{4}x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(2/3))^(1/2),x)
```

```
[Out] -2/105*(a*x+b*x^(2/3))^(1/2)*(a*x^(1/3)+b)*(30*a^2*b*x^(2/3)-24*a*b^2*x^(1/
3)-35*a^3*x+16*b^3)/x^(1/3)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax+bx^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3)), x)
```

mupad [B] time = 5.19, size = 40, normalized size = 0.37

$$\frac{3\,x\,\sqrt{a\,x+b\,x^{2/3}}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},4;\;5;\;-\frac{a\,x^{1/3}}{b}\right)}{4\,\sqrt{\frac{a\,x^{1/3}}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] (3*x*(a*x + b*x^(2/3))^(1/2)*hypergeom([-1/2, 4], 5, -(a*x^(1/3))/b))/(4*((
a*x^(1/3))/b + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax+bx^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x + b*x**(2/3)), x)
```

3.92 $\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx$

Optimal. Leaf size=23

$$\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{ax}$$

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(2/3) + a*x]/x,x]
```

```
[Out] (2*(b*x^(2/3) + a*x)^(3/2))/(a*x)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx = \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{ax}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x,x]
```

```
[Out] (2*(b*x^(2/3) + a*x)^(3/2))/(a*x)
```

IntegrateAlgebraic [A] time = 0.05, size = 23, normalized size = 1.00

$$\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b*x^(2/3) + a*x]/x,x]
```

```
[Out] (2*(b*x^(2/3) + a*x)^(3/2))/(a*x)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.21, size = 23, normalized size = 1.00

$$\frac{2\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}}{a}-\frac{2\,b^{\frac{3}{2}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*(a*x^(1/3) + b)^(3/2)/a - 2*b^(3/2)/a
```

maple [A] time = 0.04, size = 27, normalized size = 1.17

$$\frac{2\sqrt{ax+b\,x^{\frac{2}{3}}}\,\left(a\,x^{\frac{1}{3}}+b\right)}{a\,x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(2/3))^(1/2)/x,x)
```

```
[Out] 2*(a*x+b*x^(2/3))^(1/2)/x^(1/3)*(a*x^(1/3)+b)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(1/2)/x,x)
```

```
[Out] int((a*x + b*x^(2/3))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(2/3))/x, x)
```

3.93 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx$

Optimal. Leaf size=90

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{4b^{3/2}}-\frac{3a\sqrt{ax+bx^{2/3}}}{4bx^{2/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{2x}$$

Rubi [A] time = 0.14, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2029, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{4b^{3/2}}-\frac{3a\sqrt{ax+bx^{2/3}}}{4bx^{2/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(2/3) + a*x]/x^2,x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(2*x) - (3*a*Sqrt[b*x^(2/3) + a*x])/(4*b*x^(2/3)
) + (3*a^2*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(4*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx &= -\frac{3\sqrt{bx^{2/3}+ax}}{2x}+\frac{1}{4}a\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx\\ &= -\frac{3\sqrt{bx^{2/3}+ax}}{2x}-\frac{3a\sqrt{bx^{2/3}+ax}}{4bx^{2/3}}-\frac{a^2\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{8b}\\ &= -\frac{3\sqrt{bx^{2/3}+ax}}{2x}-\frac{3a\sqrt{bx^{2/3}+ax}}{4bx^{2/3}}+\frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{4b}\\ &= -\frac{3\sqrt{bx^{2/3}+ax}}{2x}-\frac{3a\sqrt{bx^{2/3}+ax}}{4bx^{2/3}}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{4b^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.06, size = 57, normalized size = 0.63

$$-\frac{2a^2\left(a\sqrt[3]{x}+b\right)\sqrt{ax+bx^{2/3}}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{b^3\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x^2,x]
```

```
[Out] (-2*a^2*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[3/2, 3, 5/2
, 1 + (a*x^(1/3))/b])/(b^3*x^(1/3))
```

IntegrateAlgebraic [A] time = 0.19, size = 76, normalized size = 0.84

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{4b^{3/2}}-\frac{3\left(a\sqrt[3]{x}+2b\right)\sqrt{ax+bx^{2/3}}}{4bx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b*x^(2/3) + a*x]/x^2,x]
```

```
[Out] (-3*(2*b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x])/(4*b*x) + (3*a^2*ArcTanh[(Sqrt
[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(4*b^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.28, size = 72, normalized size = 0.80

$$-\frac{3\left(\frac{a^3\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b}+\frac{\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^3+\sqrt{ax^{\frac{1}{3}}+b}\,a^3b}{a^2bx^{\frac{2}{3}}}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -3/4*(a^3*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) + ((a*x^(1/3) +
 b)^(3/2)*a^3 + sqrt(a*x^(1/3) + b)*a^3*b)/(a^2*b*x^(2/3)))/a
```

maple [A] time = 0.05, size = 80, normalized size = 0.89

$$\frac{3\sqrt{ax+b\,x^{\frac{2}{3}}}\left(a^2 b\,x^{\frac{2}{3}}\operatorname{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)-\sqrt{a\,x^{\frac{1}{3}}+b}\;b^{\frac{5}{2}}-\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{\frac{3}{2}}\right)}{4\sqrt{a\,x^{\frac{1}{3}}+b}\;b^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(2/3))^(1/2)/x^2,x)
```

```
[Out] 3/4*(a*x+b*x^(2/3))^(1/2)*(arctanh((a*x^(1/3)+b)^(1/2)/b^(1/2))*b*x^(2/3)*a
^2-(a*x^(1/3)+b)^(3/2)*b^(3/2)-(a*x^(1/3)+b)^(1/2)*b^(5/2))/x/(a*x^(1/3)+b)
^(1/2)/b^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(1/2)/x^2,x)
```

```
[Out] int((a*x + b*x^(2/3))^(1/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(2/3))/x**2, x)
```

3.94 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx$

Optimal. Leaf size=178

$$-\frac{21a^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{128b^{9/2}}+\frac{21a^4\sqrt{ax+bx^{2/3}}}{128b^4x^{2/3}}-\frac{7a^3\sqrt{ax+bx^{2/3}}}{64b^3x}+\frac{7a^2\sqrt{ax+bx^{2/3}}}{80b^2x^{4/3}}-\frac{3a\sqrt{ax+bx^{2/3}}}{40bx^{5/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}$$

Rubi [A] time = 0.30, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2029, 206}

$$\frac{21a^4\sqrt{ax+bx^{2/3}}}{128b^4x^{2/3}}-\frac{7a^3\sqrt{ax+bx^{2/3}}}{64b^3x}+\frac{7a^2\sqrt{ax+bx^{2/3}}}{80b^2x^{4/3}}-\frac{21a^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{128b^{9/2}}-\frac{3a\sqrt{ax+bx^{2/3}}}{40bx^{5/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(2/3) + a*x]/x^3,x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(5*x^2) - (3*a*Sqrt[b*x^(2/3) + a*x])/(40*b*x^(5
/3)) + (7*a^2*Sqrt[b*x^(2/3) + a*x])/(80*b^2*x^(4/3)) - (7*a^3*Sqrt[b*x^(2/
3) + a*x])/(64*b^3*x) + (21*a^4*Sqrt[b*x^(2/3) + a*x])/(128*b^4*x^(2/3)) -
(21*a^5*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(128*b^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2} + \frac{1}{10}a\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}} - \frac{\left(7a^2\right)\int \frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{80b}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}} + \frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}} + \frac{\left(7a^3\right)\int \frac{1}{x^{4/3}\sqrt{bx^{2/3}+ax}}\,dx}{96b^2}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}} + \frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}} - \frac{7a^3\sqrt{bx^{2/3}+ax}}{64b^3x} - \frac{\left(7a^4\right)\int \frac{1}{x\sqrt{bx^{2/3}+a}}}{128b^3}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}} + \frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}} - \frac{7a^3\sqrt{bx^{2/3}+ax}}{64b^3x} + \frac{21a^4\sqrt{bx^{2/3}+ax}}{128b^4x^{2/3}}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}} + \frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}} - \frac{7a^3\sqrt{bx^{2/3}+ax}}{64b^3x} + \frac{21a^4\sqrt{bx^{2/3}+ax}}{128b^4x^{2/3}}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}} + \frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}} - \frac{7a^3\sqrt{bx^{2/3}+ax}}{64b^3x} + \frac{21a^4\sqrt{bx^{2/3}+ax}}{128b^4x^{2/3}}$$

Mathematica [C] time = 0.07, size = 57, normalized size = 0.32

$$\frac{2a^5\left(a\sqrt[3]{x}+b\right)\sqrt{ax+bx^{2/3}}\ {}_2F_1\left(\frac{3}{2},6;\frac{5}{2};\frac{\sqrt[3]{x}\,a}{b}+1\right)}{b^6\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x^3,x]
```

```
[Out] (2*a^5*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[3/2, 6, 5/2,
 1 + (a*x^(1/3))/b])/(b^6*x^(1/3))
```

IntegrateAlgebraic [A] time = 0.26, size = 112, normalized size = 0.63

$$\frac{\sqrt{ax+bx^{2/3}}\left(105a^4x^{4/3}-70a^3bx+56a^2b^2x^{2/3}-48ab^3\sqrt[3]{x}-384b^4\right)}{640b^4x^2}-\frac{21a^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{128b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b*x^(2/3) + a*x]/x^3,x]
```

```
[Out] (Sqrt[b*x^(2/3) + a*x]*(-384*b^4 - 48*a*b^3*x^(1/3) + 56*a^2*b^2*x^(2/3) -
70*a^3*b*x + 105*a^4*x^(4/3)))/(640*b^4*x^2) - (21*a^5*ArcTanh[(Sqrt[b]*x^(
1/3))/Sqrt[b*x^(2/3) + a*x]])/(128*b^(9/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.28, size = 126, normalized size = 0.71

$$\frac{\frac{105\,a^6\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^4}+\frac{105\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^6-490\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^6b+896\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^6b^2-790\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^6b^3-105\sqrt{ax^{\frac{1}{3}}+b}\,a^6b^4}{a^5b^4x^{\frac{5}{3}}}}{640\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/640*(105*a^6*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) + (105*(
a*x^(1/3) + b)^(9/2)*a^6 - 490*(a*x^(1/3) + b)^(7/2)*a^6*b + 896*(a*x^(1/3)
 + b)^(5/2)*a^6*b^2 - 790*(a*x^(1/3) + b)^(3/2)*a^6*b^3 - 105*sqrt(a*x^(1/3
) + b)*a^6*b^4)/(a^5*b^4*x^(5/3)))/a
```

maple [A] time = 0.06, size = 125, normalized size = 0.70

$$-\frac{\sqrt{ax+b\,x^{\frac{2}{3}}}\left(105a^5b^4x^{\frac{5}{3}}\operatorname{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)+105\sqrt{a\,x^{\frac{1}{3}}+b}\;b^{\frac{17}{2}}+790\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{\frac{15}{2}}-896\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{\frac{13}{2}}+490\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^{\frac{11}{2}}-105\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^{\frac{9}{2}}\right)}{640\sqrt{a\,x^{\frac{1}{3}}+b}\;b^{\frac{17}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(2/3))^(1/2)/x^3,x)
```

```
[Out] -1/640*(a*x+b*x^(2/3))^(1/2)*(-105*(a*x^(1/3)+b)^(9/2)*b^(9/2)+490*(a*x^(1/
3)+b)^(7/2)*b^(11/2)-896*(a*x^(1/3)+b)^(5/2)*b^(13/2)+105*arctanh((a*x^(1/3
)+b)^(1/2)/b^(1/2))*b^4*x^(5/3)*a^5+790*(a*x^(1/3)+b)^(3/2)*b^(15/2)+105*(a
*x^(1/3)+b)^(1/2)*b^(17/2))/x^2/(a*x^(1/3)+b)^(1/2)/b^(17/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(1/2)/x^3,x)
```

```
[Out] int((a*x + b*x^(2/3))^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(2/3))/x**3, x)
```

3.95 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx$

Optimal. Leaf size=266

$$\frac{1287a^8\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{16384b^{15/2}}-\frac{1287a^7\sqrt{ax+bx^{2/3}}}{16384b^7x^{2/3}}+\frac{429a^6\sqrt{ax+bx^{2/3}}}{8192b^6x}-\frac{429a^5\sqrt{ax+bx^{2/3}}}{10240b^5x^{4/3}}+\frac{1287a^4\sqrt{ax+bx^{2/3}}}{35840b^4x^{5/3}}-1$$

Rubi [A] time = 0.48, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2029, 206}

$$-\frac{1287a^7\sqrt{ax+bx^{2/3}}}{16384b^7x^{2/3}}+\frac{429a^6\sqrt{ax+bx^{2/3}}}{8192b^6x}-\frac{429a^5\sqrt{ax+bx^{2/3}}}{10240b^5x^{4/3}}+\frac{1287a^4\sqrt{ax+bx^{2/3}}}{35840b^4x^{5/3}}-\frac{143a^3\sqrt{ax+bx^{2/3}}}{4480b^3x^2}+\frac{13a^2\sqrt{ax+bx^{2/3}}}{448b^2x^{7/3}}+\frac{1287a^8\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{16384b^{15/2}}-\frac{3a\sqrt{ax+bx^{2/3}}}{112bx^{8/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{8x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(2/3) + a*x]/x^4,x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(8*x^3) - (3*a*Sqrt[b*x^(2/3) + a*x])/(112*b*x^(
8/3)) + (13*a^2*Sqrt[b*x^(2/3) + a*x])/(448*b^2*x^(7/3)) - (143*a^3*Sqrt[b*
x^(2/3) + a*x])/(4480*b^3*x^2) + (1287*a^4*Sqrt[b*x^(2/3) + a*x])/(35840*b^
4*x^(5/3)) - (429*a^5*Sqrt[b*x^(2/3) + a*x])/(10240*b^5*x^(4/3)) + (429*a^6
*Sqrt[b*x^(2/3) + a*x])/(8192*b^6*x) - (1287*a^7*Sqrt[b*x^(2/3) + a*x])/(16
384*b^7*x^(2/3)) + (1287*a^8*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x
]])/(16384*b^(15/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} + \frac{1}{16}a\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} - \frac{\left(13a^2\right)\int \frac{1}{x^{8/3}\sqrt{bx^{2/3}+ax}}\,dx}{224b}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} + \frac{\left(143a^3\right)\int \frac{1}{x^{7/3}\sqrt{bx^{2/3}+ax}}\,dx}{2688b^2}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2} - \frac{\left(429a^4\right)\int\cdots}{89\cdots}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2} + \frac{1287a^4\sqrt{b\cdots}}{35840\cdots}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2} + \frac{1287a^4\sqrt{b\cdots}}{35840\cdots}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2} + \frac{1287a^4\sqrt{b\cdots}}{35840\cdots}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2} + \frac{1287a^4\sqrt{b\cdots}}{35840\cdots}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2} + \frac{1287a^4\sqrt{b\cdots}}{35840\cdots}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2} + \frac{1287a^4\sqrt{b\cdots}}{35840\cdots}$$

Mathematica [C] time = 0.05, size = 57, normalized size = 0.21

$$-\frac{2a^8\left(a\sqrt[3]{x}+b\right)\sqrt{ax+bx^{2/3}}\;{}_2F_1\left(\frac{3}{2},9;\frac{5}{2};\frac{\sqrt[3]{x}\,a}{b}+1\right)}{b^9\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x^4,x]
```

```
[Out] (-2*a^8*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[3/2, 9, 5/2
, 1 + (a*x^(1/3))/b])/(b^9*x^(1/3))
```

IntegrateAlgebraic [A] time = 0.29, size = 149, normalized size = 0.56

$$\frac{1287a^8\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{16384b^{15/2}} + \frac{\sqrt{ax+bx^{2/3}}\left(-45045a^7x^{7/3}+30030a^6bx^2-24024a^5b^2x^{5/3}+20592a^4b^3x^{4/3}-18304a^3b^4x+16640a^2b^5x^{2/3}-15360ab^6\sqrt[3]{x}-215040b^7\right)}{573440b^7x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b*x^(2/3) + a*x]/x^4,x]
```

```
[Out] (Sqrt[b*x^(2/3) + a*x]*(-215040*b^7 - 15360*a*b^6*x^(1/3) + 16640*a^2*b^5*x
^(2/3) - 18304*a^3*b^4*x + 20592*a^4*b^3*x^(4/3) - 24024*a^5*b^2*x^(5/3) +
30030*a^6*b*x^2 - 45045*a^7*x^(7/3)))/(573440*b^7*x^3) + (1287*a^8*ArcTanh[
(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(16384*b^(15/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.35, size = 177, normalized size = 0.67

$$-\frac{\frac{45045\,a^{9}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^{7}}+\frac{45045\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{9}-345345\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^{9}b+1150149\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^{9}b^{2}-2167737\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{9}b^{3}+2518087\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^{9}b^{4}-1831739\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^{9}b^{5}+801535\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^{9}b^{6}+45045\sqrt{ax^{\frac{1}{3}}+b}\,a^{9}b^{7}}{a^{8}b^{7}x^{\frac{8}{3}}}}{573440\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/573440*(45045*a^9*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7) +
(45045*(a*x^(1/3) + b)^(15/2)*a^9 - 345345*(a*x^(1/3) + b)^(13/2)*a^9*b + 1
150149*(a*x^(1/3) + b)^(11/2)*a^9*b^2 - 2167737*(a*x^(1/3) + b)^(9/2)*a^9*b
^3 + 2518087*(a*x^(1/3) + b)^(7/2)*a^9*b^4 - 1831739*(a*x^(1/3) + b)^(5/2)*
a^9*b^5 + 801535*(a*x^(1/3) + b)^(3/2)*a^9*b^6 + 45045*sqrt(a*x^(1/3) + b)*
a^9*b^7)/(a^8*b^7*x^(8/3)))/a
```

maple [A] time = 0.06, size = 167, normalized size = 0.63

$$-\frac{\sqrt{ax+b\,x^{\frac{2}{3}}}\left(-45045a^{8}b^{7}x^{\frac{8}{3}}\operatorname{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)+45045\sqrt{a\,x^{\frac{1}{3}}+b}\;b^{\frac{29}{2}}+801535\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{\frac{27}{2}}-1831739\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{\frac{25}{2}}+2518087\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^{\frac{23}{2}}-2167737\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^{\frac{21}{2}}+1150149\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^{\frac{19}{2}}-345345\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^{\frac{17}{2}}+45045\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^{\frac{15}{2}}\right)}{573440\sqrt{a\,x^{\frac{1}{3}}+b}\;b^{\frac{29}{2}}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(2/3))^(1/2)/x^4,x)
```

```
[Out] -1/573440*(a*x+b*x^(2/3))^(1/2)*(45045*b^(15/2)*(a*x^(1/3)+b)^(15/2)-345345
*b^(17/2)*(a*x^(1/3)+b)^(13/2)+1150149*b^(19/2)*(a*x^(1/3)+b)^(11/2)-216773
7*b^(21/2)*(a*x^(1/3)+b)^(9/2)+2518087*b^(23/2)*(a*x^(1/3)+b)^(7/2)-1831739
*b^(25/2)*(a*x^(1/3)+b)^(5/2)+801535*b^(27/2)*(a*x^(1/3)+b)^(3/2)-45045*arc
tanh((a*x^(1/3)+b)^(1/2)/b^(1/2))*b^7*x^(8/3)*a^8+45045*b^(29/2)*(a*x^(1/3)
+b)^(1/2))/x^3/(a*x^(1/3)+b)^(1/2)/b^(29/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax + bx^{\frac{2}{3}}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(1/2)/x^4,x)
```

```
[Out] int((a*x + b*x^(2/3))^(1/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax + bx^{\frac{2}{3}}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(2/3))/x**4, x)
```

3.96 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx$

Optimal. Leaf size=354

$$-\frac{12597a^{11}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{262144b^{21/2}}+\frac{12597a^{10}\sqrt{ax+bx^{2/3}}}{262144b^{10}x^{2/3}}-\frac{4199a^9\sqrt{ax+bx^{2/3}}}{131072b^9x}+\frac{4199a^8\sqrt{ax+bx^{2/3}}}{163840b^8x^{4/3}}-\frac{12597a^7\sqrt{ax+}}{573440b^7x}$$

Rubi [A] time = 0.66, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2029, 206}

$$\frac{12597a^{10}\sqrt{ax+bx^{2/3}}}{262144b^{10}x^{2/3}}-\frac{4199a^9\sqrt{ax+bx^{2/3}}}{131072b^9x}+\frac{4199a^8\sqrt{ax+bx^{2/3}}}{163840b^8x^{4/3}}-\frac{12597a^7\sqrt{ax+bx^{2/3}}}{573440b^7x^{5/3}}+\frac{4199a^6\sqrt{ax+bx^{2/3}}}{215040b^6x^2}-\frac{4199a^5\sqrt{ax+bx^{2/3}}}{236544b^5x^{7/3}}+\frac{323a^4\sqrt{ax+bx^{2/3}}}{19712b^4x^{8/3}}-\frac{323a^3\sqrt{ax+bx^{2/3}}}{21120b^3x^3}+\frac{19a^2\sqrt{ax+bx^{2/3}}}{1320b^2x^{10/3}}-\frac{12597a^{11}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{262144b^{21/2}}-\frac{3a\sqrt{ax+bx^{2/3}}}{220bx^{11/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{11x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(2/3) + a*x]/x^5,x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(11*x^4) - (3*a*Sqrt[b*x^(2/3) + a*x])/(220*b*x^
(11/3)) + (19*a^2*Sqrt[b*x^(2/3) + a*x])/(1320*b^2*x^(10/3)) - (323*a^3*Sqr
t[b*x^(2/3) + a*x])/(21120*b^3*x^3) + (323*a^4*Sqrt[b*x^(2/3) + a*x])/(1971
2*b^4*x^(8/3)) - (4199*a^5*Sqrt[b*x^(2/3) + a*x])/(236544*b^5*x^(7/3)) + (4
199*a^6*Sqrt[b*x^(2/3) + a*x])/(215040*b^6*x^2) - (12597*a^7*Sqrt[b*x^(2/3)
 + a*x])/(573440*b^7*x^(5/3)) + (4199*a^8*Sqrt[b*x^(2/3) + a*x])/(163840*b^
8*x^(4/3)) - (4199*a^9*Sqrt[b*x^(2/3) + a*x])/(131072*b^9*x) + (12597*a^10*
Sqrt[b*x^(2/3) + a*x])/(262144*b^10*x^(2/3)) - (12597*a^11*ArcTanh[(Sqrt[b]
*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(262144*b^(21/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx &= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}+\frac{1}{22}a\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}-\frac{\left(19a^2\right)\int \frac{1}{x^{11/3}\sqrt{bx^{2/3}+ax}}\,dx}{440b} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}+\frac{\left(323a^3\right)\int \frac{1}{x^{10/3}\sqrt{bx^{2/3}+ax}}\,dx}{7920b^2} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}-\frac{\left(323a^4\right)\int \cdots}{8\cdots} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx\cdots}}{19712b\cdots} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx\cdots}}{19712b\cdots} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx\cdots}}{19712b\cdots} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx\cdots}}{19712b\cdots} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx\cdots}}{19712b\cdots} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx\cdots}}{19712b\cdots} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx\cdots}}{19712b\cdots} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx\cdots}}{19712b\cdots} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx\cdots}}{19712b\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.05, size = 57, normalized size = 0.16

$$\frac{2a^{11}\left(a\sqrt[3]{x}+b\right)\sqrt{ax+bx^{2/3}}\,{}_2F_1\left(\frac{3}{2},12;\frac{5}{2};\frac{\sqrt[3]{x}\,a}{b}+1\right)}{b^{12}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x^5,x]
```

```
[Out] (2*a^11*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[3/2, 12, 5/
2, 1 + (a*x^(1/3))/b])/(b^12*x^(1/3))
```

IntegrateAlgebraic [A] time = 0.39, size = 186, normalized size = 0.53

$$\frac{\sqrt{ax+bx^{2/3}}\left(14549535a^{10}x^{10/3}-9699690a^9bx^3+7759752a^8b^2x^{8/3}-6651216a^7b^3x^{7/3}+5912192a^6b^4x^2-5374720a^5b^5x^{5/3}+4961280a^4b^6x^{4/3}-4630528a^3b^7x+4358144a^2b^8x^{2/3}-4128768ab^9\sqrt[3]{x}-82575360b^{10}\right)}{302776320b^{10}x^4}-\frac{12597a^{11}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{262144b^{21/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b*x^(2/3) + a*x]/x^5,x]
```

```
[Out] (Sqrt[b*x^(2/3) + a*x]*(-82575360*b^10 - 4128768*a*b^9*x^(1/3) + 4358144*a^
2*b^8*x^(2/3) - 4630528*a^3*b^7*x + 4961280*a^4*b^6*x^(4/3) - 5374720*a^5*b
^5*x^(5/3) + 5912192*a^6*b^4*x^2 - 6651216*a^7*b^3*x^(7/3) + 7759752*a^8*b^
2*x^(8/3) - 9699690*a^9*b*x^3 + 14549535*a^10*x^(10/3)))/(302776320*b^10*x^
4) - (12597*a^11*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(262144*
b^(21/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.43, size = 228, normalized size = 0.64

$$\frac{\frac{14549535\,a^{12}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^{10}}+\frac{14549535\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}a^{12}-155195040\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{12}b+749786037\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{12}b^{2}-2163862272\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{12}b^{3}+4139920070\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^{12}b^{4}-5503713280\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^{12}b^{5}+5174056250\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{12}b^{6}-3424523520\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^{12}b^{7}+1551313995\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^{12}b^{8}-450357600\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^{12}b^{9}-14549535\sqrt{ax^{\frac{1}{3}}+b}\,a^{12}b^{10}}{a^{11}b^{10}x^{\frac{11}{3}}}}{302776320\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/302776320*(14549535*a^12*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b
^10) + (14549535*(a*x^(1/3) + b)^(21/2)*a^12 - 155195040*(a*x^(1/3) + b)^(1
9/2)*a^12*b + 749786037*(a*x^(1/3) + b)^(17/2)*a^12*b^2 - 2163862272*(a*x^(
1/3) + b)^(15/2)*a^12*b^3 + 4139920070*(a*x^(1/3) + b)^(13/2)*a^12*b^4 - 55
03713280*(a*x^(1/3) + b)^(11/2)*a^12*b^5 + 5174056250*(a*x^(1/3) + b)^(9/2)
*a^12*b^6 - 3424523520*(a*x^(1/3) + b)^(7/2)*a^12*b^7 + 1551313995*(a*x^(1/
3) + b)^(5/2)*a^12*b^8 - 450357600*(a*x^(1/3) + b)^(3/2)*a^12*b^9 - 1454953
5*sqrt(a*x^(1/3) + b)*a^12*b^10)/(a^11*b^10*x^(11/3)))/a
```

maple [A] time = 0.06, size = 209, normalized size = 0.59

$$-\frac{\sqrt{ax+bx^{\frac{2}{3}}}\left(14549535a^{11}b^{10}x^{\frac{11}{3}}\operatorname{arctanh}\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{b}}\right)+14549535\sqrt{ax^{\frac{1}{3}}+b}\,b^{\frac{41}{2}}+450357600\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{\frac{39}{2}}-1551313995\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{\frac{37}{2}}+3424523520\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^{\frac{35}{2}}-5174056250\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^{\frac{33}{2}}+5503713280\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^{\frac{31}{2}}-4139920070\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^{\frac{29}{2}}+2163862272\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^{\frac{27}{2}}-749786037\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b^{\frac{25}{2}}+155195040\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}b^{\frac{23}{2}}-14549535\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}b^{\frac{21}{2}}\right)}{302776320\sqrt{ax^{\frac{1}{3}}+b}\,b^{\frac{41}{2}}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(2/3))^(1/2)/x^5,x)
```

```
[Out] -1/302776320*(a*x+b*x^(2/3))^(1/2)*(-14549535*(a*x^(1/3)+b)^(21/2)*b^(21/2)
+155195040*(a*x^(1/3)+b)^(19/2)*b^(23/2)-749786037*(a*x^(1/3)+b)^(17/2)*b^(
25/2)+2163862272*(a*x^(1/3)+b)^(15/2)*b^(27/2)-4139920070*(a*x^(1/3)+b)^(13
/2)*b^(29/2)+5503713280*(a*x^(1/3)+b)^(11/2)*b^(31/2)-5174056250*(a*x^(1/3)
+b)^(9/2)*b^(33/2)+3424523520*(a*x^(1/3)+b)^(7/2)*b^(35/2)-1551313995*(a*x^
(1/3)+b)^(5/2)*b^(37/2)+14549535*arctanh((a*x^(1/3)+b)^(1/2)/b^(1/2))*b^10*
x^(11/3)*a^11+450357600*(a*x^(1/3)+b)^(3/2)*b^(39/2)+14549535*(a*x^(1/3)+b)
^(1/2)*b^(41/2))/x^4/(a*x^(1/3)+b)^(1/2)/b^(41/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(1/2)/x^5,x)
[Out] int((a*x + b*x^(2/3))^(1/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(1/2)/x**5,x)
[Out] Integral(sqrt(a*x + b*x**(2/3))/x**5, x)
```

3.97 $\int x^2 \left(bx^{2/3}+ax\right)^{3/2}\,dx$

Optimal. Leaf size=343

$$-\frac{1048576b^{11}\left(ax+bx^{2/3}\right)^{5/2}}{152108775a^{12}x^{5/3}}+\frac{524288b^{10}\left(ax+bx^{2/3}\right)^{5/2}}{30421755a^{11}x^{4/3}}-\frac{131072b^9\left(ax+bx^{2/3}\right)^{5/2}}{4345965a^{10}x}+\frac{65536b^8\left(ax+bx^{2/3}\right)^{5/2}}{1448655a^9x^{2/3}}-\frac{901\cdots}{\cdots}$$

Rubi [A] time = 0.62, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$-\frac{1048576b^{11}\left(ax+bx^{2/3}\right)^{5/2}}{152108775a^{12}x^{5/3}}+\frac{524288b^{10}\left(ax+bx^{2/3}\right)^{5/2}}{30421755a^{11}x^{4/3}}-\frac{131072b^9\left(ax+bx^{2/3}\right)^{5/2}}{4345965a^{10}x}+\frac{65536b^8\left(ax+bx^{2/3}\right)^{5/2}}{1448655a^9x^{2/3}}-\frac{90112b^7\left(ax+bx^{2/3}\right)^{5/2}}{1448655a^8\sqrt[3]{x}}+\frac{45056b^6\left(ax+bx^{2/3}\right)^{5/2}}{557175a^7}-\frac{11264b^5\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{111435a^6}+\frac{5632b^4x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{45885a^5}-\frac{352b^3x\left(ax+bx^{2/3}\right)^{5/2}}{2415a^4}+\frac{176b^2x^{4/3}\left(ax+bx^{2/3}\right)^{5/2}}{1035a^3}-\frac{44bx^{5/3}\left(ax+bx^{2/3}\right)^{5/2}}{225a^2}+\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (45056*b^6*(b*x^(2/3) + a*x)^(5/2))/(557175*a^7) - (1048576*b^11*(b*x^(2/3)
 + a*x)^(5/2))/(152108775*a^12*x^(5/3)) + (524288*b^10*(b*x^(2/3) + a*x)^(5
/2))/(30421755*a^11*x^(4/3)) - (131072*b^9*(b*x^(2/3) + a*x)^(5/2))/(434596
5*a^10*x) + (65536*b^8*(b*x^(2/3) + a*x)^(5/2))/(1448655*a^9*x^(2/3)) - (90
112*b^7*(b*x^(2/3) + a*x)^(5/2))/(1448655*a^8*x^(1/3)) - (11264*b^5*x^(1/3)
*(b*x^(2/3) + a*x)^(5/2))/(111435*a^6) + (5632*b^4*x^(2/3)*(b*x^(2/3) + a*x
)^(5/2))/(45885*a^5) - (352*b^3*x*(b*x^(2/3) + a*x)^(5/2))/(2415*a^4) + (17
6*b^2*x^(4/3)*(b*x^(2/3) + a*x)^(5/2))/(1035*a^3) - (44*b*x^(5/3)*(b*x^(2/3
) + a*x)^(5/2))/(225*a^2) + (2*x^2*(b*x^(2/3) + a*x)^(5/2))/(9*a)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int x^2 \left(bx^{2/3}+ax\right)^{3/2} dx = \frac{2x^2\left(bx^{2/3}+ax\right)^{5/2}}{9a} - \frac{(22b)\int x^{5/3}\left(bx^{2/3}+ax\right)^{3/2} dx}{27a}$$

$$= -\frac{44bx^{5/3}\left(bx^{2/3}+ax\right)^{5/2}}{225a^2} + \frac{2x^2\left(bx^{2/3}+ax\right)^{5/2}}{9a} + \frac{\left(88b^2\right)\int x^{4/3}\left(bx^{2/3}+ax\right)^{3/2} dx}{135a^2}$$

$$= \frac{176b^2x^{4/3}\left(bx^{2/3}+ax\right)^{5/2}}{1035a^3} - \frac{44bx^{5/3}\left(bx^{2/3}+ax\right)^{5/2}}{225a^2} + \frac{2x^2\left(bx^{2/3}+ax\right)^{5/2}}{9a} - \frac{\left(176b^3\right)\ldots}{\ldots}$$

$$= -\frac{352b^3x\left(bx^{2/3}+ax\right)^{5/2}}{2415a^4} + \frac{176b^2x^{4/3}\left(bx^{2/3}+ax\right)^{5/2}}{1035a^3} - \frac{44bx^{5/3}\left(bx^{2/3}+ax\right)^{5/2}}{225a^2} + \ldots$$

$$= \frac{5632b^4x^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{45885a^5} - \frac{352b^3x\left(bx^{2/3}+ax\right)^{5/2}}{2415a^4} + \frac{176b^2x^{4/3}\left(bx^{2/3}+ax\right)^{5/2}}{1035a^3} - \ldots$$

$$= -\frac{11264b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{111435a^6} + \frac{5632b^4x^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{45885a^5} - \frac{352b^3x\left(bx^{2/3}+ax\right)^{5/2}}{2415a^4} \ldots$$

$$= \frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7} - \frac{11264b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{111435a^6} + \frac{5632b^4x^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{45885a^5} \ldots$$

$$= \frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7} - \frac{90112b^7\left(bx^{2/3}+ax\right)^{5/2}}{1448655a^8\sqrt[3]{x}} - \frac{11264b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{111435a^6} + \ldots$$

$$= \frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7} + \frac{65536b^8\left(bx^{2/3}+ax\right)^{5/2}}{1448655a^9x^{2/3}} - \frac{90112b^7\left(bx^{2/3}+ax\right)^{5/2}}{1448655a^8\sqrt[3]{x}} - \ldots$$

$$= \frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7} - \frac{131072b^9\left(bx^{2/3}+ax\right)^{5/2}}{4345965a^{10}x} + \frac{65536b^8\left(bx^{2/3}+ax\right)^{5/2}}{1448655a^9x^{2/3}} - \ldots$$

$$= \frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7} + \frac{524288b^{10}\left(bx^{2/3}+ax\right)^{5/2}}{30421755a^{11}x^{4/3}} - \frac{131072b^9\left(bx^{2/3}+ax\right)^{5/2}}{4345965a^{10}x} + \ldots$$

$$= \frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7} - \frac{1048576b^{11}\left(bx^{2/3}+ax\right)^{5/2}}{152108775a^{12}x^{5/3}} + \frac{524288b^{10}\left(bx^{2/3}+ax\right)^{5/2}}{30421755a^{11}x^{4/3}} \ldots$$

Mathematica [A] time = 0.16, size = 172, normalized size = 0.50

$$\frac{2\left(a\sqrt[3]{x}+b\right)^2\sqrt{ax+bx^{2/3}}\left(16900975a^{11}x^{11/3}-14872858a^{10}bx^{10/3}+12932920a^9b^2x^3-11085360a^8b^3x^{8/3}+9335040a^7b^4x^{7/3}-7687680a^6b^5x^2+6150144a^5b^6x^{5/3}-4730880a^4b^7x^{4/3}+3440640a^3b^8x-2293760a^2b^9x^{2/3}+1310720ab^{10}\sqrt[3]{x}-524288b^{11}\right)}{152108775a^{12}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*(-524288*b^11 + 1310720*a*b^10*x
^(1/3) - 2293760*a^2*b^9*x^(2/3) + 3440640*a^3*b^8*x - 4730880*a^4*b^7*x^(4
/3) + 6150144*a^5*b^6*x^(5/3) - 7687680*a^6*b^5*x^2 + 9335040*a^7*b^4*x^(7/
3) - 11085360*a^8*b^3*x^(8/3) + 12932920*a^9*b^2*x^3 - 14872858*a^10*b*x^(1
0/3) + 16900975*a^11*x^(11/3)))/(152108775*a^12*x^(1/3))
```

IntegrateAlgebraic [A] time = 4.51, size = 198, normalized size = 0.58

$$\frac{2\left(x^{2/3}\left(a\sqrt[3]{x}+b\right)\right)^{3/2}\left(16900975a^{13}x^{13/3}+18929092a^{12}bx^4+88179a^{11}b^2x^{11/3}-92378a^{10}b^3x^{10/3}+97240a^9b^4x^3-102960a^8b^5x^{8/3}+109824a^7b^6x^{7/3}-118272a^6b^7x^2+129024a^5b^8x^{5/3}-143360a^4b^9x^{4/3}+163840a^3b^{10}x-196608a^2b^{11}x^{2/3}+262144ab^{12}\sqrt[3]{x}-524288b^{13}\right)}{152108775a^{12}x\left(a\sqrt[3]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*((b + a*x^(1/3))*x^(2/3))^(3/2)*(-524288*b^13 + 262144*a*b^12*x^(1/3) -
196608*a^2*b^11*x^(2/3) + 163840*a^3*b^10*x - 143360*a^4*b^9*x^(4/3) + 1290
24*a^5*b^8*x^(5/3) - 118272*a^6*b^7*x^2 + 109824*a^7*b^6*x^(7/3) - 102960*a
^8*b^5*x^(8/3) + 97240*a^9*b^4*x^3 - 92378*a^10*b^3*x^(10/3) + 88179*a^11*b
^2*x^(11/3) + 18929092*a^12*b*x^4 + 16900975*a^13*x^(13/3)))/(152108775*a^1
2*(b + a*x^(1/3))*x)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.31, size = 770, normalized size = 2.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/16900975*b*(524288*b^(25/2)/a^12 + (25*(88179*(a*x^(1/3) + b)^(23/2) - 10
62347*(a*x^(1/3) + b)^(21/2)*b + 5870865*(a*x^(1/3) + b)^(19/2)*b^2 - 19684
665*(a*x^(1/3) + b)^(17/2)*b^3 + 44618574*(a*x^(1/3) + b)^(15/2)*b^4 - 7207
6158*(a*x^(1/3) + b)^(13/2)*b^5 + 85180914*(a*x^(1/3) + b)^(11/2)*b^6 - 743
64290*(a*x^(1/3) + b)^(9/2)*b^7 + 47805615*(a*x^(1/3) + b)^(7/2)*b^8 - 2230
9287*(a*x^(1/3) + b)^(5/2)*b^9 + 7436429*(a*x^(1/3) + b)^(3/2)*b^10 - 20281
17*sqrt(a*x^(1/3) + b)*b^11)*b/a^11 + 3*(676039*(a*x^(1/3) + b)^(25/2) - 88
17900*(a*x^(1/3) + b)^(23/2)*b + 53117350*(a*x^(1/3) + b)^(21/2)*b^2 - 1956
95500*(a*x^(1/3) + b)^(19/2)*b^3 + 492116625*(a*x^(1/3) + b)^(17/2)*b^4 - 8
92371480*(a*x^(1/3) + b)^(15/2)*b^5 + 1201269300*(a*x^(1/3) + b)^(13/2)*b^6
 - 1216870200*(a*x^(1/3) + b)^(11/2)*b^7 + 929553625*(a*x^(1/3) + b)^(9/2)*
b^8 - 531173500*(a*x^(1/3) + b)^(7/2)*b^9 + 223092870*(a*x^(1/3) + b)^(5/2)
*b^10 - 67603900*(a*x^(1/3) + b)^(3/2)*b^11 + 16900975*sqrt(a*x^(1/3) + b)*
b^12)/a^11)/a) - 2/152108775*a*(4194304*b^(27/2)/a^13 - (27*(676039*(a*x^(1
/3) + b)^(25/2) - 8817900*(a*x^(1/3) + b)^(23/2)*b + 53117350*(a*x^(1/3) +
b)^(21/2)*b^2 - 195695500*(a*x^(1/3) + b)^(19/2)*b^3 + 492116625*(a*x^(1/3)
 + b)^(17/2)*b^4 - 892371480*(a*x^(1/3) + b)^(15/2)*b^5 + 1201269300*(a*x^(
1/3) + b)^(13/2)*b^6 - 1216870200*(a*x^(1/3) + b)^(11/2)*b^7 + 929553625*(a
*x^(1/3) + b)^(9/2)*b^8 - 531173500*(a*x^(1/3) + b)^(7/2)*b^9 + 223092870*(
a*x^(1/3) + b)^(5/2)*b^10 - 67603900*(a*x^(1/3) + b)^(3/2)*b^11 + 16900975*
sqrt(a*x^(1/3) + b)*b^12)*b/a^12 + 13*(1300075*(a*x^(1/3) + b)^(27/2) - 182
53053*(a*x^(1/3) + b)^(25/2)*b + 119041650*(a*x^(1/3) + b)^(23/2)*b^2 - 478
056150*(a*x^(1/3) + b)^(21/2)*b^3 + 1320944625*(a*x^(1/3) + b)^(19/2)*b^4 -
 2657429775*(a*x^(1/3) + b)^(17/2)*b^5 + 4015671660*(a*x^(1/3) + b)^(15/2)*
b^6 - 4633467300*(a*x^(1/3) + b)^(13/2)*b^7 + 4106936925*(a*x^(1/3) + b)^(1
1/2)*b^8 - 2788660875*(a*x^(1/3) + b)^(9/2)*b^9 + 1434168450*(a*x^(1/3) + b
)^(7/2)*b^10 - 547591590*(a*x^(1/3) + b)^(5/2)*b^11 + 152108775*(a*x^(1/3)
+ b)^(3/2)*b^12 - 35102025*sqrt(a*x^(1/3) + b)*b^13)/a^12)/a)
```

maple [A] time = 0.05, size = 145, normalized size = 0.42

$$\frac{2\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}\left(a\,x^{\frac{1}{3}}+b\right)\left(16900975a^{11}x^{\frac{11}{3}}-14872858a^{10}b\,x^{\frac{10}{3}}+12932920a^{9}b^{2}x^{3}-11085360a^{8}b^{3}x^{\frac{8}{3}}+9335040a^{7}b^{4}x^{\frac{7}{3}}-7687680a^{6}b^{5}x^{2}+6150144a^{5}b^{6}x^{\frac{5}{3}}-4730880a^{4}b^{7}x^{\frac{4}{3}}+3440640a^{3}b^{8}x-2293760a^{2}b^{9}x^{\frac{2}{3}}+1310720a\,b^{10}x^{\frac{1}{3}}-524288b^{11}\right)}{152108775a^{12}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x+b*x^(2/3))^(3/2),x)
```

```
[Out] 2/152108775*(a*x+b*x^(2/3))^(3/2)*(a*x^(1/3)+b)*(16900975*x^(11/3)*a^11-148
72858*x^(10/3)*a^10*b+12932920*x^3*a^9*b^2-11085360*x^(8/3)*a^8*b^3+9335040
```

```
*x^(7/3)*a^7*b^4-7687680*x^2*a^6*b^5+6150144*x^(5/3)*a^5*b^6-4730880*x^(4/3
)*a^4*b^7+3440640*x*a^3*b^8-2293760*x^(2/3)*a^2*b^9+1310720*x^(1/3)*a*b^10-
524288*b^11)/x/a^12
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate((a*x + b*x^(2/3))^(3/2)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a\,x + b\,x^{2/3}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x + b*x^(2/3))^(3/2),x)
[Out] int(x^2*(a*x + b*x^(2/3))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**(2/3)+a*x)**(3/2),x)
[Out] Integral(x**2*(a*x + b*x**(2/3))**(3/2), x)
```

3.98 $\int x \left(bx^{2/3}+ax\right)^{3/2} dx$

Optimal. Leaf size=255

$$\frac{65536b^8\left(ax+bx^{2/3}\right)^{5/2}}{4849845a^9x^{5/3}}-\frac{32768b^7\left(ax+bx^{2/3}\right)^{5/2}}{969969a^8x^{4/3}}+\frac{8192b^6\left(ax+bx^{2/3}\right)^{5/2}}{138567a^7x}-\frac{4096b^5\left(ax+bx^{2/3}\right)^{5/2}}{46189a^6x^{2/3}}+\frac{512b^4\left(ax+b\right.}{4199a^5\sqrt[3]{}}$$

Rubi [A] time = 0.42, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2016, 2002, 2014}

$$\frac{65536b^8\left(ax+bx^{2/3}\right)^{5/2}}{4849845a^9x^{5/3}}-\frac{32768b^7\left(ax+bx^{2/3}\right)^{5/2}}{969969a^8x^{4/3}}+\frac{8192b^6\left(ax+bx^{2/3}\right)^{5/2}}{138567a^7x}-\frac{4096b^5\left(ax+bx^{2/3}\right)^{5/2}}{46189a^6x^{2/3}}+\frac{512b^4\left(ax+bx^{2/3}\right)^{5/2}}{4199a^5\sqrt[3]{x}}-\frac{256b^3\left(ax+bx^{2/3}\right)^{5/2}}{1615a^4}+\frac{64b^2\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{323a^3}-\frac{32bx^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{133a^2}+\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-256*b^3*(b*x^(2/3) + a*x)^(5/2))/(1615*a^4) + (65536*b^8*(b*x^(2/3) + a*x
)^(5/2))/(4849845*a^9*x^(5/3)) - (32768*b^7*(b*x^(2/3) + a*x)^(5/2))/(96996
9*a^8*x^(4/3)) + (8192*b^6*(b*x^(2/3) + a*x)^(5/2))/(138567*a^7*x) - (4096*
b^5*(b*x^(2/3) + a*x)^(5/2))/(46189*a^6*x^(2/3)) + (512*b^4*(b*x^(2/3) + a*
x)^(5/2))/(4199*a^5*x^(1/3)) + (64*b^2*x^(1/3)*(b*x^(2/3) + a*x)^(5/2))/(32
3*a^3) - (32*b*x^(2/3)*(b*x^(2/3) + a*x)^(5/2))/(133*a^2) + (2*x*(b*x^(2/3)
 + a*x)^(5/2))/(7*a)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int x\left(bx^{2/3}+ax\right)^{3/2}dx=\frac{2x\left(bx^{2/3}+ax\right)^{5/2}}{7a}-\frac{(16b)\int x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}dx}{21a}$$
$$=-\frac{32bx^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{133a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{5/2}}{7a}+\frac{\left(32b^2\right)\int \sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}dx}{57a^2}$$
$$=\frac{64b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{323a^3}-\frac{32bx^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{133a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{5/2}}{7a}-\frac{\left(128b^3\right)\int (\cdots}{3\cdots}$$
$$=-\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}+\frac{64b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{323a^3}-\frac{32bx^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{133a^2}+\frac{2x\left(bx^{\cdots}\right.}{\cdots}$$
$$=-\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}+\frac{512b^4\left(bx^{2/3}+ax\right)^{5/2}}{4199a^5\sqrt[3]{x}}+\frac{64b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{323a^3}-\frac{32bx^{2/3}\cdots}{\cdots}$$
$$=-\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}-\frac{4096b^5\left(bx^{2/3}+ax\right)^{5/2}}{46189a^6x^{2/3}}+\frac{512b^4\left(bx^{2/3}+ax\right)^{5/2}}{4199a^5\sqrt[3]{x}}+\frac{64b^2\sqrt[3]{x}\cdots}{3\cdots}$$
$$=-\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}+\frac{8192b^6\left(bx^{2/3}+ax\right)^{5/2}}{138567a^7x}-\frac{4096b^5\left(bx^{2/3}+ax\right)^{5/2}}{46189a^6x^{2/3}}+\frac{512b^4\cdots}{419\cdots}$$
$$=-\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}-\frac{32768b^7\left(bx^{2/3}+ax\right)^{5/2}}{969969a^8x^{4/3}}+\frac{8192b^6\left(bx^{2/3}+ax\right)^{5/2}}{138567a^7x}-\frac{4096b^5\cdots}{46\cdots}$$
$$=-\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}+\frac{65536b^8\left(bx^{2/3}+ax\right)^{5/2}}{4849845a^9x^{5/3}}-\frac{32768b^7\left(bx^{2/3}+ax\right)^{5/2}}{969969a^8x^{4/3}}+\frac{8192b\cdots}{\cdots}$$

Mathematica [**A**] time = 0.11, size = 135, normalized size = 0.53

$$\frac{2\left(a\sqrt[3]{x}+b\right)^2\sqrt{ax+bx^{2/3}}\left(692835a^8x^{8/3}-583440a^7bx^{7/3}+480480a^6b^2x^2-384384a^5b^3x^{5/3}+295680a^4b^4x^{4/3}-215040a^3b^5x+143360a^2b^6x^{2/3}-81920ab^7\sqrt[3]{x}+32768b^8\right)}{4849845a^9\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*(32768*b^8 - 81920*a*b^7*x^(1/3)
 + 143360*a^2*b^6*x^(2/3) - 215040*a^3*b^5*x + 295680*a^4*b^4*x^(4/3) - 384
384*a^5*b^3*x^(5/3) + 480480*a^6*b^2*x^2 - 583440*a^7*b*x^(7/3) + 692835*a^
8*x^(8/3)))/(4849845*a^9*x^(1/3))
```

IntegrateAlgebraic [**A**] time = 4.50, size = 161, normalized size = 0.63

$$\frac{2\left(x^{2/3}\left(a\sqrt[3]{x}+b\right)\right)^{3/2}\left(692835a^{10}x^{10/3}+802230a^9bx^3+6435a^8b^2x^{8/3}-6864a^7b^3x^{7/3}+7392a^6b^4x^2-8064a^5b^5x^{5/3}+8960a^4b^6x^{4/3}-10240a^3b^7x+12288a^2b^8x^{2/3}-16384ab^9\sqrt[3]{x}+32768b^{10}\right)}{4849845a^9x\left(a\sqrt[3]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*((b + a*x^(1/3))*x^(2/3))^(3/2)*(32768*b^10 - 16384*a*b^9*x^(1/3) + 1228
8*a^2*b^8*x^(2/3) - 10240*a^3*b^7*x + 8960*a^4*b^6*x^(4/3) - 8064*a^5*b^5*x
^(5/3) + 7392*a^6*b^4*x^2 - 6864*a^7*b^3*x^(7/3) + 6435*a^8*b^2*x^(8/3) + 8
02230*a^9*b*x^3 + 692835*a^10*x^(10/3)))/(4849845*a^9*(b + a*x^(1/3))*x)
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.30, size = 602, normalized size = 2.36

$$-\frac{2}{692835}\, b\left(\frac{32768\, b^{\frac{19}{2}}}{a^{9}} - \frac{\frac{19\left(6435\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}} - 58344\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b + 235620\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^{2} - 556920\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^{3} + 850850\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^{4} - 875160\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^{5} + 612612\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{6} - 291720\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{7} + 109395\sqrt{ax^{\frac{1}{3}}+b}\, b^{8}\right)b}{a^{8}} + \frac{9\left(12155\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}} - 122265\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b + 554268\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^{2} - 1492260\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^{3} + 2645370\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^{4} - 3233230\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^{5} + 2771340\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^{6} - 1662804\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{7} + 692835\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{8} - 230945\sqrt{ax^{\frac{1}{3}}+b}\, b^{9}\right)}{a^{8}}}{a}\right) + \frac{2}{1616615}\, a\left(\frac{65536\, b^{\frac{21}{2}}}{a^{10}} + \frac{\frac{21\left(12155\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}} - 122265\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b + 554268\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^{2} - 1492260\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^{3} + 2645370\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^{4} - 3233230\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^{5} + 2771340\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^{6} - 1662804\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{7} + 692835\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{8} - 230945\sqrt{ax^{\frac{1}{3}}+b}\, b^{9}\right)b}{a^{9}} + \frac{5\left(46189\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}} - 510510\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}b + 2567565\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b^{2} - 7759752\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^{3} + 15668730\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^{4} - 22221108\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^{5} + 22632610\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^{6} - 16628040\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^{7} + 8729721\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{8} - 3233230\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{9} + 969969\sqrt{ax^{\frac{1}{3}}+b}\, b^{10}\right)}{a^{9}}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/692835*b*(32768*b^(19/2)/a^9 - (19*(6435*(a*x^(1/3) + b)^(17/2) - 58344*
(a*x^(1/3) + b)^(15/2)*b + 235620*(a*x^(1/3) + b)^(13/2)*b^2 - 556920*(a*x^
(1/3) + b)^(11/2)*b^3 + 850850*(a*x^(1/3) + b)^(9/2)*b^4 - 875160*(a*x^(1/3
) + b)^(7/2)*b^5 + 612612*(a*x^(1/3) + b)^(5/2)*b^6 - 291720*(a*x^(1/3) + b
)^(3/2)*b^7 + 109395*sqrt(a*x^(1/3) + b)*b^8)*b/a^8 + 9*(12155*(a*x^(1/3) +
 b)^(19/2) - 122265*(a*x^(1/3) + b)^(17/2)*b + 554268*(a*x^(1/3) + b)^(15/2
)*b^2 - 1492260*(a*x^(1/3) + b)^(13/2)*b^3 + 2645370*(a*x^(1/3) + b)^(11/2)
*b^4 - 3233230*(a*x^(1/3) + b)^(9/2)*b^5 + 2771340*(a*x^(1/3) + b)^(7/2)*b^
6 - 1662804*(a*x^(1/3) + b)^(5/2)*b^7 + 692835*(a*x^(1/3) + b)^(3/2)*b^8 -
230945*sqrt(a*x^(1/3) + b)*b^9)/a^8)/a) + 2/1616615*a*(65536*b^(21/2)/a^10
+ (21*(12155*(a*x^(1/3) + b)^(19/2) - 122265*(a*x^(1/3) + b)^(17/2)*b + 554
268*(a*x^(1/3) + b)^(15/2)*b^2 - 1492260*(a*x^(1/3) + b)^(13/2)*b^3 + 26453
70*(a*x^(1/3) + b)^(11/2)*b^4 - 3233230*(a*x^(1/3) + b)^(9/2)*b^5 + 2771340
*(a*x^(1/3) + b)^(7/2)*b^6 - 1662804*(a*x^(1/3) + b)^(5/2)*b^7 + 692835*(a*
x^(1/3) + b)^(3/2)*b^8 - 230945*sqrt(a*x^(1/3) + b)*b^9)*b/a^9 + 5*(46189*(
a*x^(1/3) + b)^(21/2) - 510510*(a*x^(1/3) + b)^(19/2)*b + 2567565*(a*x^(1/3
) + b)^(17/2)*b^2 - 7759752*(a*x^(1/3) + b)^(15/2)*b^3 + 15668730*(a*x^(1/3
) + b)^(13/2)*b^4 - 22221108*(a*x^(1/3) + b)^(11/2)*b^5 + 22632610*(a*x^(1/
3) + b)^(9/2)*b^6 - 16628040*(a*x^(1/3) + b)^(7/2)*b^7 + 8729721*(a*x^(1/3)
 + b)^(5/2)*b^8 - 3233230*(a*x^(1/3) + b)^(3/2)*b^9 + 969969*sqrt(a*x^(1/3)
 + b)*b^10)/a^9)/a)
```

maple [A] time = 0.05, size = 112, normalized size = 0.44

$$\frac{2\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}\left(a\,x^{\frac{1}{3}}+b\right)\left(692835a^{8}x^{\frac{8}{3}}-583440a^{7}b\,x^{\frac{7}{3}}+480480a^{6}b^{2}x^{2}-384384a^{5}b^{3}x^{\frac{5}{3}}+295680a^{4}b^{4}x^{\frac{4}{3}}-215040a^{3}b^{5}x+143360a^{2}b^{6}x^{\frac{2}{3}}-81920a\,b^{7}x^{\frac{1}{3}}+32768b^{8}\right)}{4849845a^{9}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x+b*x^(2/3))^(3/2),x)
```

```
[Out] 2/4849845*(a*x+b*x^(2/3))^(3/2)*(a*x^(1/3)+b)*(692835*x^(8/3)*a^8-583440*x^
(7/3)*a^7*b+480480*a^6*b^2*x^2-384384*x^(5/3)*a^5*b^3+295680*x^(4/3)*a^4*b^
4-215040*a^3*b^5*x+143360*x^(2/3)*a^2*b^6-81920*x^(1/3)*a*b^7+32768*b^8)/x/
a^9
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a\,x+b\,x^{2/3}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x + b*x^(2/3))^(3/2),x)
[Out] int(x*(a*x + b*x^(2/3))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**(2/3)+a*x)**(3/2),x)
[Out] Integral(x*(a*x + b*x**(2/3))**(3/2), x)
```

3.99 $\int \left(bx^{2/3}+ax\right)^{3/2}\,dx$

Optimal. Leaf size=169

$$-\frac{512b^5\left(ax+bx^{2/3}\right)^{5/2}}{15015a^6x^{5/3}}+\frac{256b^4\left(ax+bx^{2/3}\right)^{5/2}}{3003a^5x^{4/3}}-\frac{64b^3\left(ax+bx^{2/3}\right)^{5/2}}{429a^4x}+\frac{32b^2\left(ax+bx^{2/3}\right)^{5/2}}{143a^3x^{2/3}}-\frac{4b\left(ax+bx^{2/3}\right)^{5/2}}{13a^2\sqrt[3]{x}}+\frac{2(}{}$$

Rubi [A] time = 0.25, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2002, 2016, 2014}

$$-\frac{512b^5\left(ax+bx^{2/3}\right)^{5/2}}{15015a^6x^{5/3}}+\frac{256b^4\left(ax+bx^{2/3}\right)^{5/2}}{3003a^5x^{4/3}}-\frac{64b^3\left(ax+bx^{2/3}\right)^{5/2}}{429a^4x}+\frac{32b^2\left(ax+bx^{2/3}\right)^{5/2}}{143a^3x^{2/3}}-\frac{4b\left(ax+bx^{2/3}\right)^{5/2}}{13a^2\sqrt[3]{x}}+\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(b*x^(2/3) + a*x)^(5/2))/(5*a) - (512*b^5*(b*x^(2/3) + a*x)^(5/2))/(1501
5*a^6*x^(5/3)) + (256*b^4*(b*x^(2/3) + a*x)^(5/2))/(3003*a^5*x^(4/3)) - (64
*b^3*(b*x^(2/3) + a*x)^(5/2))/(429*a^4*x) + (32*b^2*(b*x^(2/3) + a*x)^(5/2)
)/(143*a^3*x^(2/3)) - (4*b*(b*x^(2/3) + a*x)^(5/2))/(13*a^2*x^(1/3))
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \left(bx^{2/3}+ax\right)^{3/2}\,dx &= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a} - \frac{(2b)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{\sqrt[3]{x}}\,dx}{3a} \\
&= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a} - \frac{4b\left(bx^{2/3}+ax\right)^{5/2}}{13a^2\sqrt[3]{x}} + \frac{\left(16b^2\right)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^{2/3}}\,dx}{39a^2} \\
&= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a} + \frac{32b^2\left(bx^{2/3}+ax\right)^{5/2}}{143a^3x^{2/3}} - \frac{4b\left(bx^{2/3}+ax\right)^{5/2}}{13a^2\sqrt[3]{x}} - \frac{\left(32b^3\right)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx}{143a^3} \\
&= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a} - \frac{64b^3\left(bx^{2/3}+ax\right)^{5/2}}{429a^4x} + \frac{32b^2\left(bx^{2/3}+ax\right)^{5/2}}{143a^3x^{2/3}} - \frac{4b\left(bx^{2/3}+ax\right)^{5/2}}{13a^2\sqrt[3]{x}} + \cdots \\
&= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a} + \frac{256b^4\left(bx^{2/3}+ax\right)^{5/2}}{3003a^5x^{4/3}} - \frac{64b^3\left(bx^{2/3}+ax\right)^{5/2}}{429a^4x} + \frac{32b^2\left(bx^{2/3}+ax\right)^{5/2}}{143a^3x^{2/3}} \cdots \\
&= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a} - \frac{512b^5\left(bx^{2/3}+ax\right)^{5/2}}{15015a^6x^{5/3}} + \frac{256b^4\left(bx^{2/3}+ax\right)^{5/2}}{3003a^5x^{4/3}} - \frac{64b^3\left(bx^{2/3}+ax\right)^{5/2}}{429a^4x} \cdots
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 98, normalized size = 0.58

$$\frac{2\left(a\sqrt[3]{x}+b\right)^2\sqrt{ax+bx^{2/3}}\left(3003a^5x^{5/3}-2310a^4bx^{4/3}+1680a^3b^2x-1120a^2b^3x^{2/3}+640ab^4\sqrt[3]{x}-256b^5\right)}{15015a^6\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*(-256*b^5 + 640*a*b^4*x^(1/3) -
1120*a^2*b^3*x^(2/3) + 1680*a^3*b^2*x - 2310*a^4*b*x^(4/3) + 3003*a^5*x^(5/
3)))/(15015*a^6*x^(1/3))
```

IntegrateAlgebraic [A] time = 4.47, size = 98, normalized size = 0.58

$$\frac{2\left(a\sqrt[3]{x}+b\right)\left(x^{2/3}\left(a\sqrt[3]{x}+b\right)\right)^{3/2}\left(3003a^5x^{5/3}-2310a^4bx^{4/3}+1680a^3b^2x-1120a^2b^3x^{2/3}+640ab^4\sqrt[3]{x}-256b^5\right)}{15015a^6x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(b + a*x^(1/3))*((b + a*x^(1/3))*x^(2/3))^(3/2)*(-256*b^5 + 640*a*b^4*x^
(1/3) - 1120*a^2*b^3*x^(2/3) + 1680*a^3*b^2*x - 2310*a^4*b*x^(4/3) + 3003*a
^5*x^(5/3)))/(15015*a^6*x)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.23, size = 434, normalized size = 2.57

$$\frac{2}{3003}b\left(\frac{256b^{\frac{13}{2}}}{a^6}+\frac{\frac{13\left(63\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}-385\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b+990\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^2-1386\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^3+1155\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^4-693\sqrt{ax^{\frac{1}{3}}+b}\,b^5\right)b}{a^5}+\frac{3\left(231\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}-1638\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b+5005\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^2-8580\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^3+9009\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^4-6006\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^5+3003\sqrt{ax^{\frac{1}{3}}+b}\,b^6\right)}{a^5}}{a}\right)-\frac{2}{15015}a\left(\frac{1024b^{\frac{15}{2}}}{a^7}-\frac{\frac{15\left(231\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}-1638\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b+5005\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^2-8580\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^3+9009\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^4-6006\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^5+3003\sqrt{ax^{\frac{1}{3}}+b}\,b^6\right)b}{a^6}+\frac{7\left(429\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}-3465\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b+12285\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^2-25025\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^3+32175\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^4-27027\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^5+15015\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^6-6435\sqrt{ax^{\frac{1}{3}}+b}\,b^7\right)}{a^6}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3003*b*(256*b^(13/2)/a^6 + (13*(63*(a*x^(1/3) + b)^(11/2) - 385*(a*x^(1/3
) + b)^(9/2)*b + 990*(a*x^(1/3) + b)^(7/2)*b^2 - 1386*(a*x^(1/3) + b)^(5/2)
*b^3 + 1155*(a*x^(1/3) + b)^(3/2)*b^4 - 693*sqrt(a*x^(1/3) + b)*b^5)*b/a^5
+ 3*(231*(a*x^(1/3) + b)^(13/2) - 1638*(a*x^(1/3) + b)^(11/2)*b + 5005*(a*x
^(1/3) + b)^(9/2)*b^2 - 8580*(a*x^(1/3) + b)^(7/2)*b^3 + 9009*(a*x^(1/3) +
b)^(5/2)*b^4 - 6006*(a*x^(1/3) + b)^(3/2)*b^5 + 3003*sqrt(a*x^(1/3) + b)*b^
6)/a^5)/a) - 2/15015*a*(1024*b^(15/2)/a^7 - (15*(231*(a*x^(1/3) + b)^(13/2)
 - 1638*(a*x^(1/3) + b)^(11/2)*b + 5005*(a*x^(1/3) + b)^(9/2)*b^2 - 8580*(a
*x^(1/3) + b)^(7/2)*b^3 + 9009*(a*x^(1/3) + b)^(5/2)*b^4 - 6006*(a*x^(1/3)
+ b)^(3/2)*b^5 + 3003*sqrt(a*x^(1/3) + b)*b^6)*b/a^6 + 7*(429*(a*x^(1/3) +
b)^(15/2) - 3465*(a*x^(1/3) + b)^(13/2)*b + 12285*(a*x^(1/3) + b)^(11/2)*b^
2 - 25025*(a*x^(1/3) + b)^(9/2)*b^3 + 32175*(a*x^(1/3) + b)^(7/2)*b^4 - 270
27*(a*x^(1/3) + b)^(5/2)*b^5 + 15015*(a*x^(1/3) + b)^(3/2)*b^6 - 6435*sqrt(
a*x^(1/3) + b)*b^7)/a^6)/a)
```

maple [A] time = 0.05, size = 79, normalized size = 0.47

$$\frac{2\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}\left(a\,x^{\frac{1}{3}}+b\right)\left(3003a^5x^{\frac{5}{3}}-2310a^4b\,x^{\frac{4}{3}}+1680a^3b^2x-1120a^2b^3x^{\frac{2}{3}}+640a\,b^4x^{\frac{1}{3}}-256b^5\right)}{15015a^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(2/3))^(3/2),x)
```

```
[Out] 2/15015*(a*x+b*x^(2/3))^(3/2)*(a*x^(1/3)+b)*(3003*x^(5/3)*a^5-2310*a^4*b*x^
(4/3)+1680*a^3*b^2*x-1120*x^(2/3)*a^2*b^3+640*a*b^4*x^(1/3)-256*b^5)/x/a^6
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2), x)
```

mupad [B] time = 5.14, size = 40, normalized size = 0.24

$$\frac{x\left(a\,x+b\,x^{2/3}\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},6;\,7;\,-\frac{a\,x^{1/3}}{b}\right)}{2\left(\frac{a\,x^{1/3}}{b}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(3/2),x)
```

```
[Out] (x*(a*x + b*x^(2/3))^(3/2)*hypergeom([-3/2, 6], 7, -(a*x^(1/3))/b))/(2*((a*
x^(1/3))/b + 1)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2),x)
[Out] Integral((a*x + b*x**(2/3))**(3/2), x)
```

3.100 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=84

$$\frac{16b^2\left(ax+bx^{2/3}\right)^{5/2}}{105a^3x^{5/3}}-\frac{8b\left(ax+bx^{2/3}\right)^{5/2}}{21a^2x^{4/3}}+\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{3ax}$$

Rubi [A] time = 0.14, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{16b^2\left(ax+bx^{2/3}\right)^{5/2}}{105a^3x^{5/3}}-\frac{8b\left(ax+bx^{2/3}\right)^{5/2}}{21a^2x^{4/3}}+\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{3ax}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x,x]
```

```
[Out] (16*b^2*(b*x^(2/3) + a*x)^(5/2))/(105*a^3*x^(5/3)) - (8*b*(b*x^(2/3) + a*x)
^(5/2))/(21*a^2*x^(4/3)) + (2*(b*x^(2/3) + a*x)^(5/2))/(3*a*x)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx &= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{3ax}-\frac{(4b)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^{4/3}}\,dx}{9a}\\ &= -\frac{8b\left(bx^{2/3}+ax\right)^{5/2}}{21a^2x^{4/3}}+\frac{2\left(bx^{2/3}+ax\right)^{5/2}}{3ax}+\frac{\left(8b^2\right)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^{5/3}}\,dx}{63a^2}\\ &= \frac{16b^2\left(bx^{2/3}+ax\right)^{5/2}}{105a^3x^{5/3}}-\frac{8b\left(bx^{2/3}+ax\right)^{5/2}}{21a^2x^{4/3}}+\frac{2\left(bx^{2/3}+ax\right)^{5/2}}{3ax}\end{aligned}$$

Mathematica [A] time = 0.08, size = 63, normalized size = 0.75

$$\frac{2\left(a\sqrt[3]{x}+b\right)^2\left(35a^2x^{2/3}-20ab\sqrt[3]{x}+8b^2\right)\sqrt{ax+bx^{2/3}}}{105a^3\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x,x]
```

```
[Out] (2*(b + a*x^(1/3))^2*(8*b^2 - 20*a*b*x^(1/3) + 35*a^2*x^(2/3))*Sqrt[b*x^(2/
3) + a*x])/(105*a^3*x^(1/3))
```

IntegrateAlgebraic [A] time = 4.54, size = 87, normalized size = 1.04

$$\frac{2\left(x^{2/3}\left(a\sqrt[3]{x}+b\right)\right)^{3/2}\left(35a^4x^{4/3}+50a^3bx+3a^2b^2x^{2/3}-4ab^3\sqrt[3]{x}+8b^4\right)}{105a^3x\left(a\sqrt[3]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^(2/3) + a*x)^(3/2)/x,x]
```

```
[Out] (2*((b + a*x^(1/3))*x^(2/3))^(3/2)*(8*b^4 - 4*a*b^3*x^(1/3) + 3*a^2*b^2*x^(
2/3) + 50*a^3*b*x + 35*a^4*x^(4/3)))/(105*a^3*(b + a*x^(1/3))*x)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.21, size = 265, normalized size = 3.15

$$-\frac{2}{35}\,b\left(\frac{8\,b^{\frac{7}{2}}}{a^{3}}-\frac{\frac{7\left(3\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}-10\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b+15\sqrt{ax^{\frac{1}{3}}+b}\,b^{2}\right)b}{a^{2}}+\frac{3\left(5\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}-21\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b+35\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{2}-35\sqrt{ax^{\frac{1}{3}}+b}\,b^{3}\right)}{a^{2}}}{a}\right)+\frac{2}{105}\,a\left(\frac{16\,b^{\frac{9}{2}}}{a^{4}}+\frac{\frac{9\left(5\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}-21\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b+35\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{2}-35\sqrt{ax^{\frac{1}{3}}+b}\,b^{3}\right)b}{a^{3}}+\frac{35\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}-180\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b+378\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{2}-420\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{3}+315\sqrt{ax^{\frac{1}{3}}+b}\,b^{4}}{a^{3}}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -2/35*b*(8*b^(7/2)/a^3 - (7*(3*(a*x^(1/3) + b)^(5/2) - 10*(a*x^(1/3) + b)^(
3/2)*b + 15*sqrt(a*x^(1/3) + b)*b^2)*b/a^2 + 3*(5*(a*x^(1/3) + b)^(7/2) - 2
1*(a*x^(1/3) + b)^(5/2)*b + 35*(a*x^(1/3) + b)^(3/2)*b^2 - 35*sqrt(a*x^(1/3
) + b)*b^3)/a^2)/a) + 2/105*a*(16*b^(9/2)/a^4 + (9*(5*(a*x^(1/3) + b)^(7/2)
 - 21*(a*x^(1/3) + b)^(5/2)*b + 35*(a*x^(1/3) + b)^(3/2)*b^2 - 35*sqrt(a*x^
(1/3) + b)*b^3)*b/a^3 + (35*(a*x^(1/3) + b)^(9/2) - 180*(a*x^(1/3) + b)^(7/
2)*b + 378*(a*x^(1/3) + b)^(5/2)*b^2 - 420*(a*x^(1/3) + b)^(3/2)*b^3 + 315*
sqrt(a*x^(1/3) + b)*b^4)/a^3)/a)
```

maple [A] time = 0.05, size = 48, normalized size = 0.57

$$\frac{2\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}\left(a\,x^{\frac{1}{3}}+b\right)\left(35a^{2}x^{\frac{2}{3}}-20ab\,x^{\frac{1}{3}}+8b^{2}\right)}{105a^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(2/3))^(3/2)/x,x)
```

```
[Out] 2/105*(a*x+b*x^(2/3))^(3/2)*(a*x^(1/3)+b)*(35*a^2*x^(2/3)-20*a*b*x^(1/3)+8*
b^2)/x/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x+b\,x^{2/3}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(3/2)/x,x)
```

```
[Out] int((a*x + b*x^(2/3))^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x,x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(3/2)/x, x)
```

3.101 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=78

$$-6b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)+\frac{6b\sqrt{ax+bx^{2/3}}}{\sqrt[3]{x}}+\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{x}$$

Rubi [A] time = 0.14, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2021, 2029, 206}

$$-6b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)+\frac{6b\sqrt{ax+bx^{2/3}}}{\sqrt[3]{x}}+\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^2,x]
```

```
[Out] (6*b*Sqrt[b*x^(2/3) + a*x])/x^(1/3) + (2*(b*x^(2/3) + a*x)^(3/2))/x - 6*b^(
3/2)*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx &= \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{x}+b\int \frac{\sqrt{bx^{2/3}+ax}}{x^{4/3}}\,dx\\
&= \frac{6b\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}+\frac{2\left(bx^{2/3}+ax\right)^{3/2}}{x}+b^2\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx\\
&= \frac{6b\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}+\frac{2\left(bx^{2/3}+ax\right)^{3/2}}{x}-\left(6b^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)\\
&= \frac{6b\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}+\frac{2\left(bx^{2/3}+ax\right)^{3/2}}{x}-6b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)
\end{aligned}$$

Mathematica [A] time = 0.10, size = 88, normalized size = 1.13

$$\frac{2\sqrt{ax+bx^{2/3}}\left(\sqrt{a\sqrt[3]{x}+b}\left(a\sqrt[3]{x}+4b\right)-3b^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sqrt[3]{x}+b}}{\sqrt{b}}\right)\right)}{\sqrt[3]{x}\sqrt{a\sqrt[3]{x}+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^2,x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(Sqrt[b + a*x^(1/3)]*(4*b + a*x^(1/3)) - 3*b^(3/2)
*ArcTanh[Sqrt[b + a*x^(1/3)]/Sqrt[b]]))/(Sqrt[b + a*x^(1/3)]*x^(1/3))
```

IntegrateAlgebraic [A] time = 10.82, size = 90, normalized size = 1.15

$$\frac{\left(x^{2/3}\left(a\sqrt[3]{x}+b\right)\right)^{3/2}\left(2\sqrt{a\sqrt[3]{x}+b}\left(a\sqrt[3]{x}+4b\right)-6b^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sqrt[3]{x}+b}}{\sqrt{b}}\right)\right)}{x\left(a\sqrt[3]{x}+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^(2/3) + a*x)^(3/2)/x^2,x]
```

```
[Out] (((b + a*x^(1/3))*x^(2/3))^(3/2)*(2*Sqrt[b + a*x^(1/3)]*(4*b + a*x^(1/3)) -
 6*b^(3/2)*ArcTanh[Sqrt[b + a*x^(1/3)]/Sqrt[b]]))/((b + a*x^(1/3))^(3/2)*x)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.23, size = 83, normalized size = 1.06

$$\frac{6\,b^2\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+2\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}+6\sqrt{ax^{\frac{1}{3}}+b}\,b-\frac{2\left(3\,b^2\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+4\sqrt{-b}\,b^{\frac{3}{2}}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 6*b^2*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/sqrt(-b) + 2*(a*x^(1/3) + b)^(3/
2) + 6*sqrt(a*x^(1/3) + b)*b - 2*(3*b^2*arctan(sqrt(b)/sqrt(-b)) + 4*sqrt(-
b)*b^(3/2))/sqrt(-b)
```

maple [A] time = 0.05, size = 69, normalized size = 0.88

$$-\frac{2\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}\left(3b^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)-3\sqrt{a\,x^{\frac{1}{3}}+b}\;b-\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{3}{2}}\right)}{\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(2/3))^(3/2)/x^2,x)
```

```
[Out] -2*(a*x+b*x^(2/3))^(3/2)*(3*b^(3/2)*arctanh((a*x^(1/3)+b)^(1/2)/b^(1/2))-(a
*x^(1/3)+b)^(3/2)-3*(a*x^(1/3)+b)^(1/2)*b)/x/(a*x^(1/3)+b)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x + b\,x^{2/3}\right)^{3/2}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(3/2)/x^2,x)
```

```
[Out] int((a*x + b*x^(2/3))^(3/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**2,x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(3/2)/x**2, x)
```

3.102 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=113

$$\frac{3a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{8b^{3/2}}-\frac{3a^2\sqrt{ax+bx^{2/3}}}{8bx^{2/3}}-\frac{3a\sqrt{ax+bx^{2/3}}}{4x}-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^2}$$

Rubi [A] time = 0.18, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2029, 206}

$$\frac{3a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{8b^{3/2}}-\frac{3a^2\sqrt{ax+bx^{2/3}}}{8bx^{2/3}}-\frac{3a\sqrt{ax+bx^{2/3}}}{4x}-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^3,x]
```

```
[Out] (-3*a*Sqrt[b*x^(2/3) + a*x])/(4*x) - (3*a^2*Sqrt[b*x^(2/3) + a*x])/(8*b*x^(
2/3)) - (b*x^(2/3) + a*x)^(3/2)/x^2 + (3*a^3*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt
[b*x^(2/3) + a*x]])/(8*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx = -\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}+\frac{1}{2}a\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx$$
$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{4x}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}+\frac{1}{8}a^2\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx$$
$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{4x}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{8bx^{2/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}-\frac{a^3\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{16b}$$
$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{4x}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{8bx^{2/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}+\frac{\left(3a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt[3]{x}}{\sqrt{b}\ldots}\right)}{8b}$$
$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{4x}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{8bx^{2/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}+\frac{3a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{8b^{3/2}}$$

Mathematica [C] time = 0.07, size = 61, normalized size = 0.54

$$\frac{6a^3\left(a\sqrt[3]{x}+b\right)^2\sqrt{ax+bx^{2/3}}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{5b^4\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^3,x]
```

```
[Out] (6*a^3*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 4, 7/
2, 1 + (a*x^(1/3))/b])/(5*b^4*x^(1/3))
```

IntegrateAlgebraic [A] time = 16.12, size = 117, normalized size = 1.04

$$\frac{\left(x^{2/3}\left(a\sqrt[3]{x}+b\right)\right)^{3/2}\left(\frac{3a^3\tanh^{-1}\left(\frac{\sqrt{a\sqrt[3]{x}+b}}{\sqrt{b}}\right)}{8b^{3/2}}+\frac{\sqrt{a\sqrt[3]{x}+b}\left(-3a^2x^{2/3}-14ab\sqrt[3]{x}-8b^2\right)}{8bx}\right)}{x\left(a\sqrt[3]{x}+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^(2/3) + a*x)^(3/2)/x^3,x]
```

```
[Out] (((b + a*x^(1/3))*x^(2/3))^(3/2)*((Sqrt[b + a*x^(1/3)]*(-8*b^2 - 14*a*b*x^(
1/3) - 3*a^2*x^(2/3)))/(8*b*x) + (3*a^3*ArcTanh[Sqrt[b + a*x^(1/3)]/Sqrt[b]
])/(8*b^(3/2))))/((b + a*x^(1/3))^(3/2)*x)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.26, size = 92, normalized size = 0.81

$$-\frac{\frac{3\,a^4\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b}+\frac{3\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^4+8\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^4b-3\sqrt{ax^{\frac{1}{3}}+b}\,a^4b^2}{a^3bx}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*a^4*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) + (3*(a*x^(1/
3) + b)^(5/2)*a^4 + 8*(a*x^(1/3) + b)^(3/2)*a^4*b - 3*sqrt(a*x^(1/3) + b)*a
^4*b^2)/(a^3*b*x))/a
```

maple [A] time = 0.06, size = 93, normalized size = 0.82

$$-\frac{\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}\left(-3a^{3}bx\,\mathrm{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)-3\sqrt{a\,x^{\frac{1}{3}}+b}\;b^{\frac{7}{2}}+8\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{\frac{5}{2}}+3\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{\frac{3}{2}}\right)}{8\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{\frac{5}{2}}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(2/3))^(3/2)/x^3,x)
```

```
[Out] -1/8*(a*x+b*x^(2/3))^(3/2)*(3*(a*x^(1/3)+b)^(5/2)*b^(3/2)+8*(a*x^(1/3)+b)^(
3/2)*b^(5/2)-3*(a*x^(1/3)+b)^(1/2)*b^(7/2)-3*arctanh((a*x^(1/3)+b)^(1/2)/b^
(1/2))*x*a^3*b)/x^2/(a*x^(1/3)+b)^(3/2)/b^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x+b\,x^{2/3}\right)^{3/2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(3/2)/x^3,x)
```

```
[Out] int((a*x + b*x^(2/3))^(3/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**3,x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(3/2)/x**3, x)
```

3.103 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=203

$$-\frac{21a^6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{512b^{9/2}}+\frac{21a^5\sqrt{ax+bx^{2/3}}}{512b^4x^{2/3}}-\frac{7a^4\sqrt{ax+bx^{2/3}}}{256b^3x}+\frac{7a^3\sqrt{ax+bx^{2/3}}}{320b^2x^{4/3}}-\frac{3a^2\sqrt{ax+bx^{2/3}}}{160bx^{5/3}}-\frac{\left(ax+bx^{2}\right.}{2x^3}$$

Rubi [A] time = 0.34, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2029, 206}

$$\frac{21a^5\sqrt{ax+bx^{2/3}}}{512b^4x^{2/3}}-\frac{7a^4\sqrt{ax+bx^{2/3}}}{256b^3x}+\frac{7a^3\sqrt{ax+bx^{2/3}}}{320b^2x^{4/3}}-\frac{21a^6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{512b^{9/2}}-\frac{3a^2\sqrt{ax+bx^{2/3}}}{160bx^{5/3}}-\frac{3a\sqrt{ax+bx^{2/3}}}{20x^2}-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^4,x]
```

```
[Out] (-3*a*Sqrt[b*x^(2/3) + a*x])/(20*x^2) - (3*a^2*Sqrt[b*x^(2/3) + a*x])/(160*
b*x^(5/3)) + (7*a^3*Sqrt[b*x^(2/3) + a*x])/(320*b^2*x^(4/3)) - (7*a^4*Sqrt[
b*x^(2/3) + a*x])/(256*b^3*x) + (21*a^5*Sqrt[b*x^(2/3) + a*x])/(512*b^4*x^(
2/3)) - (b*x^(2/3) + a*x)^(3/2)/(2*x^3) - (21*a^6*ArcTanh[(Sqrt[b]*x^(1/3))
/Sqrt[b*x^(2/3) + a*x]])/(512*b^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx = -\frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3} + \frac{1}{4}a\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2} - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3} + \frac{1}{40}a^2\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}} - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3} - \frac{\left(7a^3\right)\int \frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{320b}$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}} + \frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}} - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3} + \frac{\left(7a^4\right)\int \frac{}{x^{4/3}\sqrt{b}}}{384b}$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}} + \frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}} - \frac{7a^4\sqrt{bx^{2/3}+ax}}{256b^3x} - \frac{\left(bx^{2/3}+ax\right)}{2x^3}$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}} + \frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}} - \frac{7a^4\sqrt{bx^{2/3}+ax}}{256b^3x} + \frac{21a^5\sqrt{bx^{2/3}}}{512b^4x^{2/}}$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}} + \frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}} - \frac{7a^4\sqrt{bx^{2/3}+ax}}{256b^3x} + \frac{21a^5\sqrt{bx^{2/3}}}{512b^4x^{2/}}$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}} + \frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}} - \frac{7a^4\sqrt{bx^{2/3}+ax}}{256b^3x} + \frac{21a^5\sqrt{bx^{2/3}}}{512b^4x^{2/}}$$

Mathematica [C] time = 0.05, size = 61, normalized size = 0.30

$$-\frac{6a^6\left(a\sqrt[3]{x}+b\right)^2\sqrt{ax+bx^{2/3}}\,{}_2F_1\left(\frac{5}{2},7;\frac{7}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{5b^7\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^4,x]
```

```
[Out] (-6*a^6*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 7, 7
/2, 1 + (a*x^(1/3))/b])/(5*b^7*x^(1/3))
```

IntegrateAlgebraic [A] time = 17.13, size = 152, normalized size = 0.75

$$\frac{\left(x^{2/3}\left(a\sqrt[3]{x}+b\right)\right)^{3/2}\left(\frac{\sqrt{a\sqrt[3]{x}+b}\left(105a^5x^{5/3}-70a^4bx^{4/3}+56a^3b^2x-48a^2b^3x^{2/3}-1664ab^4\sqrt[3]{x}-1280b^5\right)}{2560b^4x^2}-\frac{21a^6\tanh^{-1}\left(\frac{\sqrt{a\sqrt[3]{x}+b}}{\sqrt{b}}\right)}{512b^{9/2}}\right)}{x\left(a\sqrt[3]{x}+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^(2/3) + a*x)^(3/2)/x^4,x]
```

```
[Out] (((b + a*x^(1/3))*x^(2/3))^(3/2)*((Sqrt[b + a*x^(1/3)]*(-1280*b^5 - 1664*a*
b^4*x^(1/3) - 48*a^2*b^3*x^(2/3) + 56*a^3*b^2*x - 70*a^4*b*x^(4/3) + 105*a^
5*x^(5/3)))/(2560*b^4*x^2) - (21*a^6*ArcTanh[Sqrt[b + a*x^(1/3)]/Sqrt[b]])/
(512*b^(9/2))))/((b + a*x^(1/3))^(3/2)*x)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.37, size = 143, normalized size = 0.70

$$\frac{\frac{105\,a^{7}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^{4}}+\frac{105\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^{7}-595\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{7}b+1386\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^{7}b^{2}-1686\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^{7}b^{3}-595\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^{7}b^{4}+105\sqrt{ax^{\frac{1}{3}}+b}\,a^{7}b^{5}}{a^{6}b^{4}x^{2}}}{2560\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2560*(105*a^7*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) + (105*
(a*x^(1/3) + b)^(11/2)*a^7 - 595*(a*x^(1/3) + b)^(9/2)*a^7*b + 1386*(a*x^(1
/3) + b)^(7/2)*a^7*b^2 - 1686*(a*x^(1/3) + b)^(5/2)*a^7*b^3 - 595*(a*x^(1/3
) + b)^(3/2)*a^7*b^4 + 105*sqrt(a*x^(1/3) + b)*a^7*b^5)/(a^6*b^4*x^2))/a
```

maple [A] time = 0.06, size = 139, normalized size = 0.68

$$\frac{\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}\left(-105a^{6}b^{4}x^{2}\operatorname{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)+105\sqrt{a\,x^{\frac{1}{3}}+b}\;b^{\frac{19}{2}}-595\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{\frac{17}{2}}-1686\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{\frac{15}{2}}+1386\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^{\frac{13}{2}}-595\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^{\frac{11}{2}}+105\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^{\frac{9}{2}}\right)}{2560\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{\frac{17}{2}}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(2/3))^(3/2)/x^4,x)
```

```
[Out] 1/2560*(a*x+b*x^(2/3))^(3/2)*(105*b^(9/2)*(a*x^(1/3)+b)^(11/2)-595*b^(11/2)
*(a*x^(1/3)+b)^(9/2)+1386*b^(13/2)*(a*x^(1/3)+b)^(7/2)-1686*b^(15/2)*(a*x^(
1/3)+b)^(5/2)-595*b^(17/2)*(a*x^(1/3)+b)^(3/2)+105*b^(19/2)*(a*x^(1/3)+b)^(
1/2)-105*arctanh((a*x^(1/3)+b)^(1/2)/b^(1/2))*b^4*x^2*a^6)/x^3/(a*x^(1/3)+b
)^(3/2)/b^(17/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a\,x+b\,x^{2/3}\right)}^{3/2}}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(3/2)/x^4,x)
```

```
[Out] int((a*x + b*x^(2/3))^(3/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**4,x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(3/2)/x**4, x)
```

3.104 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=291

$$\frac{429a^9\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{32768b^{15/2}}-\frac{429a^8\sqrt{ax+bx^{2/3}}}{32768b^7x^{2/3}}+\frac{143a^7\sqrt{ax+bx^{2/3}}}{16384b^6x}-\frac{143a^6\sqrt{ax+bx^{2/3}}}{20480b^5x^{4/3}}+\frac{429a^5\sqrt{ax+bx^{2/3}}}{71680b^4x^{5/3}}-1$$

Rubi [A] time = 0.52, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2029, 206}

$$-\frac{429a^8\sqrt{ax+bx^{2/3}}}{32768b^7x^{2/3}}+\frac{143a^7\sqrt{ax+bx^{2/3}}}{16384b^6x}-\frac{143a^6\sqrt{ax+bx^{2/3}}}{20480b^5x^{4/3}}+\frac{429a^5\sqrt{ax+bx^{2/3}}}{71680b^4x^{5/3}}-\frac{143a^4\sqrt{ax+bx^{2/3}}}{26880b^3x^2}+\frac{13a^3\sqrt{ax+bx^{2/3}}}{2688b^2x^{7/3}}+\frac{429a^9\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{32768b^{15/2}}-\frac{a^2\sqrt{ax+bx^{2/3}}}{224bx^{8/3}}-\frac{a\sqrt{ax+bx^{2/3}}}{16x^3}-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^5,x]
```

```
[Out] -(a*Sqrt[b*x^(2/3) + a*x])/(16*x^3) - (a^2*Sqrt[b*x^(2/3) + a*x])/(224*b*x^
(8/3)) + (13*a^3*Sqrt[b*x^(2/3) + a*x])/(2688*b^2*x^(7/3)) - (143*a^4*Sqrt[
b*x^(2/3) + a*x])/(26880*b^3*x^2) + (429*a^5*Sqrt[b*x^(2/3) + a*x])/(71680*
b^4*x^(5/3)) - (143*a^6*Sqrt[b*x^(2/3) + a*x])/(20480*b^5*x^(4/3)) + (143*a
^7*Sqrt[b*x^(2/3) + a*x])/(16384*b^6*x) - (429*a^8*Sqrt[b*x^(2/3) + a*x])/(
32768*b^7*x^(2/3)) - (b*x^(2/3) + a*x)^(3/2)/(3*x^4) + (429*a^9*ArcTanh[(Sq
rt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(32768*b^(15/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx = -\frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4}+\frac{1}{6}a\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx$$

$$= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4}+\frac{1}{96}a^2\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx$$

$$= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4}-\frac{\left(13a^3\right)\int \frac{1}{x^{8/3}\sqrt{bx^{2/3}+ax}}\,dx}{1344b}$$

$$= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4}+\frac{\left(143a^4\right)\int \frac{}{x^{7/3}\sqrt{}}}{16128}$$

$$= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}-\frac{\left(bx^{2/3}+ax\right.}{3x^4}$$

$$= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}+\frac{429a^5\sqrt{bx^2}}{71680b^4}$$

$$= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}+\frac{429a^5\sqrt{bx^2}}{71680b^4}$$

$$= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}+\frac{429a^5\sqrt{bx^2}}{71680b^4}$$

$$= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}+\frac{429a^5\sqrt{bx^2}}{71680b^4}$$

$$= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}+\frac{429a^5\sqrt{bx^2}}{71680b^4}$$

$$= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}+\frac{429a^5\sqrt{bx^2}}{71680b^4}$$

Mathematica [C] time = 0.06, size = 61, normalized size = 0.21

$$\frac{6a^9\left(a\sqrt[3]{x}+b\right)^2\sqrt{ax+bx^{2/3}}\;{}_2F_1\left(\frac{5}{2},10;\frac{7}{2};\frac{\sqrt[3]{x}\,a}{b}+1\right)}{5b^{10}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^5,x]
```

```
[Out] (6*a^9*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 10, 7
/2, 1 + (a*x^(1/3))/b])/(5*b^10*x^(1/3))
```

IntegrateAlgebraic [A] time = 18.45, size = 189, normalized size = 0.65

$$\frac{\left(x^{2/3}\left(a\sqrt[3]{x}+b\right)\right)^{3/2}\left(\frac{429a^9\tanh^{-1}\left(\frac{\sqrt{a\sqrt[3]{x}+b}}{\sqrt{b}}\right)}{32768b^{15/2}}+\frac{\sqrt{a\sqrt[3]{x}+b}\left(-45045a^8x^{8/3}+30030a^7bx^{7/3}-24024a^6b^2x^2+20592a^5b^3x^{5/3}-18304a^4b^4x^{4/3}+16640a^3b^5x-15360a^2b^6x^{2/3}-1361920ab^7\sqrt[3]{x}-1146880b^8\right)}{3440640b^7x^3}\right)}{x\left(a\sqrt[3]{x}+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^(2/3) + a*x)^(3/2)/x^5,x]
```

```
[Out] (((b + a*x^(1/3))*x^(2/3))^(3/2)*((Sqrt[b + a*x^(1/3)]*(-1146880*b^8 - 1361
920*a*b^7*x^(1/3) - 15360*a^2*b^6*x^(2/3) + 16640*a^3*b^5*x - 18304*a^4*b^4
```

```
*x^(4/3) + 20592*a^5*b^3*x^(5/3) - 24024*a^6*b^2*x^2 + 30030*a^7*b*x^(7/3)
- 45045*a^8*x^(8/3)))/(3440640*b^7*x^3) + (429*a^9*ArcTanh[Sqrt[b + a*x^(1/
3)]/Sqrt[b]])/(32768*b^(15/2))))/((b + a*x^(1/3))^(3/2)*x)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.38, size = 194, normalized size = 0.67

$$-\frac{\frac{45045\,a^{10}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^{7}}+\frac{45045\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{10}-390390\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{10}b+1495494\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^{10}b^{2}-3317886\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^{10}b^{3}+4685824\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{10}b^{4}-4349826\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^{10}b^{5}+2633274\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^{10}b^{6}+390390\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^{10}b^{7}-45045\sqrt{ax^{\frac{1}{3}}+b}\,a^{10}b^{8}}{a^{9}b^{7}x^{3}}}{3440640\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/3440640*(45045*a^10*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7)
+ (45045*(a*x^(1/3) + b)^(17/2)*a^10 - 390390*(a*x^(1/3) + b)^(15/2)*a^10*b
 + 1495494*(a*x^(1/3) + b)^(13/2)*a^10*b^2 - 3317886*(a*x^(1/3) + b)^(11/2)
*a^10*b^3 + 4685824*(a*x^(1/3) + b)^(9/2)*a^10*b^4 - 4349826*(a*x^(1/3) + b
)^(7/2)*a^10*b^5 + 2633274*(a*x^(1/3) + b)^(5/2)*a^10*b^6 + 390390*(a*x^(1/
3) + b)^(3/2)*a^10*b^7 - 45045*sqrt(a*x^(1/3) + b)*a^10*b^8)/(a^9*b^7*x^3))
/a
```

maple [A] time = 0.06, size = 181, normalized size = 0.62

$$-\frac{\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}\left(-45045a^{9}b^{7}x^{3}\,\mathrm{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)-45045\sqrt{a\,x^{\frac{1}{3}}+b}\;b^{\frac{31}{2}}+390390\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{\frac{29}{2}}+2633274\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{\frac{27}{2}}-4349826\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^{\frac{25}{2}}+4685824\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^{\frac{23}{2}}-3317886\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^{\frac{21}{2}}+1495494\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^{\frac{19}{2}}-390390\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^{\frac{17}{2}}+45045\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b^{\frac{15}{2}}\right)}{3440640\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{\frac{29}{2}}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(2/3))^(3/2)/x^5,x)
```

```
[Out] -1/3440640*(a*x+b*x^(2/3))^(3/2)*(45045*(a*x^(1/3)+b)^(17/2)*b^(15/2)-39039
0*(a*x^(1/3)+b)^(15/2)*b^(17/2)+1495494*(a*x^(1/3)+b)^(13/2)*b^(19/2)-33178
86*(a*x^(1/3)+b)^(11/2)*b^(21/2)+4685824*(a*x^(1/3)+b)^(9/2)*b^(23/2)-43498
26*(a*x^(1/3)+b)^(7/2)*b^(25/2)+2633274*(a*x^(1/3)+b)^(5/2)*b^(27/2)+390390
*(a*x^(1/3)+b)^(3/2)*b^(29/2)-45045*(a*x^(1/3)+b)^(1/2)*b^(31/2)-45045*arct
anh((a*x^(1/3)+b)^(1/2)/b^(1/2))*b^7*x^3*a^9)/x^4/(a*x^(1/3)+b)^(3/2)/b^(29
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a\,x+b\,x^{2/3}\right)}^{3/2}}{x^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(3/2)/x^5,x)
```

```
[Out] int((a*x + b*x^(2/3))^(3/2)/x^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**5,x)
```

```
[Out] Timed out
```

3.105 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=379

$$-\frac{12597a^{12}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{2097152b^{21/2}}+\frac{12597a^{11}\sqrt{ax+bx^{2/3}}}{2097152b^{10}x^{2/3}}-\frac{4199a^{10}\sqrt{ax+bx^{2/3}}}{1048576b^9x}+\frac{4199a^9\sqrt{ax+bx^{2/3}}}{1310720b^8x^{4/3}}-\frac{12597a^8\sqrt{\ }}{458752}$$

Rubi [A] time = 0.72, antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2029, 206}

$$\frac{12597a^{11}\sqrt{ax+bx^{2/3}}}{2097152b^{10}x^{2/3}}-\frac{4199a^{10}\sqrt{ax+bx^{2/3}}}{1048576b^9x}+\frac{4199a^9\sqrt{ax+bx^{2/3}}}{1310720b^8x^{4/3}}-\frac{12597a^8\sqrt{ax+bx^{2/3}}}{4587520b^7x^{5/3}}+\frac{4199a^7\sqrt{ax+bx^{2/3}}}{1720320b^6x^2}-\frac{4199a^6\sqrt{ax+bx^{2/3}}}{1892352b^5x^{7/3}}+\frac{323a^5\sqrt{ax+bx^{2/3}}}{157696b^4x^{8/3}}-\frac{323a^4\sqrt{ax+bx^{2/3}}}{168960b^3x^3}+\frac{19a^3\sqrt{ax+bx^{2/3}}}{10560b^2x^{10/3}}-\frac{12597a^{12}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{2097152b^{21/2}}-\frac{3a^2\sqrt{ax+bx^{2/3}}}{1760bx^{11/3}}-\frac{3a\sqrt{ax+bx^{2/3}}}{88x^4}-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{4x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^6,x]
```

```
[Out] (-3*a*Sqrt[b*x^(2/3) + a*x])/(88*x^4) - (3*a^2*Sqrt[b*x^(2/3) + a*x])/(1760
*b*x^(11/3)) + (19*a^3*Sqrt[b*x^(2/3) + a*x])/(10560*b^2*x^(10/3)) - (323*a
^4*Sqrt[b*x^(2/3) + a*x])/(168960*b^3*x^3) + (323*a^5*Sqrt[b*x^(2/3) + a*x]
)/(157696*b^4*x^(8/3)) - (4199*a^6*Sqrt[b*x^(2/3) + a*x])/(1892352*b^5*x^(7
/3)) + (4199*a^7*Sqrt[b*x^(2/3) + a*x])/(1720320*b^6*x^2) - (12597*a^8*Sqrt
[b*x^(2/3) + a*x])/(4587520*b^7*x^(5/3)) + (4199*a^9*Sqrt[b*x^(2/3) + a*x])
/(1310720*b^8*x^(4/3)) - (4199*a^10*Sqrt[b*x^(2/3) + a*x])/(1048576*b^9*x)
+ (12597*a^11*Sqrt[b*x^(2/3) + a*x])/(2097152*b^10*x^(2/3)) - (b*x^(2/3) +
a*x)^(3/2)/(4*x^5) - (12597*a^12*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) +
 a*x]])/(2097152*b^(21/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx &= -\frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5}+\frac{1}{8}a\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5}+\frac{1}{176}a^2\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5}-\frac{\left(19a^3\right)\int \frac{1}{x^{11/3}\sqrt{bx^{2/3}+ax}}\,dx}{3520b} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5}+\frac{\left(323a^4\right)\int \frac{\cdots}{x^{1\cdots}}}{633\cdots} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}-\frac{\left(bx^{2/3}+\cdots\right.}{4x^5\cdots} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\sqrt{b\cdots}}{15769\cdots} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\sqrt{b\cdots}}{15769\cdots} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\sqrt{b\cdots}}{15769\cdots} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\sqrt{b\cdots}}{15769\cdots} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\sqrt{b\cdots}}{15769\cdots} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\sqrt{b\cdots}}{15769\cdots} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\sqrt{b\cdots}}{15769\cdots} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\sqrt{b\cdots}}{15769\cdots} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\sqrt{b\cdots}}{15769\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.10, size = 61, normalized size = 0.16

$$
-\frac{6a^{12}\left(a\sqrt[3]{x}+b\right)^2\sqrt{ax+bx^{2/3}}\;{}_2F_1\left(\frac{5}{2},13;\frac{7}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{5b^{13}\sqrt[3]{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^6,x]
```

```
[Out] (-6*a^12*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 13,
 7/2, 1 + (a*x^(1/3))/b])/(5*b^13*x^(1/3))
```

IntegrateAlgebraic [A] time = 19.89, size = 226, normalized size = 0.60

$$
\frac{\left(x^{2/3}\left(a\sqrt[3]{x}+b\right)\right)^{3/2}\left(\frac{\sqrt{a\sqrt[3]{x}+b}\left(14549535a^{11}x^{11/3}-9699690a^{10}bx^{10/3}+7759752a^9b^2x^3-6651216a^8b^3x^{8/3}+5912192a^7b^4x^{7/3}-5374720a^6b^5x^2+4961280a^5b^6x^{5/3}-4630528a^4b^7x^{4/3}+4358144a^3b^8x-4128768a^2b^9x^{2/3}-688128000ab^{10}\sqrt[3]{x}-605552640b^{11}\right)}{2422210560b^{10}x^4}-\frac{12597a^{12}\tanh^{-1}\left(\frac{\sqrt{a\sqrt[3]{x}+b}}{\sqrt{b}}\right)}{2097152b^{21/2}}\right)}{x\left(a\sqrt[3]{x}+b\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^(2/3) + a*x)^(3/2)/x^6,x]
```

```
[Out] (((b + a*x^(1/3))*x^(2/3))^(3/2)*((Sqrt[b + a*x^(1/3)]*(-605552640*b^11 - 6
88128000*a*b^10*x^(1/3) - 4128768*a^2*b^9*x^(2/3) + 4358144*a^3*b^8*x - 463
0528*a^4*b^7*x^(4/3) + 4961280*a^5*b^6*x^(5/3) - 5374720*a^6*b^5*x^2 + 5912
192*a^7*b^4*x^(7/3) - 6651216*a^8*b^3*x^(8/3) + 7759752*a^9*b^2*x^3 - 96996
90*a^10*b*x^(10/3) + 14549535*a^11*x^(11/3)))/(2422210560*b^10*x^4) - (1259
7*a^12*ArcTanh[Sqrt[b + a*x^(1/3)]/Sqrt[b]])/(2097152*b^(21/2))))/((b + a*x
^(1/3))^(3/2)*x)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.47, size = 245, normalized size = 0.65

$$\frac{\frac{14549535\,a^{13}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^{10}}+\frac{14549535\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}}a^{13}-169744575\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}a^{13}b+904981077\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{13}b^{2}-2913648309\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{13}b^{3}+6303782342\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{13}b^{4}-9643633350\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^{13}b^{5}+10677769530\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^{13}b^{6}-8598579770\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{13}b^{7}+4975837515\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^{13}b^{8}-2001671595\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^{13}b^{9}-169744575\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^{13}b^{10}+14549535\sqrt{ax^{\frac{1}{3}}+b}\,a^{13}b^{11}}{a^{12}b^{10}x^{4}}}{2422210560\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/2422210560*(14549535*a^13*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*
b^10) + (14549535*(a*x^(1/3) + b)^(23/2)*a^13 - 169744575*(a*x^(1/3) + b)^(
21/2)*a^13*b + 904981077*(a*x^(1/3) + b)^(19/2)*a^13*b^2 - 2913648309*(a*x^
(1/3) + b)^(17/2)*a^13*b^3 + 6303782342*(a*x^(1/3) + b)^(15/2)*a^13*b^4 - 9
643633350*(a*x^(1/3) + b)^(13/2)*a^13*b^5 + 10677769530*(a*x^(1/3) + b)^(11
/2)*a^13*b^6 - 8598579770*(a*x^(1/3) + b)^(9/2)*a^13*b^7 + 4975837515*(a*x^
(1/3) + b)^(7/2)*a^13*b^8 - 2001671595*(a*x^(1/3) + b)^(5/2)*a^13*b^9 - 169
744575*(a*x^(1/3) + b)^(3/2)*a^13*b^10 + 14549535*sqrt(a*x^(1/3) + b)*a^13*
b^11)/(a^12*b^10*x^4))/a
```

maple [A] time = 0.07, size = 223, normalized size = 0.59

$$\frac{\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}\left(-14549535a^{12}b^{10}x^{4}\,\mathrm{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)+14549535\sqrt{a\,x^{\frac{1}{3}}+b}\;b^{\frac{43}{2}}-169744575\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{\frac{41}{2}}-2001671595\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{\frac{39}{2}}+4975837515\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^{\frac{37}{2}}-8598579770\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^{\frac{35}{2}}+10677769530\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^{\frac{33}{2}}-9643633350\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^{\frac{31}{2}}+6303782342\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^{\frac{29}{2}}-2913648309\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b^{\frac{27}{2}}+904981077\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{19}{2}}b^{\frac{25}{2}}-169744575\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{21}{2}}b^{\frac{23}{2}}+14549535\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{23}{2}}b^{\frac{21}{2}}\right)}{2422210560\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{\frac{41}{2}}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(2/3))^(3/2)/x^6,x)
```

```
[Out] 1/2422210560*(a*x+b*x^(2/3))^(3/2)*(14549535*(a*x^(1/3)+b)^(23/2)*b^(21/2)-
169744575*(a*x^(1/3)+b)^(21/2)*b^(23/2)+904981077*(a*x^(1/3)+b)^(19/2)*b^(2
5/2)-2913648309*(a*x^(1/3)+b)^(17/2)*b^(27/2)+6303782342*(a*x^(1/3)+b)^(15/
2)*b^(29/2)-9643633350*(a*x^(1/3)+b)^(13/2)*b^(31/2)+10677769530*(a*x^(1/3)
+b)^(11/2)*b^(33/2)-8598579770*(a*x^(1/3)+b)^(9/2)*b^(35/2)+4975837515*(a*x
^(1/3)+b)^(7/2)*b^(37/2)-2001671595*(a*x^(1/3)+b)^(5/2)*b^(39/2)-169744575*
(a*x^(1/3)+b)^(3/2)*b^(41/2)+14549535*(a*x^(1/3)+b)^(1/2)*b^(43/2)-14549535
*arctanh((a*x^(1/3)+b)^(1/2)/b^(1/2))*b^10*x^4*a^12)/x^5/(a*x^(1/3)+b)^(3/2
)/b^(41/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^6,x, algorithm="maxima")
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x+b\,x^{2/3}\right)^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(3/2)/x^6,x)
[Out] int((a*x + b*x^(2/3))^(3/2)/x^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**6,x)
[Out] Timed out
```

3.106 $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx$

Optimal. Leaf size=401

$$-\frac{16777216b^{13}\sqrt{ax+bx^{2/3}}}{11700675a^{14}\sqrt[3]{x}}+\frac{8388608b^{12}\sqrt{ax+bx^{2/3}}}{11700675a^{13}}-\frac{2097152b^{11}\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{3900225a^{12}}+\frac{1048576b^{10}x^{2/3}\sqrt{ax+bx}}{2340135a^{11}}$$

Rubi [A] time = 0.73, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$-\frac{16777216b^{13}\sqrt{ax+bx^{2/3}}}{11700675a^{14}\sqrt[3]{x}}+\frac{8388608b^{12}\sqrt{ax+bx^{2/3}}}{11700675a^{13}}-\frac{2097152b^{11}\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{3900225a^{12}}+\frac{1048576b^{10}x^{2/3}\sqrt{ax+bx^{2/3}}}{2340135a^{11}}-\frac{131072b^{9}x\sqrt{ax+bx^{2/3}}}{334305a^{10}}+\frac{65536b^{8}x^{4/3}\sqrt{ax+bx^{2/3}}}{185725a^{9}}-\frac{180224b^{7}x^{5/3}\sqrt{ax+bx^{2/3}}}{557175a^{8}}+\frac{1171456b^{6}x^{2}\sqrt{ax+bx^{2/3}}}{3900225a^{7}}-\frac{73216b^{5}x^{7/3}\sqrt{ax+bx^{2/3}}}{260015a^{6}}+\frac{36608b^{4}x^{8/3}\sqrt{ax+bx^{2/3}}}{137655a^{5}}-\frac{9152b^{3}x^{3}\sqrt{ax+bx^{2/3}}}{36225a^{4}}+\frac{416b^{2}x^{10/3}\sqrt{ax+bx^{2/3}}}{1725a^{3}}-\frac{52bx^{11/3}\sqrt{ax+bx^{2/3}}}{225a^{2}}+\frac{2x^{4}\sqrt{ax+bx^{2/3}}}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (8388608*b^12*Sqrt[b*x^(2/3) + a*x])/(11700675*a^13) - (16777216*b^13*Sqrt[
b*x^(2/3) + a*x])/(11700675*a^14*x^(1/3)) - (2097152*b^11*x^(1/3)*Sqrt[b*x^
(2/3) + a*x])/(3900225*a^12) + (1048576*b^10*x^(2/3)*Sqrt[b*x^(2/3) + a*x])
/(2340135*a^11) - (131072*b^9*x*Sqrt[b*x^(2/3) + a*x])/(334305*a^10) + (655
36*b^8*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(185725*a^9) - (180224*b^7*x^(5/3)*Sq
rt[b*x^(2/3) + a*x])/(557175*a^8) + (1171456*b^6*x^2*Sqrt[b*x^(2/3) + a*x])
/(3900225*a^7) - (73216*b^5*x^(7/3)*Sqrt[b*x^(2/3) + a*x])/(260015*a^6) + (
36608*b^4*x^(8/3)*Sqrt[b*x^(2/3) + a*x])/(137655*a^5) - (9152*b^3*x^3*Sqrt[
b*x^(2/3) + a*x])/(36225*a^4) + (416*b^2*x^(10/3)*Sqrt[b*x^(2/3) + a*x])/(1
725*a^3) - (52*b*x^(11/3)*Sqrt[b*x^(2/3) + a*x])/(225*a^2) + (2*x^4*Sqrt[b*
x^(2/3) + a*x])/(9*a)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2x^4\sqrt{bx^{2/3}+ax}}{9a} - \frac{(26b)\int \frac{x^{11/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{27a}$$

$$= -\frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^2} + \frac{2x^4\sqrt{bx^{2/3}+ax}}{9a} + \frac{\left(208b^2\right)\int \frac{x^{10/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{225a^2}$$

$$= \frac{416b^2x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^3} - \frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^2} + \frac{2x^4\sqrt{bx^{2/3}+ax}}{9a} - \frac{\left(4576b^3\right)\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}}{5175a^3}$$

$$= -\frac{9152b^3x^3\sqrt{bx^{2/3}+ax}}{36225a^4} + \frac{416b^2x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^3} - \frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^2} +$$

$$= \frac{36608b^4x^{8/3}\sqrt{bx^{2/3}+ax}}{137655a^5} - \frac{9152b^3x^3\sqrt{bx^{2/3}+ax}}{36225a^4} + \frac{416b^2x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^3} -$$

$$= -\frac{73216b^5x^{7/3}\sqrt{bx^{2/3}+ax}}{260015a^6} + \frac{36608b^4x^{8/3}\sqrt{bx^{2/3}+ax}}{137655a^5} - \frac{9152b^3x^3\sqrt{bx^{2/3}+ax}}{36225a^4} +$$

$$= \frac{1171456b^6x^2\sqrt{bx^{2/3}+ax}}{3900225a^7} - \frac{73216b^5x^{7/3}\sqrt{bx^{2/3}+ax}}{260015a^6} + \frac{36608b^4x^{8/3}\sqrt{bx^{2/3}+ax}}{137655a^5} -$$

$$= -\frac{180224b^7x^{5/3}\sqrt{bx^{2/3}+ax}}{557175a^8} + \frac{1171456b^6x^2\sqrt{bx^{2/3}+ax}}{3900225a^7} - \frac{73216b^5x^{7/3}\sqrt{bx^{2/3}+ax}}{260015a^6} +$$

$$= \frac{65536b^8x^{4/3}\sqrt{bx^{2/3}+ax}}{185725a^9} - \frac{180224b^7x^{5/3}\sqrt{bx^{2/3}+ax}}{557175a^8} + \frac{1171456b^6x^2\sqrt{bx^{2/3}+ax}}{3900225a^7} -$$

$$= -\frac{131072b^9x\sqrt{bx^{2/3}+ax}}{334305a^{10}} + \frac{65536b^8x^{4/3}\sqrt{bx^{2/3}+ax}}{185725a^9} - \frac{180224b^7x^{5/3}\sqrt{bx^{2/3}+ax}}{557175a^8} +$$

$$= \frac{1048576b^{10}x^{2/3}\sqrt{bx^{2/3}+ax}}{2340135a^{11}} - \frac{131072b^9x\sqrt{bx^{2/3}+ax}}{334305a^{10}} + \frac{65536b^8x^{4/3}\sqrt{bx^{2/3}+ax}}{185725a^9} -$$

$$= -\frac{2097152b^{11}\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{3900225a^{12}} + \frac{1048576b^{10}x^{2/3}\sqrt{bx^{2/3}+ax}}{2340135a^{11}} - \frac{131072b^9x\sqrt{bx^{2/3}+ax}}{334305a^{10}} +$$

$$= \frac{8388608b^{12}\sqrt{bx^{2/3}+ax}}{11700675a^{13}} - \frac{2097152b^{11}\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{3900225a^{12}} + \frac{1048576b^{10}x^{2/3}\sqrt{bx^{2/3}+ax}}{2340135a^{11}} -$$

$$= \frac{8388608b^{12}\sqrt{bx^{2/3}+ax}}{11700675a^{13}} - \frac{16777216b^{13}\sqrt{bx^{2/3}+ax}}{11700675a^{14}\sqrt[3]{x}} - \frac{2097152b^{11}\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{3900225a^{12}} +$$

Mathematica [A] time = 0.23, size = 185, normalized size = 0.46

$$\frac{2\sqrt{ax+bx^{2/3}}\left(1300075a^{13}x^{13/3}-1352078a^{12}bx^4+1410864a^{11}b^2x^{11/3}-1478048a^{10}b^3x^{10/3}+1555840a^9b^4x^3-1647360a^8b^5x^{8/3}+1757184a^7b^6x^{7/3}-1892352a^6b^7x^2+2064384a^5b^8x^{5/3}-2293760a^4b^9x^{4/3}+2621440a^3b^{10}x-3145728a^2b^{11}x^{2/3}+4194304ab^{12}\sqrt[3]{x}-8388608b^{13}\right)}{11700675a^{14}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(-8388608*b^13 + 4194304*a*b^12*x^(1/3) - 3145728*
a^2*b^11*x^(2/3) + 2621440*a^3*b^10*x - 2293760*a^4*b^9*x^(4/3) + 2064384*a
^5*b^8*x^(5/3) - 1892352*a^6*b^7*x^2 + 1757184*a^7*b^6*x^(7/3) - 1647360*a^
8*b^5*x^(8/3) + 1555840*a^9*b^4*x^3 - 1478048*a^10*b^3*x^(10/3) + 1410864*a
^11*b^2*x^(11/3) - 1352078*a^12*b*x^4 + 1300075*a^13*x^(13/3)))/(11700675*a
^14*x^(1/3))
```

IntegrateAlgebraic [A] time = 0.13, size = 185, normalized size = 0.46

$$\frac{2\sqrt{ax+bx^{2/3}}\left(1300075a^{13}x^{13/3}-1352078a^{12}bx^4+1410864a^{11}b^2x^{11/3}-1478048a^{10}b^3x^{10/3}+1555840a^9b^4x^3-1647360a^8b^5x^{8/3}+1757184a^7b^6x^{7/3}-1892352a^6b^7x^2+2064384a^5b^8x^{5/3}-2293760a^4b^9x^{4/3}+2621440a^3b^{10}x-3145728a^2b^{11}x^{2/3}+4194304ab^{12}\sqrt[3]{x}-8388608b^{13}\right)}{11700675a^{14}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(-8388608*b^13 + 4194304*a*b^12*x^(1/3) - 3145728*
a^2*b^11*x^(2/3) + 2621440*a^3*b^10*x - 2293760*a^4*b^9*x^(4/3) + 2064384*a
^5*b^8*x^(5/3) - 1892352*a^6*b^7*x^2 + 1757184*a^7*b^6*x^(7/3) - 1647360*a^
8*b^5*x^(8/3) + 1555840*a^9*b^4*x^3 - 1478048*a^10*b^3*x^(10/3) + 1410864*a
^11*b^2*x^(11/3) - 1352078*a^12*b*x^4 + 1300075*a^13*x^(13/3)))/(11700675*a
^14*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.30, size = 206, normalized size = 0.51

$$\frac{16777216\, b^{\frac{27}{2}}}{11700675\, a^{14}} + \frac{2 \left(1300075 \left(ax^{\frac{1}{3}} + b\right)^{\frac{27}{2}} - 18253053 \left(ax^{\frac{1}{3}} + b\right)^{\frac{25}{2}} b + 119041650 \left(ax^{\frac{1}{3}} + b\right)^{\frac{23}{2}} b^{2} - 478056150 \left(ax^{\frac{1}{3}} + b\right)^{\frac{21}{2}} b^{3} + 1320944625 \left(ax^{\frac{1}{3}} + b\right)^{\frac{19}{2}} b^{4} - 2657429775 \left(ax^{\frac{1}{3}} + b\right)^{\frac{17}{2}} b^{5} + 4015671660 \left(ax^{\frac{1}{3}} + b\right)^{\frac{15}{2}} b^{6} - 4633467300 \left(ax^{\frac{1}{3}} + b\right)^{\frac{13}{2}} b^{7} + 4106936925 \left(ax^{\frac{1}{3}} + b\right)^{\frac{11}{2}} b^{8} - 2788660875 \left(ax^{\frac{1}{3}} + b\right)^{\frac{9}{2}} b^{9} + 1434168450 \left(ax^{\frac{1}{3}} + b\right)^{\frac{7}{2}} b^{10} - 547591590 \left(ax^{\frac{1}{3}} + b\right)^{\frac{5}{2}} b^{11} + 152108775 \left(ax^{\frac{1}{3}} + b\right)^{\frac{3}{2}} b^{12} - 35102025 \sqrt{ax^{\frac{1}{3}} + b}\, b^{13}\right)}{11700675\, a^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 16777216/11700675*b^(27/2)/a^14 + 2/11700675*(1300075*(a*x^(1/3) + b)^(27/2
) - 18253053*(a*x^(1/3) + b)^(25/2)*b + 119041650*(a*x^(1/3) + b)^(23/2)*b^
2 - 478056150*(a*x^(1/3) + b)^(21/2)*b^3 + 1320944625*(a*x^(1/3) + b)^(19/2
)*b^4 - 2657429775*(a*x^(1/3) + b)^(17/2)*b^5 + 4015671660*(a*x^(1/3) + b)^
(15/2)*b^6 - 4633467300*(a*x^(1/3) + b)^(13/2)*b^7 + 4106936925*(a*x^(1/3)
+ b)^(11/2)*b^8 - 2788660875*(a*x^(1/3) + b)^(9/2)*b^9 + 1434168450*(a*x^(1
/3) + b)^(7/2)*b^10 - 547591590*(a*x^(1/3) + b)^(5/2)*b^11 + 152108775*(a*x
^(1/3) + b)^(3/2)*b^12 - 35102025*sqrt(a*x^(1/3) + b)*b^13)/a^14
```

maple [A] time = 0.05, size = 167, normalized size = 0.42

$$\frac{2 \left(a\,x^{\frac{1}{3}} + b\right) \left(1300075 a^{13} x^{\frac{13}{3}} - 1352078 a^{12} b\, x^{4} + 1410864 a^{11} b^{2} x^{\frac{11}{3}} - 1478048 a^{10} b^{3} x^{\frac{10}{3}} + 1555840 a^{9} b^{4} x^{3} - 1647360 a^{8} b^{5} x^{\frac{8}{3}} + 1757184 a^{7} b^{6} x^{\frac{7}{3}} - 1892352 a^{6} b^{7} x^{2} + 2064384 a^{5} b^{8} x^{\frac{5}{3}} - 2293760 a^{4} b^{9} x^{\frac{4}{3}} + 2621440 a^{3} b^{10} x - 3145728 a^{2} b^{11} x^{\frac{2}{3}} + 4194304 a\, b^{12} x^{\frac{1}{3}} - 8388608 b^{13}\right) x^{\frac{1}{3}}}{11700675 \sqrt{ax + b\,x^{\frac{2}{3}}}\; a^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x+b*x^(2/3))^(1/2),x)
```

```
[Out] 2/11700675*x^(1/3)*(a*x^(1/3)+b)*(1300075*x^(13/3)*a^13-1352078*x^4*a^12*b+
1410864*x^(11/3)*a^11*b^2-1478048*x^(10/3)*a^10*b^3+1555840*x^3*a^9*b^4-164
7360*x^(8/3)*a^8*b^5+1757184*x^(7/3)*a^7*b^6-1892352*x^2*a^6*b^7+2064384*x^
(5/3)*a^5*b^8-2293760*x^(4/3)*a^4*b^9+2621440*x*a^3*b^10-3145728*x^(2/3)*a^
2*b^11+4194304*x^(1/3)*a*b^12-8388608*b^13)/(a*x+b*x^(2/3))^(1/2)/a^14
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\sqrt{ax + bx^{\frac{2}{3}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(a*x + b*x^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{a\,x + b\,x^{2/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x + b*x^(2/3))^(1/2),x)
[Out] int(x^4/(a*x + b*x^(2/3))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{ax + bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**(2/3)+a*x)**(1/2),x)
[Out] Integral(x**4/sqrt(a*x + b*x**(2/3)), x)
```

3.107 $\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx$

Optimal. Leaf size=313

$$\frac{524288b^{10}\sqrt{ax+bx^{2/3}}}{323323a^{11}\sqrt[3]{x}}-\frac{262144b^9\sqrt{ax+bx^{2/3}}}{323323a^{10}}+\frac{196608b^8\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{323323a^9}-\frac{163840b^7x^{2/3}\sqrt{ax+bx^{2/3}}}{323323a^8}+\frac{2048(}{}$$

Rubi [A] time = 0.53, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$\frac{524288b^{10}\sqrt{ax+bx^{2/3}}}{323323a^{11}\sqrt[3]{x}}-\frac{262144b^9\sqrt{ax+bx^{2/3}}}{323323a^{10}}+\frac{196608b^8\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{323323a^9}-\frac{163840b^7x^{2/3}\sqrt{ax+bx^{2/3}}}{323323a^8}+\frac{20480b^6x\sqrt{ax+bx^{2/3}}}{46189a^7}-\frac{18432b^5x^{4/3}\sqrt{ax+bx^{2/3}}}{46189a^6}+\frac{1536b^4x^{5/3}\sqrt{ax+bx^{2/3}}}{4199a^5}-\frac{768b^3x^2\sqrt{ax+bx^{2/3}}}{2261a^4}+\frac{720b^2x^{7/3}\sqrt{ax+bx^{2/3}}}{2261a^3}-\frac{40bx^{8/3}\sqrt{ax+bx^{2/3}}}{133a^2}+\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (-262144*b^9*Sqrt[b*x^(2/3) + a*x])/(323323*a^10) + (524288*b^10*Sqrt[b*x^(
2/3) + a*x])/(323323*a^11*x^(1/3)) + (196608*b^8*x^(1/3)*Sqrt[b*x^(2/3) + a
*x])/(323323*a^9) - (163840*b^7*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(323323*a^8)
 + (20480*b^6*x*Sqrt[b*x^(2/3) + a*x])/(46189*a^7) - (18432*b^5*x^(4/3)*Sqr
t[b*x^(2/3) + a*x])/(46189*a^6) + (1536*b^4*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/
(4199*a^5) - (768*b^3*x^2*Sqrt[b*x^(2/3) + a*x])/(2261*a^4) + (720*b^2*x^(7
/3)*Sqrt[b*x^(2/3) + a*x])/(2261*a^3) - (40*b*x^(8/3)*Sqrt[b*x^(2/3) + a*x]
)/(133*a^2) + (2*x^3*Sqrt[b*x^(2/3) + a*x])/(7*a)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2x^3\sqrt{bx^{2/3}+ax}}{7a} - \frac{(20b)\int \frac{x^{8/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{21a}$$

$$= -\frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2} + \frac{2x^3\sqrt{bx^{2/3}+ax}}{7a} + \frac{\left(120b^2\right)\int \frac{x^{7/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{133a^2}$$

$$= \frac{720b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^3} - \frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2} + \frac{2x^3\sqrt{bx^{2/3}+ax}}{7a} - \frac{\left(1920b^3\right)\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx}{2261a^3}$$

$$= -\frac{768b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^4} + \frac{720b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^3} - \frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2} + \frac{2x^3\sqrt{bx^{2/3}+ax}}{7a} \cdots$$

$$= \frac{1536b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^5} - \frac{768b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^4} + \frac{720b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^3} - \cdots$$

$$= -\frac{18432b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{46189a^6} + \frac{1536b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^5} - \frac{768b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^4} + \cdots$$

$$= \frac{20480b^6x\sqrt{bx^{2/3}+ax}}{46189a^7} - \frac{18432b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{46189a^6} + \frac{1536b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^5} - \cdots$$

$$= -\frac{163840b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{323323a^8} + \frac{20480b^6x\sqrt{bx^{2/3}+ax}}{46189a^7} - \frac{18432b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{46189a^6} + \cdots$$

$$= \frac{196608b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{323323a^9} - \frac{163840b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{323323a^8} + \frac{20480b^6x\sqrt{bx^{2/3}+ax}}{46189a^7} - \cdots$$

$$= -\frac{262144b^9\sqrt{bx^{2/3}+ax}}{323323a^{10}} + \frac{196608b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{323323a^9} - \frac{163840b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{323323a^8} + \cdots$$

$$= -\frac{262144b^9\sqrt{bx^{2/3}+ax}}{323323a^{10}} + \frac{524288b^{10}\sqrt{bx^{2/3}+ax}}{323323a^{11}\sqrt[3]{x}} + \frac{196608b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{323323a^9} - \cdots$$

Mathematica [A] time = 0.14, size = 148, normalized size = 0.47

$$\frac{2\sqrt{ax+bx^{2/3}}\left(46189a^{10}x^{10/3}-48620a^9bx^3+51480a^8b^2x^{8/3}-54912a^7b^3x^{7/3}+59136a^6b^4x^2-64512a^5b^5x^{5/3}+71680a^4b^6x^{4/3}-81920a^3b^7x+98304a^2b^8x^{2/3}-131072ab^9\sqrt[3]{x}+262144b^{10}\right)}{323323a^{11}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(262144*b^10 - 131072*a*b^9*x^(1/3) + 98304*a^2*b^
8*x^(2/3) - 81920*a^3*b^7*x + 71680*a^4*b^6*x^(4/3) - 64512*a^5*b^5*x^(5/3)
 + 59136*a^6*b^4*x^2 - 54912*a^7*b^3*x^(7/3) + 51480*a^8*b^2*x^(8/3) - 4862
0*a^9*b*x^3 + 46189*a^10*x^(10/3)))/(323323*a^11*x^(1/3))
```

IntegrateAlgebraic [A] time = 0.11, size = 148, normalized size = 0.47

$$\frac{2\sqrt{ax+bx^{2/3}}\left(46189a^{10}x^{10/3}-48620a^9bx^3+51480a^8b^2x^{8/3}-54912a^7b^3x^{7/3}+59136a^6b^4x^2-64512a^5b^5x^{5/3}+71680a^4b^6x^{4/3}-81920a^3b^7x+98304a^2b^8x^{2/3}-131072ab^9\sqrt[3]{x}+262144b^{10}\right)}{323323a^{11}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(262144*b^10 - 131072*a*b^9*x^(1/3) + 98304*a^2*b^
8*x^(2/3) - 81920*a^3*b^7*x + 71680*a^4*b^6*x^(4/3) - 64512*a^5*b^5*x^(5/3)
 + 59136*a^6*b^4*x^2 - 54912*a^7*b^3*x^(7/3) + 51480*a^8*b^2*x^(8/3) - 4862
0*a^9*b*x^3 + 46189*a^10*x^(10/3)))/(323323*a^11*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.19, size = 164, normalized size = 0.52

$$-\frac{524288\, b^{\frac{21}{2}}}{323323\, a^{11}} + \frac{2\left(46189\left(ax^{\frac{1}{3}} + b\right)^{\frac{21}{2}} - 510510\left(ax^{\frac{1}{3}} + b\right)^{\frac{19}{2}} b + 2567565\left(ax^{\frac{1}{3}} + b\right)^{\frac{17}{2}} b^2 - 7759752\left(ax^{\frac{1}{3}} + b\right)^{\frac{15}{2}} b^3 + 15668730\left(ax^{\frac{1}{3}} + b\right)^{\frac{13}{2}} b^4 - 22221108\left(ax^{\frac{1}{3}} + b\right)^{\frac{11}{2}} b^5 + 22632610\left(ax^{\frac{1}{3}} + b\right)^{\frac{9}{2}} b^6 - 16628040\left(ax^{\frac{1}{3}} + b\right)^{\frac{7}{2}} b^7 + 8729721\left(ax^{\frac{1}{3}} + b\right)^{\frac{5}{2}} b^8 - 3233230\left(ax^{\frac{1}{3}} + b\right)^{\frac{3}{2}} b^9 + 969969\sqrt{ax^{\frac{1}{3}} + b}\, b^{10}\right)}{323323\, a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -524288/323323*b^(21/2)/a^11 + 2/323323*(46189*(a*x^(1/3) + b)^(21/2) - 510
510*(a*x^(1/3) + b)^(19/2)*b + 2567565*(a*x^(1/3) + b)^(17/2)*b^2 - 7759752
*(a*x^(1/3) + b)^(15/2)*b^3 + 15668730*(a*x^(1/3) + b)^(13/2)*b^4 - 2222110
8*(a*x^(1/3) + b)^(11/2)*b^5 + 22632610*(a*x^(1/3) + b)^(9/2)*b^6 - 1662804
0*(a*x^(1/3) + b)^(7/2)*b^7 + 8729721*(a*x^(1/3) + b)^(5/2)*b^8 - 3233230*(
a*x^(1/3) + b)^(3/2)*b^9 + 969969*sqrt(a*x^(1/3) + b)*b^10)/a^11
```

maple [A] time = 0.04, size = 134, normalized size = 0.43

$$\frac{2\left(a\,x^{\frac{1}{3}} + b\right)\left(46189 a^{10} x^{\frac{10}{3}} - 48620 a^{9} b\, x^{3} + 51480 a^{8} b^{2} x^{\frac{8}{3}} - 54912 a^{7} b^{3} x^{\frac{7}{3}} + 59136 a^{6} b^{4} x^{2} - 64512 a^{5} b^{5} x^{\frac{5}{3}} + 71680 a^{4} b^{6} x^{\frac{4}{3}} - 81920 a^{3} b^{7} x + 98304 a^{2} b^{8} x^{\frac{2}{3}} - 131072 a\, b^{9} x^{\frac{1}{3}} + 262144 b^{10}\right) x^{\frac{1}{3}}}{323323\sqrt{ax + b\,x^{\frac{2}{3}}}\; a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x+b*x^(2/3))^(1/2),x)
```

```
[Out] 2/323323*x^(1/3)*(a*x^(1/3)+b)*(46189*x^(10/3)*a^10-48620*x^3*a^9*b+51480*a
^8*b^2*x^(8/3)-54912*x^(7/3)*a^7*b^3+59136*a^6*b^4*x^2-64512*a^5*b^5*x^(5/3
)+71680*x^(4/3)*a^4*b^6-81920*x*a^3*b^7+98304*a^2*b^8*x^(2/3)-131072*x^(1/3
)*a*b^9+262144*b^10)/(a*x+b*x^(2/3))^(1/2)/a^11
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{ax + bx^{\frac{2}{3}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(a*x + b*x^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{a\,x + b\,x^{2/3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] int(x^3/(a*x + b*x^(2/3))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{ax + bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a*x + b*x**(2/3)), x)
```

3.108 $\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx$

Optimal. Leaf size=225

$$-\frac{4096b^7\sqrt{ax+bx^{2/3}}}{2145a^8\sqrt[3]{x}}+\frac{2048b^6\sqrt{ax+bx^{2/3}}}{2145a^7}-\frac{512b^5\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{715a^6}+\frac{256b^4x^{2/3}\sqrt{ax+bx^{2/3}}}{429a^5}-\frac{224b^3x\sqrt{ax+bx^{2/3}}}{429a^4}$$

Rubi [A] time = 0.35, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$-\frac{4096b^7\sqrt{ax+bx^{2/3}}}{2145a^8\sqrt[3]{x}}+\frac{2048b^6\sqrt{ax+bx^{2/3}}}{2145a^7}-\frac{512b^5\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{715a^6}+\frac{256b^4x^{2/3}\sqrt{ax+bx^{2/3}}}{429a^5}-\frac{224b^3x\sqrt{ax+bx^{2/3}}}{429a^4}+\frac{336b^2x^{4/3}\sqrt{ax+bx^{2/3}}}{715a^3}-\frac{28bx^{5/3}\sqrt{ax+bx^{2/3}}}{65a^2}+\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2048*b^6*Sqrt[b*x^(2/3) + a*x])/(2145*a^7) - (4096*b^7*Sqrt[b*x^(2/3) + a*
x])/(2145*a^8*x^(1/3)) - (512*b^5*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^6)
+ (256*b^4*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(429*a^5) - (224*b^3*x*Sqrt[b*x^(
2/3) + a*x])/(429*a^4) + (336*b^2*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^3)
- (28*b*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/(65*a^2) + (2*x^2*Sqrt[b*x^(2/3) + a
*x])/(5*a)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2x^2\sqrt{bx^{2/3}+ax}}{5a} - \frac{(14b)\int \frac{x^{5/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{15a}$$

$$= -\frac{28bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3}+ax}}{5a} + \frac{\left(56b^2\right)\int \frac{x^{4/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{65a^2}$$

$$= \frac{336b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^3} - \frac{28bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3}+ax}}{5a} - \frac{\left(112b^3\right)\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx}{143a^3}$$

$$= -\frac{224b^3x\sqrt{bx^{2/3}+ax}}{429a^4} + \frac{336b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^3} - \frac{28bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3}+ax}}{5a} +$$

$$= \frac{256b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^5} - \frac{224b^3x\sqrt{bx^{2/3}+ax}}{429a^4} + \frac{336b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^3} - \frac{28bx^{5/3}\sqrt{bx^{2/3}+a}}{65a^2}$$

$$= -\frac{512b^5\sqrt[3]{x}\,\sqrt{bx^{2/3}+ax}}{715a^6} + \frac{256b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^5} - \frac{224b^3x\sqrt{bx^{2/3}+ax}}{429a^4} + \frac{336b^2x^{4/3}\sqrt{bx^{2/3}}}{715a^3}$$

$$= \frac{2048b^6\sqrt{bx^{2/3}+ax}}{2145a^7} - \frac{512b^5\sqrt[3]{x}\,\sqrt{bx^{2/3}+ax}}{715a^6} + \frac{256b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^5} - \frac{224b^3x\sqrt{bx^{2/3}+a}}{429a^4}$$

$$= \frac{2048b^6\sqrt{bx^{2/3}+ax}}{2145a^7} - \frac{4096b^7\sqrt{bx^{2/3}+ax}}{2145a^8\sqrt[3]{x}} - \frac{512b^5\sqrt[3]{x}\,\sqrt{bx^{2/3}+ax}}{715a^6} + \frac{256b^4x^{2/3}\sqrt{bx^{2/3}+a}}{429a^5}$$

Mathematica [A] time = 0.13, size = 111, normalized size = 0.49

$$\frac{2\sqrt{ax+bx^{2/3}}\left(429a^7x^{7/3}-462a^6bx^2+504a^5b^2x^{5/3}-560a^4b^3x^{4/3}+640a^3b^4x-768a^2b^5x^{2/3}+1024ab^6\sqrt[3]{x}-2048b^7\right)}{2145a^8\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(-2048*b^7 + 1024*a*b^6*x^(1/3) - 768*a^2*b^5*x^(2
/3) + 640*a^3*b^4*x - 560*a^4*b^3*x^(4/3) + 504*a^5*b^2*x^(5/3) - 462*a^6*b
*x^2 + 429*a^7*x^(7/3)))/(2145*a^8*x^(1/3))
```

IntegrateAlgebraic [A] time = 0.10, size = 111, normalized size = 0.49

$$\frac{2\sqrt{ax+bx^{2/3}}\left(429a^7x^{7/3}-462a^6bx^2+504a^5b^2x^{5/3}-560a^4b^3x^{4/3}+640a^3b^4x-768a^2b^5x^{2/3}+1024ab^6\sqrt[3]{x}-2048b^7\right)}{2145a^8\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(-2048*b^7 + 1024*a*b^6*x^(1/3) - 768*a^2*b^5*x^(2
/3) + 640*a^3*b^4*x - 560*a^4*b^3*x^(4/3) + 504*a^5*b^2*x^(5/3) - 462*a^6*b
*x^2 + 429*a^7*x^(7/3)))/(2145*a^8*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.27, size = 122, normalized size = 0.54

$$\frac{4096\,b^{\frac{15}{2}}}{2145\,a^8}+\frac{2\left(429\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}-3465\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b+12285\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^2-25025\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^3+32175\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^4-27027\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^5+15015\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^6-6435\sqrt{ax^{\frac{1}{3}}+b}\,b^7\right)}{2145\,a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4096/2145*b^(15/2)/a^8 + 2/2145*(429*(a*x^(1/3) + b)^(15/2) - 3465*(a*x^(1/
3) + b)^(13/2)*b + 12285*(a*x^(1/3) + b)^(11/2)*b^2 - 25025*(a*x^(1/3) + b)
^(9/2)*b^3 + 32175*(a*x^(1/3) + b)^(7/2)*b^4 - 27027*(a*x^(1/3) + b)^(5/2)*
b^5 + 15015*(a*x^(1/3) + b)^(3/2)*b^6 - 6435*sqrt(a*x^(1/3) + b)*b^7)/a^8
```

maple [A] time = 0.05, size = 101, normalized size = 0.45

$$\frac{2\left(a\,x^{\frac{1}{3}}+b\right)\left(429a^7x^{\frac{7}{3}}-462a^6b\,x^2+504a^5b^2x^{\frac{5}{3}}-560a^4b^3x^{\frac{4}{3}}+640a^3b^4x-768a^2b^5x^{\frac{2}{3}}+1024a\,b^6x^{\frac{1}{3}}-2048b^7\right)x^{\frac{1}{3}}}{2145\sqrt{ax+b\,x^{\frac{2}{3}}}\;a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+b*x^(2/3))^(1/2),x)
```

```
[Out] 2/2145*x^(1/3)*(a*x^(1/3)+b)*(429*x^(7/3)*a^7-462*x^2*a^6*b+504*x^(5/3)*a^5
*b^2-560*a^4*x^(4/3)*b^3+640*x*a^3*b^4-768*x^(2/3)*a^2*b^5+1024*x^(1/3)*a*b
^6-2048*b^7)/(a*x+b*x^(2/3))^(1/2)/a^8
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(a*x + b*x^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^2}{\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] int(x^2/(a*x + b*x^(2/3))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a*x + b*x**(2/3)), x)
```

3.109 $\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx$

Optimal. Leaf size=137

$$\frac{256b^4\sqrt{ax+bx^{2/3}}}{105a^5\sqrt[3]{x}}-\frac{128b^3\sqrt{ax+bx^{2/3}}}{105a^4}+\frac{32b^2\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{35a^3}-\frac{16bx^{2/3}\sqrt{ax+bx^{2/3}}}{21a^2}+\frac{2x\sqrt{ax+bx^{2/3}}}{3a}$$

Rubi [A] time = 0.18, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2016, 2002, 2014}

$$\frac{256b^4\sqrt{ax+bx^{2/3}}}{105a^5\sqrt[3]{x}}-\frac{128b^3\sqrt{ax+bx^{2/3}}}{105a^4}+\frac{32b^2\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{35a^3}-\frac{16bx^{2/3}\sqrt{ax+bx^{2/3}}}{21a^2}+\frac{2x\sqrt{ax+bx^{2/3}}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (-128*b^3*Sqrt[b*x^(2/3) + a*x])/(105*a^4) + (256*b^4*Sqrt[b*x^(2/3) + a*x]
)/(105*a^5*x^(1/3)) + (32*b^2*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(35*a^3) - (16
*b*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(21*a^2) + (2*x*Sqrt[b*x^(2/3) + a*x])/(3
*a)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2x\sqrt{bx^{2/3}+ax}}{3a} - \frac{(8b)\int \frac{x^{2/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{9a}$$
$$= -\frac{16bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^2} + \frac{2x\sqrt{bx^{2/3}+ax}}{3a} + \frac{\left(16b^2\right)\int \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\,dx}{21a^2}$$
$$= \frac{32b^2\sqrt[3]{x}\,\sqrt{bx^{2/3}+ax}}{35a^3} - \frac{16bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^2} + \frac{2x\sqrt{bx^{2/3}+ax}}{3a} - \frac{\left(64b^3\right)\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx}{105a^3}$$
$$= -\frac{128b^3\sqrt{bx^{2/3}+ax}}{105a^4} + \frac{32b^2\sqrt[3]{x}\,\sqrt{bx^{2/3}+ax}}{35a^3} - \frac{16bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^2} + \frac{2x\sqrt{bx^{2/3}+ax}}{3a} + \cdot$$
$$= -\frac{128b^3\sqrt{bx^{2/3}+ax}}{105a^4} + \frac{256b^4\sqrt{bx^{2/3}+ax}}{105a^5\sqrt[3]{x}} + \frac{32b^2\sqrt[3]{x}\,\sqrt{bx^{2/3}+ax}}{35a^3} - \frac{16bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^2}$$

Mathematica [A] time = 0.08, size = 74, normalized size = 0.54

$$\frac{2\sqrt{ax+bx^{2/3}}\left(35a^4x^{4/3}-40a^3bx+48a^2b^2x^{2/3}-64ab^3\sqrt[3]{x}+128b^4\right)}{105a^5\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[b*x^(2/3) + a*x],x]
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(128*b^4 - 64*a*b^3*x^(1/3) + 48*a^2*b^2*x^(2/3) -
 40*a^3*b*x + 35*a^4*x^(4/3)))/(105*a^5*x^(1/3))
```

IntegrateAlgebraic [A] time = 0.06, size = 74, normalized size = 0.54

$$\frac{2\sqrt{ax+bx^{2/3}}\left(35a^4x^{4/3}-40a^3bx+48a^2b^2x^{2/3}-64ab^3\sqrt[3]{x}+128b^4\right)}{105a^5\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[b*x^(2/3) + a*x],x]
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(128*b^4 - 64*a*b^3*x^(1/3) + 48*a^2*b^2*x^(2/3) -
 40*a^3*b*x + 35*a^4*x^(4/3)))/(105*a^5*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.18, size = 80, normalized size = 0.58

$$-\frac{256\,b^{\frac{9}{2}}}{105\,a^5} + \frac{2\left(35\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}} - 180\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b + 378\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^2 - 420\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^3 + 315\sqrt{ax^{\frac{1}{3}}+b}\,b^4\right)}{105\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -256/105*b^(9/2)/a^5 + 2/105*(35*(a*x^(1/3) + b)^(9/2) - 180*(a*x^(1/3) + b
)^(7/2)*b + 378*(a*x^(1/3) + b)^(5/2)*b^2 - 420*(a*x^(1/3) + b)^(3/2)*b^3 +
 315*sqrt(a*x^(1/3) + b)*b^4)/a^5
```

maple [A] time = 0.05, size = 68, normalized size = 0.50

$$\frac{2\left(a\,x^{\frac{1}{3}}+b\right)\left(35a^{4}x^{\frac{4}{3}}-40a^{3}bx+48a^{2}b^{2}x^{\frac{2}{3}}-64a\,b^{3}x^{\frac{1}{3}}+128b^{4}\right)x^{\frac{1}{3}}}{105\sqrt{ax+b\,x^{\frac{2}{3}}}\;a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x+b*x^(2/3))^(1/2),x)
```

```
[Out] 2/105*x^(1/3)*(a*x^(1/3)+b)*(35*x^(4/3)*a^4-40*x*a^3*b+48*x^(2/3)*a^2*b^2-6
4*x^(1/3)*a*b^3+128*b^4)/(a*x+b*x^(2/3))^(1/2)/a^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(a*x + b*x^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] int(x/(a*x + b*x^(2/3))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a*x + b*x**(2/3)), x)
```

3.110 $\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx$

Optimal. Leaf size=47

$$\frac{2\sqrt{ax+bx^{2/3}}}{a}-\frac{4b\sqrt{ax+bx^{2/3}}}{a^2\sqrt[3]{x}}$$

Rubi [A] time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2002, 2014}

$$\frac{2\sqrt{ax+bx^{2/3}}}{a}-\frac{4b\sqrt{ax+bx^{2/3}}}{a^2\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x])/a - (4*b*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx &= \frac{2\sqrt{bx^{2/3}+ax}}{a}-\frac{(2b)\int \frac{1}{\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}\,dx}{3a}\\ &= \frac{2\sqrt{bx^{2/3}+ax}}{a}-\frac{4b\sqrt{bx^{2/3}+ax}}{a^2\sqrt[3]{x}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 36, normalized size = 0.77

$$\frac{2\left(a\sqrt[3]{x}-2b\right)\sqrt{ax+bx^{2/3}}}{a^2\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(-2*b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))
```

IntegrateAlgebraic [A] time = 0.04, size = 36, normalized size = 0.77

$$\frac{2\left(a\sqrt[3]{x}-2b\right)\sqrt{ax+bx^{2/3}}}{a^2\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(-2*b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.49, size = 36, normalized size = 0.77

$$\frac{4\,b^{\frac{3}{2}}}{a^2}+\frac{2\left(\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}-3\,\sqrt{ax^{\frac{1}{3}}+b}\,b\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4*b^(3/2)/a^2 + 2*((a*x^(1/3) + b)^(3/2) - 3*sqrt(a*x^(1/3) + b)*b)/a^2
```

maple [A] time = 0.04, size = 36, normalized size = 0.77

$$\frac{2\left(a\,x^{\frac{1}{3}}+b\right)\left(a\,x^{\frac{1}{3}}-2b\right)x^{\frac{1}{3}}}{\sqrt{ax+b\,x^{\frac{2}{3}}}\;a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x^(2/3))^(1/2),x)
```

```
[Out] 2*x^(1/3)*(a*x^(1/3)+b)*(a*x^(1/3)-2*b)/(a*x+b*x^(2/3))^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*x + b*x^(2/3)), x)
```

mupad [B] time = 5.22, size = 40, normalized size = 0.85

$$\frac{3\,x\,\sqrt{\frac{a\,x^{1/3}}{b}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{2},2;\;3;\;-\frac{a\,x^{1/3}}{b}\right)}{2\,\sqrt{a\,x+b\,x^{2/3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] (3*x*((a*x^(1/3))/b + 1)^(1/2)*hypergeom([1/2, 2], 3, -(a*x^(1/3))/b))/(2*(
a*x + b*x^(2/3))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + bx^{\frac{2}{3}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*x**(2/3)), x)
```

3.111 $\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx$

Optimal. Leaf size=61

$$\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}$$

Rubi [A] time = 0.09, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2029, 206}

$$\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqr
t[b*x^(2/3) + a*x]])/b^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx &= -\frac{3\sqrt{bx^{2/3}+ax}}{bx^{2/3}}-\frac{a\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{2b}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{bx^{2/3}}+\frac{(3a)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{bx^{2/3}}+\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 90, normalized size = 1.48

$$\frac{6a\sqrt[3]{x}\left(a\sqrt[3]{x}+b\right)\left(\frac{\tanh^{-1}\left(\sqrt{\frac{a\sqrt[3]{x}}{b}+1}\right)}{2\sqrt{\frac{a\sqrt[3]{x}}{b}+1}}-\frac{b}{2a\sqrt[3]{x}}\right)}{b^2\sqrt{x^{2/3}\left(a\sqrt[3]{x}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (6*a*(b + a*x^(1/3))*x^(1/3)*(-1/2*b/(a*x^(1/3)) + ArcTanh[Sqrt[1 + (a*x^(1
/3))/b]]/(2*Sqrt[1 + (a*x^(1/3))/b])))/(b^2*Sqrt[(b + a*x^(1/3))*x^(2/3)])
```

IntegrateAlgebraic [A] time = 0.14, size = 61, normalized size = 1.00

$$\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqr
t[b*x^(2/3) + a*x]])/b^(3/2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.22, size = 51, normalized size = 0.84

$$-\frac{3\left(\frac{a^2\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b}+\frac{\sqrt{ax^{\frac{1}{3}}+b}\,a}{bx^{\frac{1}{3}}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -3*(a^2*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) + sqrt(a*x^(1/3)
+ b)*a/(b*x^(1/3)))/a
```

maple [A] time = 0.05, size = 61, normalized size = 1.00

$$\frac{3\sqrt{a\,x^{\frac{1}{3}}+b}\,\left(ab\,x^{\frac{1}{3}}\,\mathrm{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)-\sqrt{a\,x^{\frac{1}{3}}+b}\,b^{\frac{3}{2}}\right)}{\sqrt{ax+b\,x^{\frac{2}{3}}}\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x+b*x^(2/3))^(1/2),x)
```

```
[Out] 3*(a*x^(1/3)+b)^(1/2)*(arctanh((a*x^(1/3)+b)^(1/2)/b^(1/2))*b*x^(1/3)*a-(a*
x^(1/3)+b)^(1/2)*b^(3/2))/(a*x+b*x^(2/3))^(1/2)/b^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + bx^{\frac{2}{3}}}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(2/3))*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\sqrt{a\,x + b\,x^{2/3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^(2/3))^(1/2)),x)
```

```
[Out] int(1/(x*(a*x + b*x^(2/3))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{ax + bx^{\frac{2}{3}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a*x + b*x**(2/3))), x)
```

3.112 $\int \frac{1}{x^2 \sqrt{bx^{2/3}+ax}}\,dx$

Optimal. Leaf size=153

$$-\frac{105a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{64b^{9/2}}+\frac{105a^3\sqrt{ax+bx^{2/3}}}{64b^4x^{2/3}}-\frac{35a^2\sqrt{ax+bx^{2/3}}}{32b^3x}+\frac{7a\sqrt{ax+bx^{2/3}}}{8b^2x^{4/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}$$

Rubi [A] time = 0.24, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2029, 206}

$$\frac{105a^3\sqrt{ax+bx^{2/3}}}{64b^4x^{2/3}}-\frac{35a^2\sqrt{ax+bx^{2/3}}}{32b^3x}-\frac{105a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{64b^{9/2}}+\frac{7a\sqrt{ax+bx^{2/3}}}{8b^2x^{4/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(4*b*x^(5/3)) + (7*a*Sqrt[b*x^(2/3) + a*x])/(8*b
^2*x^(4/3)) - (35*a^2*Sqrt[b*x^(2/3) + a*x])/(32*b^3*x) + (105*a^3*Sqrt[b*x
^(2/3) + a*x])/(64*b^4*x^(2/3)) - (105*a^4*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b
*x^(2/3) + a*x]])/(64*b^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} - \frac{(7a)\int \frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{8b}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} + \frac{(35a^2)\int \frac{1}{x^{4/3}\sqrt{bx^{2/3}+ax}}\,dx}{48b^2}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} - \frac{35a^2\sqrt{bx^{2/3}+ax}}{32b^3x} - \frac{(35a^3)\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx}{64b^3}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} - \frac{35a^2\sqrt{bx^{2/3}+ax}}{32b^3x} + \frac{105a^3\sqrt{bx^{2/3}+ax}}{64b^4x^{2/3}} + \frac{(35a^4)\int \frac{}{x}}{1}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} - \frac{35a^2\sqrt{bx^{2/3}+ax}}{32b^3x} + \frac{105a^3\sqrt{bx^{2/3}+ax}}{64b^4x^{2/3}} - \frac{(105a^4)\,\mathrm{Su}}{}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} - \frac{35a^2\sqrt{bx^{2/3}+ax}}{32b^3x} + \frac{105a^3\sqrt{bx^{2/3}+ax}}{64b^4x^{2/3}} - \frac{105a^4\tanh}{6}$$

Mathematica [C] time = 0.06, size = 48, normalized size = 0.31

$$-\frac{6a^4\sqrt{ax+bx^{2/3}}\,{}_2F_1\left(\frac{1}{2},5;\frac{3}{2};\frac{\sqrt[3]{x}\,a}{b}+1\right)}{b^5\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-6*a^4*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[1/2, 5, 3/2, 1 + (a*x^(1/3)
)/b])/(b^5*x^(1/3))
```

IntegrateAlgebraic [A] time = 0.21, size = 101, normalized size = 0.66

$$\frac{\sqrt{ax+bx^{2/3}}\left(105a^3x-70a^2bx^{2/3}+56ab^2\sqrt[3]{x}-48b^3\right)}{64b^4x^{5/3}} - \frac{105a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{64b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (Sqrt[b*x^(2/3) + a*x]*(-48*b^3 + 56*a*b^2*x^(1/3) - 70*a^2*b*x^(2/3) + 105
*a^3*x))/(64*b^4*x^(5/3)) - (105*a^4*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/
3) + a*x]])/(64*b^(9/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.26, size = 109, normalized size = 0.71

$$\frac{\frac{105\,a^5\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^4} + \frac{105\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^5-385\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^5b+511\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^5b^2-279\sqrt{ax^{\frac{1}{3}}+b}\,a^5b^3}{a^4b^4x^{\frac{4}{3}}}}{64\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/64*(105*a^5*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) + (105*(a
*x^(1/3) + b)^(7/2)*a^5 - 385*(a*x^(1/3) + b)^(5/2)*a^5*b + 511*(a*x^(1/3)
+ b)^(3/2)*a^5*b^2 - 279*sqrt(a*x^(1/3) + b)*a^5*b^3)/(a^4*b^4*x^(4/3)))/a
```

maple [A] time = 0.05, size = 126, normalized size = 0.82

$$-\frac{\sqrt{a\,x^{\frac{1}{3}}+b}\,\left(105 a^{4} b\,x^{\frac{7}{3}} \operatorname{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)-105\sqrt{a\,x^{\frac{1}{3}}+b}\;a^{3} b^{\frac{3}{2}} x^{2}+70\sqrt{a\,x^{\frac{1}{3}}+b}\;a^{2} b^{\frac{5}{2}} x^{\frac{5}{3}}-56\sqrt{a\,x^{\frac{1}{3}}+b}\;a\,b^{\frac{7}{2}} x^{\frac{4}{3}}+48\sqrt{a\,x^{\frac{1}{3}}+b}\;b^{\frac{9}{2}} x\right)}{64\sqrt{ax+b\,x^{\frac{2}{3}}}\;b^{\frac{11}{2}} x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a*x+b*x^(2/3))^(1/2),x)
```

```
[Out] -1/64*(a*x^(1/3)+b)^(1/2)*(105*x^(7/3)*arctanh((a*x^(1/3)+b)^(1/2)/b^(1/2))
*a^4*b+70*x^(5/3)*(a*x^(1/3)+b)^(1/2)*b^(5/2)*a^2-56*x^(4/3)*(a*x^(1/3)+b)^
(1/2)*b^(7/2)*a+48*(a*x^(1/3)+b)^(1/2)*b^(9/2)*x-105*x^2*(a*x^(1/3)+b)^(1/2
)*b^(3/2)*a^3)/x^2/(a*x+b*x^(2/3))^(1/2)/b^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}\;x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(2/3))*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{2}\,\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^(2/3))^(1/2)),x)
```

```
[Out] int(1/(x^2*(a*x + b*x^(2/3))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a*x + b*x**(2/3))), x)
```

3.113 $\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx$

Optimal. Leaf size=241

$$\frac{1287a^7\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{1024b^{15/2}}-\frac{1287a^6\sqrt{ax+bx^{2/3}}}{1024b^7x^{2/3}}+\frac{429a^5\sqrt{ax+bx^{2/3}}}{512b^6x}-\frac{429a^4\sqrt{ax+bx^{2/3}}}{640b^5x^{4/3}}+\frac{1287a^3\sqrt{ax+bx^{2/3}}}{2240b^4x^{5/3}}-1$$

Rubi [A] time = 0.41, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2029, 206}

$$-\frac{1287a^6\sqrt{ax+bx^{2/3}}}{1024b^7x^{2/3}}+\frac{429a^5\sqrt{ax+bx^{2/3}}}{512b^6x}-\frac{429a^4\sqrt{ax+bx^{2/3}}}{640b^5x^{4/3}}+\frac{1287a^3\sqrt{ax+bx^{2/3}}}{2240b^4x^{5/3}}-\frac{143a^2\sqrt{ax+bx^{2/3}}}{280b^3x^2}+\frac{1287a^7\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{1024b^{15/2}}+\frac{13a\sqrt{ax+bx^{2/3}}}{28b^2x^{7/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(7*b*x^(8/3)) + (13*a*Sqrt[b*x^(2/3) + a*x])/(28
*b^2*x^(7/3)) - (143*a^2*Sqrt[b*x^(2/3) + a*x])/(280*b^3*x^2) + (1287*a^3*S
qrt[b*x^(2/3) + a*x])/(2240*b^4*x^(5/3)) - (429*a^4*Sqrt[b*x^(2/3) + a*x])/
(640*b^5*x^(4/3)) + (429*a^5*Sqrt[b*x^(2/3) + a*x])/(512*b^6*x) - (1287*a^6
*Sqrt[b*x^(2/3) + a*x])/(1024*b^7*x^(2/3)) + (1287*a^7*ArcTanh[(Sqrt[b]*x^(
1/3))/Sqrt[b*x^(2/3) + a*x]])/(1024*b^(15/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} - \frac{(13a)\int \frac{1}{x^{8/3}\sqrt{bx^{2/3}+ax}}\,dx}{14b}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} + \frac{\left(143a^2\right)\int \frac{1}{x^{7/3}\sqrt{bx^{2/3}+ax}}\,dx}{168b^2}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} - \frac{\left(429a^3\right)\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx}{560b^3}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}} + \frac{(429}{}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}} - 429a$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}} - 429a$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}} - 429a$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}} - 429a$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}} - 429a$$

Mathematica [C] time = 0.08, size = 48, normalized size = 0.20

$$\frac{6a^7\sqrt{ax+bx^{2/3}}\,{}_2F_1\left(\frac{1}{2},8;\frac{3}{2};\frac{\sqrt[3]{x}\,a}{b}+1\right)}{b^8\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (6*a^7*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[1/2, 8, 3/2, 1 + (a*x^(1/3))
/b])/(b^8*x^(1/3))
```

IntegrateAlgebraic [A] time = 0.22, size = 138, normalized size = 0.57

$$\frac{1287a^7\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{1024b^{15/2}} + \frac{\sqrt{ax+bx^{2/3}}\left(-45045a^6x^2+30030a^5bx^{5/3}-24024a^4b^2x^{4/3}+20592a^3b^3x-18304a^2b^4x^{2/3}+16640ab^5\sqrt[3]{x}-15360b^6\right)}{35840b^7x^{8/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (Sqrt[b*x^(2/3) + a*x]*(-15360*b^6 + 16640*a*b^5*x^(1/3) - 18304*a^2*b^4*x^
(2/3) + 20592*a^3*b^3*x - 24024*a^4*b^2*x^(4/3) + 30030*a^5*b*x^(5/3) - 450
45*a^6*x^2))/(35840*b^7*x^(8/3)) + (1287*a^7*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt
[b*x^(2/3) + a*x]])/(1024*b^(15/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.31, size = 160, normalized size = 0.66

$$-\frac{\frac{45045\,a^{8}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^{7}}+\frac{45045\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^{8}-300300\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^{8}b+849849\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{8}b^{2}-1317888\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^{8}b^{3}+1200199\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^{8}b^{4}-631540\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^{8}b^{5}+169995\sqrt{ax^{\frac{1}{3}}+b}\,a^{8}b^{6}}{a^{7}b^{7}x^{\frac{7}{3}}}}{35840\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")

[Out] -1/35840*(45045*a^8*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7) + (45045*(a*x^(1/3) + b)^(13/2)*a^8 - 300300*(a*x^(1/3) + b)^(11/2)*a^8*b + 84 9849*(a*x^(1/3) + b)^(9/2)*a^8*b^2 - 1317888*(a*x^(1/3) + b)^(7/2)*a^8*b^3 + 1200199*(a*x^(1/3) + b)^(5/2)*a^8*b^4 - 631540*(a*x^(1/3) + b)^(3/2)*a^8* b^5 + 169995*sqrt(a*x^(1/3) + b)*a^8*b^6)/(a^7*b^7*x^(7/3)))/a

maple [A] time = 0.05, size = 188, normalized size = 0.78

$$\frac{\sqrt{a x^{\frac{1}{3}}+b}\,\left(45045 a^{7} b x^{\frac{13}{3}} \operatorname{arctanh}\left(\frac{\sqrt{a x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)-45045 \sqrt{a x^{\frac{1}{3}}+b}\, a^{6} b^{\frac{3}{2}} x^{4}+30030 \sqrt{a x^{\frac{1}{3}}+b}\, a^{5} b^{\frac{5}{2}} x^{\frac{11}{3}}-24024 \sqrt{a x^{\frac{1}{3}}+b}\, a^{4} b^{\frac{7}{2}} x^{\frac{10}{3}}+20592 \sqrt{a x^{\frac{1}{3}}+b}\, a^{3} b^{\frac{9}{2}} x^{3}-18304 \sqrt{a x^{\frac{1}{3}}+b}\, a^{2} b^{\frac{11}{2}} x^{\frac{8}{3}}+16640 \sqrt{a x^{\frac{1}{3}}+b}\, a b^{\frac{13}{2}} x^{\frac{7}{3}}-15360 \sqrt{a x^{\frac{1}{3}}+b}\, b^{\frac{15}{2}} x^{2}\right)}{35840 \sqrt{a x+b x^{\frac{2}{3}}}\, b^{\frac{17}{2}} x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(a*x+b*x^(2/3))^(1/2),x)

[Out] 1/35840*(a*x^(1/3)+b)^(1/2)*(45045*x^(13/3)*arctanh((a*x^(1/3)+b)^(1/2)/b^(1/2))*a^7*b+30030*x^(11/3)*(a*x^(1/3)+b)^(1/2)*b^(5/2)*a^5-24024*x^(10/3)*(a*x^(1/3)+b)^(1/2)*b^(7/2)*a^4-18304*x^(8/3)*(a*x^(1/3)+b)^(1/2)*b^(11/2)*a ^2+16640*x^(7/3)*(a*x^(1/3)+b)^(1/2)*b^(13/2)*a-15360*(a*x^(1/3)+b)^(1/2)*b ^(15/2)*x^2+20592*x^3*(a*x^(1/3)+b)^(1/2)*b^(9/2)*a^3-45045*x^4*(a*x^(1/3)+ b)^(1/2)*b^(3/2)*a^6)/x^4/(a*x+b*x^(2/3))^(1/2)/b^(17/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}\,x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(a*x + b*x^(2/3))*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{3}\,\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(a*x + b*x^(2/3))^(1/2)),x)

[Out] int(1/(x^3*(a*x + b*x^(2/3))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a*x + b*x**(2/3))), x)
```

3.114 $\int \frac{1}{x^4 \sqrt{bx^{2/3}+ax}} \, dx$

Optimal. Leaf size=329

$$-\frac{138567a^{10}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{131072b^{21/2}}+\frac{138567a^9\sqrt{ax+bx^{2/3}}}{131072b^{10}x^{2/3}}-\frac{46189a^8\sqrt{ax+bx^{2/3}}}{65536b^9x}+\frac{46189a^7\sqrt{ax+bx^{2/3}}}{81920b^8x^{4/3}}-\frac{138567a^6\cdots}{28672\cdots}$$

Rubi [A] time = 0.58, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2025, 2029, 206}

$$\frac{138567a^9\sqrt{ax+bx^{2/3}}}{131072b^{10}x^{2/3}}-\frac{46189a^8\sqrt{ax+bx^{2/3}}}{65536b^9x}+\frac{46189a^7\sqrt{ax+bx^{2/3}}}{81920b^8x^{4/3}}-\frac{138567a^6\sqrt{ax+bx^{2/3}}}{286720b^7x^{5/3}}+\frac{46189a^5\sqrt{ax+bx^{2/3}}}{107520b^6x^2}-\frac{4199a^4\sqrt{ax+bx^{2/3}}}{10752b^5x^{7/3}}+\frac{323a^3\sqrt{ax+bx^{2/3}}}{896b^4x^{8/3}}-\frac{323a^2\sqrt{ax+bx^{2/3}}}{960b^3x^3}-\frac{138567a^{10}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{131072b^{21/2}}+\frac{19a\sqrt{ax+bx^{2/3}}}{60b^2x^{10/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(10*b*x^(11/3)) + (19*a*Sqrt[b*x^(2/3) + a*x])/(
60*b^2*x^(10/3)) - (323*a^2*Sqrt[b*x^(2/3) + a*x])/(960*b^3*x^3) + (323*a^3
*Sqrt[b*x^(2/3) + a*x])/(896*b^4*x^(8/3)) - (4199*a^4*Sqrt[b*x^(2/3) + a*x]
)/(10752*b^5*x^(7/3)) + (46189*a^5*Sqrt[b*x^(2/3) + a*x])/(107520*b^6*x^2)
- (138567*a^6*Sqrt[b*x^(2/3) + a*x])/(286720*b^7*x^(5/3)) + (46189*a^7*Sqrt
[b*x^(2/3) + a*x])/(81920*b^8*x^(4/3)) - (46189*a^8*Sqrt[b*x^(2/3) + a*x])/
(65536*b^9*x) + (138567*a^9*Sqrt[b*x^(2/3) + a*x])/(131072*b^10*x^(2/3)) -
(138567*a^10*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(131072*b^(2
1/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} - \frac{(19a)\int \frac{1}{x^{11/3}\sqrt{bx^{2/3}+ax}}\,dx}{20b}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} + \frac{\left(323a^2\right)\int \frac{1}{x^{10/3}\sqrt{bx^{2/3}+ax}}\,dx}{360b^2}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} - \frac{\left(323a^3\right)\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx}{384b^3}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}} + (4199$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}} - 4199$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}} - 4199$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}} - 4199$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}} - 4199$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}} - 4199$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}} - 4199$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}} - 4199$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}} - 4199$$

Mathematica [C] time = 0.07, size = 48, normalized size = 0.15

$$-\frac{6a^{10}\sqrt{ax+bx^{2/3}}\;{}_2F_1\left(\frac{1}{2},11;\frac{3}{2};\frac{\sqrt[3]{x}\,a}{b}+1\right)}{b^{11}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-6*a^10*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[1/2, 11, 3/2, 1 + (a*x^(1/
3))/b])/(b^11*x^(1/3))
```

IntegrateAlgebraic [A] time = 0.25, size = 175, normalized size = 0.53

$$\frac{\sqrt{ax+bx^{2/3}}\left(14549535a^9x^3-9699690a^8bx^{8/3}+7759752a^7b^2x^{7/3}-6651216a^6b^3x^2+5912192a^5b^4x^{5/3}-5374720a^4b^5x^{4/3}+4961280a^3b^6x-4630528a^2b^7x^{2/3}+4358144ab^8\sqrt[3]{x}-4128768b^9\right)}{13762560b^{10}x^{11/3}}-\frac{138567a^{10}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{131072b^{21/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (Sqrt[b*x^(2/3) + a*x]*(-4128768*b^9 + 4358144*a*b^8*x^(1/3) - 4630528*a^2*
b^7*x^(2/3) + 4961280*a^3*b^6*x - 5374720*a^4*b^5*x^(4/3) + 5912192*a^5*b^4
```

```
*x^(5/3) - 6651216*a^6*b^3*x^2 + 7759752*a^7*b^2*x^(7/3) - 9699690*a^8*b*x^
(8/3) + 14549535*a^9*x^3))/(13762560*b^10*x^(11/3)) - (138567*a^10*ArcTanh[
(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(131072*b^(21/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.36, size = 211, normalized size = 0.64

$$\frac{\frac{14549535\,a^{11}\arctan\left(\frac{\sqrt{a x^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^{10}}+\frac{14549535\left(a x^{\frac{1}{3}}+b\right)^{\frac{19}{2}} a^{11}-140645505\left(a x^{\frac{1}{3}}+b\right)^{\frac{17}{2}} a^{11} b+609140532\left(a x^{\frac{1}{3}}+b\right)^{\frac{15}{2}} a^{11} b^{2}-1554721740\left(a x^{\frac{1}{3}}+b\right)^{\frac{13}{2}} a^{11} b^{3}+2585198330\left(a x^{\frac{1}{3}}+b\right)^{\frac{11}{2}} a^{11} b^{4}-2918514950\left(a x^{\frac{1}{3}}+b\right)^{\frac{9}{2}} a^{11} b^{5}+2255541300\left(a x^{\frac{1}{3}}+b\right)^{\frac{7}{2}} a^{11} b^{6}-1168982220\left(a x^{\frac{1}{3}}+b\right)^{\frac{5}{2}} a^{11} b^{7}+382331775\left(a x^{\frac{1}{3}}+b\right)^{\frac{3}{2}} a^{11} b^{8}-68025825\sqrt{a x^{\frac{1}{3}}+b}\,a^{11} b^{9}}{a^{10} b^{10} x^{\frac{10}{3}}}}{13762560\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/13762560*(14549535*a^11*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^
10) + (14549535*(a*x^(1/3) + b)^(19/2)*a^11 - 140645505*(a*x^(1/3) + b)^(17
/2)*a^11*b + 609140532*(a*x^(1/3) + b)^(15/2)*a^11*b^2 - 1554721740*(a*x^(1
/3) + b)^(13/2)*a^11*b^3 + 2585198330*(a*x^(1/3) + b)^(11/2)*a^11*b^4 - 291
8514950*(a*x^(1/3) + b)^(9/2)*a^11*b^5 + 2255541300*(a*x^(1/3) + b)^(7/2)*a
^11*b^6 - 1168982220*(a*x^(1/3) + b)^(5/2)*a^11*b^7 + 382331775*(a*x^(1/3)
+ b)^(3/2)*a^11*b^8 - 68025825*sqrt(a*x^(1/3) + b)*a^11*b^9)/(a^10*b^10*x^(
10/3)))/a
```

maple [A] time = 0.05, size = 248, normalized size = 0.75

$$-\frac{\sqrt{a x^{\frac{1}{3}}+b}\,\left(14549535 a^{10} b\, x^{\frac{19}{3}} \operatorname{arctanh}\left(\frac{\sqrt{a x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)-14549535\sqrt{a x^{\frac{1}{3}}+b}\, a^{9} b^{\frac{3}{2}} x^{6}+9699690\sqrt{a x^{\frac{1}{3}}+b}\, a^{8} b^{\frac{5}{2}} x^{\frac{17}{3}}-7759752\sqrt{a x^{\frac{1}{3}}+b}\, a^{7} b^{\frac{7}{2}} x^{\frac{16}{3}}+6651216\sqrt{a x^{\frac{1}{3}}+b}\, a^{6} b^{\frac{9}{2}} x^{5}-5912192\sqrt{a x^{\frac{1}{3}}+b}\, a^{5} b^{\frac{11}{2}} x^{\frac{14}{3}}+5374720\sqrt{a x^{\frac{1}{3}}+b}\, a^{4} b^{\frac{13}{2}} x^{\frac{13}{3}}-4961280\sqrt{a x^{\frac{1}{3}}+b}\, a^{3} b^{\frac{15}{2}} x^{4}+4630528\sqrt{a x^{\frac{1}{3}}+b}\, a^{2} b^{\frac{17}{2}} x^{\frac{11}{3}}-4358144\sqrt{a x^{\frac{1}{3}}+b}\, a\, b^{\frac{19}{2}} x^{\frac{10}{3}}+4128768\sqrt{a x^{\frac{1}{3}}+b}\, b^{\frac{21}{2}} x^{3}\right)}{13762560\sqrt{a x+b x^{\frac{2}{3}}}\, b^{\frac{23}{2}} x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a*x+b*x^(2/3))^(1/2),x)
```

```
[Out] -1/13762560*(a*x^(1/3)+b)^(1/2)*(14549535*x^(19/3)*arctanh((a*x^(1/3)+b)^(1
/2)/b^(1/2))*a^10*b+9699690*x^(17/3)*(a*x^(1/3)+b)^(1/2)*b^(5/2)*a^8-775975
2*x^(16/3)*(a*x^(1/3)+b)^(1/2)*b^(7/2)*a^7-5912192*x^(14/3)*(a*x^(1/3)+b)^(
1/2)*b^(11/2)*a^5+5374720*x^(13/3)*(a*x^(1/3)+b)^(1/2)*b^(13/2)*a^4+4630528
*x^(11/3)*(a*x^(1/3)+b)^(1/2)*b^(17/2)*a^2-4358144*x^(10/3)*(a*x^(1/3)+b)^(
1/2)*b^(19/2)*a+4128768*(a*x^(1/3)+b)^(1/2)*b^(21/2)*x^3-4961280*x^4*(a*x^(
1/3)+b)^(1/2)*b^(15/2)*a^3+6651216*x^5*(a*x^(1/3)+b)^(1/2)*b^(9/2)*a^6-1454
9535*x^6*(a*x^(1/3)+b)^(1/2)*b^(3/2)*a^9)/x^6/(a*x+b*x^(2/3))^(1/2)/b^(23/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + bx^{\frac{2}{3}}}\, x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(2/3))*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a*x + b*x^(2/3))^(1/2)),x)
[Out] int(1/(x^4*(a*x + b*x^(2/3))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**(2/3)+a*x)**(1/2),x)
[Out] Integral(1/(x**4*sqrt(a*x + b*x**(2/3))), x)
```

3.115 $\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=336

$$\frac{1048576b^{10}\sqrt{ax+bx^{2/3}}}{29393a^{12}\sqrt[3]{x}}-\frac{524288b^9\sqrt{ax+bx^{2/3}}}{29393a^{11}}+\frac{393216b^8\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{29393a^{10}}-\frac{327680b^7x^{2/3}\sqrt{ax+bx^{2/3}}}{29393a^9}+\frac{40960b}{}$$

Rubi [A] time = 0.60, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2015, 2016, 2002, 2014}

$$\frac{1048576b^{10}\sqrt{ax+bx^{2/3}}}{29393a^{12}\sqrt[3]{x}}-\frac{524288b^9\sqrt{ax+bx^{2/3}}}{29393a^{11}}+\frac{393216b^8\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{29393a^{10}}-\frac{327680b^7x^{2/3}\sqrt{ax+bx^{2/3}}}{29393a^9}+\frac{40960b^6x\sqrt{ax+bx^{2/3}}}{4199a^8}-\frac{36864b^5x^{4/3}\sqrt{ax+bx^{2/3}}}{4199a^7}+\frac{33792b^4x^{5/3}\sqrt{ax+bx^{2/3}}}{4199a^6}-\frac{16896b^3x^2\sqrt{ax+bx^{2/3}}}{2261a^5}+\frac{15840b^2x^{7/3}\sqrt{ax+bx^{2/3}}}{2261a^4}-\frac{880bx^{8/3}\sqrt{ax+bx^{2/3}}}{133a^3}+\frac{44x^3\sqrt{ax+bx^{2/3}}}{7a^2}-\frac{6x^4}{a\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-6*x^4)/(a*Sqrt[b*x^(2/3) + a*x]) - (524288*b^9*Sqrt[b*x^(2/3) + a*x])/(29
393*a^11) + (1048576*b^10*Sqrt[b*x^(2/3) + a*x])/(29393*a^12*x^(1/3)) + (39
3216*b^8*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(29393*a^10) - (327680*b^7*x^(2/3)*
Sqrt[b*x^(2/3) + a*x])/(29393*a^9) + (40960*b^6*x*Sqrt[b*x^(2/3) + a*x])/(4
199*a^8) - (36864*b^5*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(4199*a^7) + (33792*b^
4*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/(4199*a^6) - (16896*b^3*x^2*Sqrt[b*x^(2/3)
 + a*x])/(2261*a^5) + (15840*b^2*x^(7/3)*Sqrt[b*x^(2/3) + a*x])/(2261*a^4)
- (880*b*x^(8/3)*Sqrt[b*x^(2/3) + a*x])/(133*a^3) + (44*x^3*Sqrt[b*x^(2/3)
+ a*x])/(7*a^2)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
```

```
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx &= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}+\frac{22\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx}{a}\\
&= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}+\frac{44x^3\sqrt{bx^{2/3}+ax}}{7a^2}-\frac{(440b)\int \frac{x^{8/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{21a^2}\\
&= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}-\frac{880bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^3}+\frac{44x^3\sqrt{bx^{2/3}+ax}}{7a^2}+\frac{\left(2640b^2\right)\int \frac{x^{7/3}}{\sqrt{bx^{2/3}+ax}}\,d\ldots}{133a^3}\\
&= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}+\frac{15840b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^4}-\frac{880bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^3}+\frac{44x^3\sqrt{bx^{2/3}+a\ldots}}{7a^2}\\
&= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}-\frac{16896b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^5}+\frac{15840b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^4}-\frac{880bx^{8/3}\sqrt{bx^{2\ldots}}}{133a^3}\\
&= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}+\frac{33792b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^6}-\frac{16896b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^5}+\frac{15840b^2x^{7/3}\ldots}{2261\ldots}\\
&= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}-\frac{36864b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{4199a^7}+\frac{33792b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^6}-\frac{16896b^3x^2\ldots}{2261\ldots}\\
&= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}+\frac{40960b^6x\sqrt{bx^{2/3}+ax}}{4199a^8}-\frac{36864b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{4199a^7}+\frac{33792b^4x^{5/3}\ldots}{4199\ldots}\\
&= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}-\frac{327680b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{29393a^9}+\frac{40960b^6x\sqrt{bx^{2/3}+ax}}{4199a^8}-\frac{36864b^5x^{4/3}\ldots}{4199\ldots}\\
&= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}+\frac{393216b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{29393a^{10}}-\frac{327680b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{29393a^9}+\frac{40960b^6x\ldots}{41\ldots}\\
&= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}-\frac{524288b^9\sqrt{bx^{2/3}+ax}}{29393a^{11}}+\frac{393216b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{29393a^{10}}-\frac{327680b^7x^{2/3}\ldots}{2939\ldots}\\
&= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}-\frac{524288b^9\sqrt{bx^{2/3}+ax}}{29393a^{11}}+\frac{1048576b^{10}\sqrt{bx^{2/3}+ax}}{29393a^{12}\sqrt[3]{x}}+\frac{393216b^8\sqrt[3]{x}\ldots}{29393\ldots}
\end{aligned}
$$

Mathematica [A] time = 0.19, size = 161, normalized size = 0.48

$$\frac{2\sqrt[3]{x}\left(4199a^{11}x^{11/3}-4862a^{10}bx^{10/3}+5720a^9b^2x^3-6864a^8b^3x^{8/3}+8448a^7b^4x^{7/3}-10752a^6b^5x^2+14336a^5b^6x^{5/3}-20480a^4b^7x^{4/3}+32768a^3b^8x-65536a^2b^9x^{2/3}+262144ab^{10}\sqrt[3]{x}+524288b^{11}\right)}{29393a^{12}\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*x^(1/3)*(524288*b^11 + 262144*a*b^10*x^(1/3) - 65536*a^2*b^9*x^(2/3) + 3
2768*a^3*b^8*x - 20480*a^4*b^7*x^(4/3) + 14336*a^5*b^6*x^(5/3) - 10752*a^6*
b^5*x^2 + 8448*a^7*b^4*x^(7/3) - 6864*a^8*b^3*x^(8/3) + 5720*a^9*b^2*x^3 -
4862*a^10*b*x^(10/3) + 4199*a^11*x^(11/3)))/(29393*a^12*Sqrt[b*x^(2/3) + a*
x])
```

IntegrateAlgebraic [A] time = 3.99, size = 165, normalized size = 0.49

$$\frac{2\sqrt[3]{x}\left(4199a^{11}x^{11/3}-4862a^{10}bx^{10/3}+5720a^{9}b^{2}x^{3}-6864a^{8}b^{3}x^{8/3}+8448a^{7}b^{4}x^{7/3}-10752a^{6}b^{5}x^{2}+14336a^{5}b^{6}x^{5/3}-20480a^{4}b^{7}x^{4/3}+32768a^{3}b^{8}x-65536a^{2}b^{9}x^{2/3}+262144ab^{10}\sqrt[3]{x}+524288b^{11}\right)}{29393a^{12}\sqrt{x^{2/3}\left(a\sqrt[3]{x}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*x^(1/3)*(524288*b^11 + 262144*a*b^10*x^(1/3) - 65536*a^2*b^9*x^(2/3) + 3
2768*a^3*b^8*x - 20480*a^4*b^7*x^(4/3) + 14336*a^5*b^6*x^(5/3) - 10752*a^6*
b^5*x^2 + 8448*a^7*b^4*x^(7/3) - 6864*a^8*b^3*x^(8/3) + 5720*a^9*b^2*x^3 -
4862*a^10*b*x^(10/3) + 4199*a^11*x^(11/3)))/(29393*a^12*Sqrt[(b + a*x^(1/3)
)*x^(2/3)])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.25, size = 214, normalized size = 0.64

$$-\frac{1048576b^{\frac{21}{2}}}{29393a^{12}}+\frac{6b^{11}}{\sqrt{ax^{\frac{1}{3}}+b}\,a^{12}}+\frac{2\left(4199\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}a^{240}-51051\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{240}b+285285\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{240}b^{2}-969969\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{240}b^{3}+2238390\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^{240}b^{4}-3703518\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^{240}b^{5}+4526522\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{240}b^{6}-4157010\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^{240}b^{7}+2909907\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^{240}b^{8}-1616615\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^{240}b^{9}+969969\sqrt{ax^{\frac{1}{3}}+b}\,a^{240}b^{10}\right)}{29393a^{252}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -1048576/29393*b^(21/2)/a^12 + 6*b^11/(sqrt(a*x^(1/3) + b)*a^12) + 2/29393*
(4199*(a*x^(1/3) + b)^(21/2)*a^240 - 51051*(a*x^(1/3) + b)^(19/2)*a^240*b +
 285285*(a*x^(1/3) + b)^(17/2)*a^240*b^2 - 969969*(a*x^(1/3) + b)^(15/2)*a^
240*b^3 + 2238390*(a*x^(1/3) + b)^(13/2)*a^240*b^4 - 3703518*(a*x^(1/3) + b
)^(11/2)*a^240*b^5 + 4526522*(a*x^(1/3) + b)^(9/2)*a^240*b^6 - 4157010*(a*x
^(1/3) + b)^(7/2)*a^240*b^7 + 2909907*(a*x^(1/3) + b)^(5/2)*a^240*b^8 - 161
6615*(a*x^(1/3) + b)^(3/2)*a^240*b^9 + 969969*sqrt(a*x^(1/3) + b)*a^240*b^1
0)/a^252
```

maple [A] time = 0.05, size = 143, normalized size = 0.43

$$\frac{2\left(ax^{\frac{1}{3}}+b\right)\left(4199a^{11}x^{\frac{11}{3}}-4862a^{10}bx^{\frac{10}{3}}+5720a^{9}b^{2}x^{3}-6864a^{8}b^{3}x^{\frac{8}{3}}+8448a^{7}b^{4}x^{\frac{7}{3}}-10752a^{6}b^{5}x^{2}+14336a^{5}b^{6}x^{\frac{5}{3}}-20480a^{4}b^{7}x^{\frac{4}{3}}+32768a^{3}b^{8}x-65536a^{2}b^{9}x^{\frac{2}{3}}+262144ab^{10}x^{\frac{1}{3}}+524288b^{11}\right)x}{29393\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x+b*x^(2/3))^(3/2),x)
```

```
[Out] 2/29393*x*(a*x^(1/3)+b)*(4199*a^11*x^(11/3)-4862*a^10*b*x^(10/3)+5720*a^9*b
^2*x^3-6864*a^8*b^3*x^(8/3)+8448*a^7*b^4*x^(7/3)-10752*a^6*b^5*x^2+14336*a^
5*b^6*x^(5/3)-20480*a^4*b^7*x^(4/3)+32768*a^3*b^8*x-65536*a^2*b^9*x^(2/3)+2
62144*a*b^10*x^(1/3)+524288*b^11)/(a*x+b*x^(2/3))^(3/2)/a^12
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(a*x + b*x^(2/3))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(a\,x+b\,x^{2/3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x + b*x^(2/3))^(3/2),x)
```

```
[Out] int(x^4/(a*x + b*x^(2/3))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x**4/(a*x + b*x**(2/3))**(3/2), x)
```

3.116 $\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=248

$$-\frac{65536b^7\sqrt{ax+bx^{2/3}}}{2145a^9\sqrt[3]{x}}+\frac{32768b^6\sqrt{ax+bx^{2/3}}}{2145a^8}-\frac{8192b^5\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{715a^7}+\frac{4096b^4x^{2/3}\sqrt{ax+bx^{2/3}}}{429a^6}-\frac{3584b^3x\sqrt{ax+}}{429a^5}$$

Rubi [A] time = 0.41, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2015, 2016, 2002, 2014}

$$-\frac{65536b^7\sqrt{ax+bx^{2/3}}}{2145a^9\sqrt[3]{x}}+\frac{32768b^6\sqrt{ax+bx^{2/3}}}{2145a^8}-\frac{8192b^5\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{715a^7}+\frac{4096b^4x^{2/3}\sqrt{ax+bx^{2/3}}}{429a^6}-\frac{3584b^3x\sqrt{ax+bx^{2/3}}}{429a^5}+\frac{5376b^2x^{4/3}\sqrt{ax+bx^{2/3}}}{715a^4}-\frac{448bx^{5/3}\sqrt{ax+bx^{2/3}}}{65a^3}+\frac{32x^2\sqrt{ax+bx^{2/3}}}{5a^2}-\frac{6x^3}{a\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-6*x^3)/(a*Sqrt[b*x^(2/3) + a*x]) + (32768*b^6*Sqrt[b*x^(2/3) + a*x])/(214
5*a^8) - (65536*b^7*Sqrt[b*x^(2/3) + a*x])/(2145*a^9*x^(1/3)) - (8192*b^5*x
^(1/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^7) + (4096*b^4*x^(2/3)*Sqrt[b*x^(2/3)
+ a*x])/(429*a^6) - (3584*b^3*x*Sqrt[b*x^(2/3) + a*x])/(429*a^5) + (5376*b^
2*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^4) - (448*b*x^(5/3)*Sqrt[b*x^(2/3)
+ a*x])/(65*a^3) + (32*x^2*Sqrt[b*x^(2/3) + a*x])/(5*a^2)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} + \frac{16\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx}{a}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} + \frac{32x^2\sqrt{bx^{2/3}+ax}}{5a^2} - \frac{(224b)\int \frac{x^{5/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{15a^2}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} - \frac{448bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^3} + \frac{32x^2\sqrt{bx^{2/3}+ax}}{5a^2} + \frac{\left(896b^2\right)\int \frac{x^{4/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{65a^3}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} + \frac{5376b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^4} - \frac{448bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^3} + \frac{32x^2\sqrt{bx^{2/3}+ax}}{5a^2}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} - \frac{3584b^3x\sqrt{bx^{2/3}+ax}}{429a^5} + \frac{5376b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^4} - \frac{448bx^{5/3}\sqrt{bx^{2/3}+}}{65a^3}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} + \frac{4096b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^6} - \frac{3584b^3x\sqrt{bx^{2/3}+ax}}{429a^5} + \frac{5376b^2x^{4/3}\sqrt{bx^{2/3}}}{715a^4}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} - \frac{8192b^5\sqrt[3]{x}\,\sqrt{bx^{2/3}+ax}}{715a^7} + \frac{4096b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^6} - \frac{3584b^3x\sqrt{bx^{2/3}}}{429a^5}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} + \frac{32768b^6\sqrt{bx^{2/3}+ax}}{2145a^8} - \frac{8192b^5\sqrt[3]{x}\,\sqrt{bx^{2/3}+ax}}{715a^7} + \frac{4096b^4x^{2/3}\sqrt{bx^{2/3}}}{429a^6}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} + \frac{32768b^6\sqrt{bx^{2/3}+ax}}{2145a^8} - \frac{65536b^7\sqrt{bx^{2/3}+ax}}{2145a^9\sqrt[3]{x}} - \frac{8192b^5\sqrt[3]{x}\,\sqrt{bx^{2/3}+}}{715a^7}$$

Mathematica [A] time = 0.13, size = 122, normalized size = 0.49

$$\frac{2\left(429a^8x^3-528a^7bx^{8/3}+672a^6b^2x^{7/3}-896a^5b^3x^2+1280a^4b^4x^{5/3}-2048a^3b^5x^{4/3}+4096a^2b^6x-16384ab^7x^{2/3}-32768b^8\sqrt[3]{x}\right)}{2145a^9\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(-32768*b^8*x^(1/3) - 16384*a*b^7*x^(2/3) + 4096*a^2*b^6*x - 2048*a^3*b^
5*x^(4/3) + 1280*a^4*b^4*x^(5/3) - 896*a^5*b^3*x^2 + 672*a^6*b^2*x^(7/3) -
528*a^7*b*x^(8/3) + 429*a^8*x^3))/(2145*a^9*Sqrt[b*x^(2/3) + a*x])
```

IntegrateAlgebraic [A] time = 3.94, size = 128, normalized size = 0.52

$$-\frac{2\sqrt[3]{x}\left(-429a^8x^{8/3}+528a^7bx^{7/3}-672a^6b^2x^2+896a^5b^3x^{5/3}-1280a^4b^4x^{4/3}+2048a^3b^5x-4096a^2b^6x^{2/3}+16384ab^7\sqrt[3]{x}+32768b^8\right)}{2145a^9\sqrt{x^{2/3}\left(a\sqrt[3]{x}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-2*x^(1/3)*(32768*b^8 + 16384*a*b^7*x^(1/3) - 4096*a^2*b^6*x^(2/3) + 2048*
a^3*b^5*x - 1280*a^4*b^4*x^(4/3) + 896*a^5*b^3*x^(5/3) - 672*a^6*b^2*x^2 +
528*a^7*b*x^(7/3) - 429*a^8*x^(8/3)))/(2145*a^9*Sqrt[(b + a*x^(1/3))*x^(2/3
)])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.21, size = 163, normalized size = 0.66

$$\frac{65536\,b^{\frac{15}{2}}}{2145\,a^{9}}-\frac{6\,b^{8}}{\sqrt{ax^{\frac{1}{3}}+b}\,a^{9}}+\frac{2\left(429\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{126}-3960\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^{126}b+16380\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^{126}b^{2}-40040\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{126}b^{3}+64350\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^{126}b^{4}-72072\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^{126}b^{5}+60060\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^{126}b^{6}-51480\sqrt{ax^{\frac{1}{3}}+b}\,a^{126}b^{7}\right)}{2145\,a^{135}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 65536/2145*b^(15/2)/a^9 - 6*b^8/(sqrt(a*x^(1/3) + b)*a^9) + 2/2145*(429*(a*
x^(1/3) + b)^(15/2)*a^126 - 3960*(a*x^(1/3) + b)^(13/2)*a^126*b + 16380*(a*
x^(1/3) + b)^(11/2)*a^126*b^2 - 40040*(a*x^(1/3) + b)^(9/2)*a^126*b^3 + 643
50*(a*x^(1/3) + b)^(7/2)*a^126*b^4 - 72072*(a*x^(1/3) + b)^(5/2)*a^126*b^5
+ 60060*(a*x^(1/3) + b)^(3/2)*a^126*b^6 - 51480*sqrt(a*x^(1/3) + b)*a^126*b
^7)/a^135
```

maple [A] time = 0.05, size = 110, normalized size = 0.44

$$\frac{2\left(a\,x^{\frac{1}{3}}+b\right)\left(429a^{8}x^{\frac{8}{3}}-528a^{7}b\,x^{\frac{7}{3}}+672a^{6}b^{2}x^{2}-896a^{5}b^{3}x^{\frac{5}{3}}+1280a^{4}b^{4}x^{\frac{4}{3}}-2048a^{3}b^{5}x+4096a^{2}b^{6}x^{\frac{2}{3}}-16384a\,b^{7}x^{\frac{1}{3}}-32768b^{8}\right)x}{2145\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}a^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x+b*x^(2/3))^(3/2),x)
```

```
[Out] 2/2145*x*(a*x^(1/3)+b)*(429*a^8*x^(8/3)-528*a^7*b*x^(7/3)+672*a^6*b^2*x^2-8
96*a^5*b^3*x^(5/3)+1280*x^(4/3)*a^4*b^4-2048*a^3*b^5*x+4096*x^(2/3)*a^2*b^6
-16384*x^(1/3)*a*b^7-32768*b^8)/(a*x+b*x^(2/3))^(3/2)/a^9
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{3}}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(a*x + b*x^(2/3))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^{3}}{{\left(a\,x+b\,x^{2/3}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^(2/3))^(3/2),x)
```

```
[Out] int(x^3/(a*x + b*x^(2/3))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{3}}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x**3/(a*x + b*x**(2/3))**(3/2), x)
```

3.117 $\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=160

$$\frac{512b^4\sqrt{ax+bx^{2/3}}}{21a^6\sqrt[3]{x}}-\frac{256b^3\sqrt{ax+bx^{2/3}}}{21a^5}+\frac{64b^2\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{7a^4}-\frac{160bx^{2/3}\sqrt{ax+bx^{2/3}}}{21a^3}+\frac{20x\sqrt{ax+bx^{2/3}}}{3a^2}-\frac{6x^2}{a\sqrt{ax+}}$$

Rubi [A] time = 0.24, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2015, 2016, 2002, 2014}

$$\frac{512b^4\sqrt{ax+bx^{2/3}}}{21a^6\sqrt[3]{x}}-\frac{256b^3\sqrt{ax+bx^{2/3}}}{21a^5}+\frac{64b^2\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{7a^4}-\frac{160bx^{2/3}\sqrt{ax+bx^{2/3}}}{21a^3}+\frac{20x\sqrt{ax+bx^{2/3}}}{3a^2}-\frac{6x^2}{a\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-6*x^2)/(a*Sqrt[b*x^(2/3) + a*x]) - (256*b^3*Sqrt[b*x^(2/3) + a*x])/(21*a^
5) + (512*b^4*Sqrt[b*x^(2/3) + a*x])/(21*a^6*x^(1/3)) + (64*b^2*x^(1/3)*Sqr
t[b*x^(2/3) + a*x])/(7*a^4) - (160*b*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(21*a^3
) + (20*x*Sqrt[b*x^(2/3) + a*x])/(3*a^2)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}} + \frac{10\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx}{a}$$

$$= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}} + \frac{20x\sqrt{bx^{2/3}+ax}}{3a^2} - \frac{(80b)\int \frac{x^{2/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{9a^2}$$

$$= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}} - \frac{160bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^3} + \frac{20x\sqrt{bx^{2/3}+ax}}{3a^2} + \frac{\left(160b^2\right)\int \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\,dx}{21a^3}$$

$$= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}} + \frac{64b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{7a^4} - \frac{160bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^3} + \frac{20x\sqrt{bx^{2/3}+ax}}{3a^2} -$$

$$= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}} - \frac{256b^3\sqrt{bx^{2/3}+ax}}{21a^5} + \frac{64b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{7a^4} - \frac{160bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^3} -$$

$$= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}} - \frac{256b^3\sqrt{bx^{2/3}+ax}}{21a^5} + \frac{512b^4\sqrt{bx^{2/3}+ax}}{21a^6\sqrt[3]{x}} + \frac{64b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{7a^4} -$$

Mathematica [A] time = 0.10, size = 85, normalized size = 0.53

$$\frac{14a^5x^2 - 20a^4bx^{5/3} + 32a^3b^2x^{4/3} - 64a^2b^3x + 256ab^4x^{2/3} + 512b^5\sqrt[3]{x}}{21a^6\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (512*b^5*x^(1/3) + 256*a*b^4*x^(2/3) - 64*a^2*b^3*x + 32*a^3*b^2*x^(4/3) -
20*a^4*b*x^(5/3) + 14*a^5*x^2)/(21*a^6*Sqrt[b*x^(2/3) + a*x])
```

IntegrateAlgebraic [A] time = 4.18, size = 91, normalized size = 0.57

$$\frac{2\sqrt[3]{x}\left(7a^5x^{5/3} - 10a^4bx^{4/3} + 16a^3b^2x - 32a^2b^3x^{2/3} + 128ab^4\sqrt[3]{x} + 256b^5\right)}{21a^6\sqrt{x^{2/3}\left(a\sqrt[3]{x}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*x^(1/3)*(256*b^5 + 128*a*b^4*x^(1/3) - 32*a^2*b^3*x^(2/3) + 16*a^3*b^2*x
 - 10*a^4*b*x^(4/3) + 7*a^5*x^(5/3)))/(21*a^6*Sqrt[(b + a*x^(1/3))*x^(2/3)]
)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.23, size = 112, normalized size = 0.70

$$-\frac{512\,b^{\frac{9}{2}}}{21\,a^6} + \frac{6\,b^5}{\sqrt{ax^{\frac{1}{3}}+b}\,a^6} + \frac{2\left(7\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{48} - 45\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^{48}b + 126\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^{48}b^2 - 210\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^{48}b^3 + 315\sqrt{ax^{\frac{1}{3}}+b}\,a^{48}b^4\right)}{21\,a^{54}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -512/21*b^(9/2)/a^6 + 6*b^5/(sqrt(a*x^(1/3) + b)*a^6) + 2/21*(7*(a*x^(1/3)
+ b)^(9/2)*a^48 - 45*(a*x^(1/3) + b)^(7/2)*a^48*b + 126*(a*x^(1/3) + b)^(5/
2)*a^48*b^2 - 210*(a*x^(1/3) + b)^(3/2)*a^48*b^3 + 315*sqrt(a*x^(1/3) + b)*
a^48*b^4)/a^54
```

maple [A] time = 0.05, size = 77, normalized size = 0.48

$$\frac{2\left(a\,x^{\frac{1}{3}}+b\right)\left(7a^{5}x^{\frac{5}{3}}-10a^{4}b\,x^{\frac{4}{3}}+16a^{3}b^{2}x-32a^{2}b^{3}x^{\frac{2}{3}}+128a\,b^{4}x^{\frac{1}{3}}+256b^{5}\right)x}{21\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+b*x^(2/3))^(3/2),x)
```

```
[Out] 2/21*x*(a*x^(1/3)+b)*(7*a^5*x^(5/3)-10*a^4*b*x^(4/3)+16*a^3*b^2*x-32*a^2*b^
3*x^(2/3)+128*a*b^4*x^(1/3)+256*b^5)/(a*x+b*x^(2/3))^(3/2)/a^6
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{2}}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(a*x + b*x^(2/3))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^{2}}{\left(a\,x+b\,x^{2/3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^(2/3))^(3/2),x)
```

```
[Out] int(x^2/(a*x + b*x^(2/3))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{2}}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x**2/(a*x + b*x**(2/3))**(3/2), x)
```

3.118 $\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{16b\sqrt{ax+bx^{2/3}}}{a^3\sqrt[3]{x}}+\frac{8\sqrt{ax+bx^{2/3}}}{a^2}-\frac{6x}{a\sqrt{ax+bx^{2/3}}}$$

Rubi [A] time = 0.08, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2015, 2002, 2014}

$$-\frac{16b\sqrt{ax+bx^{2/3}}}{a^3\sqrt[3]{x}}+\frac{8\sqrt{ax+bx^{2/3}}}{a^2}-\frac{6x}{a\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-6*x)/(a*Sqrt[b*x^(2/3) + a*x]) + (8*Sqrt[b*x^(2/3) + a*x])/a^2 - (16*b*Sq
rt[b*x^(2/3) + a*x])/(a^3*x^(1/3))
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx &= -\frac{6x}{a\sqrt{bx^{2/3}+ax}}+\frac{4\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx}{a}\\ &= -\frac{6x}{a\sqrt{bx^{2/3}+ax}}+\frac{8\sqrt{bx^{2/3}+ax}}{a^2}-\frac{(8b)\int \frac{1}{\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}\,dx}{3a^2}\\ &= -\frac{6x}{a\sqrt{bx^{2/3}+ax}}+\frac{8\sqrt{bx^{2/3}+ax}}{a^2}-\frac{16b\sqrt{bx^{2/3}+ax}}{a^3\sqrt[3]{x}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 60, normalized size = 0.88

$$\frac{2\left(a^2x^{2/3}-4ab\sqrt[3]{x}-8b^2\right)\sqrt{ax+bx^{2/3}}}{a^3\sqrt[3]{x}\left(a\sqrt[3]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(-8*b^2 - 4*a*b*x^(1/3) + a^2*x^(2/3))*Sqrt[b*x^(2/3) + a*x])/(a^3*(b +
a*x^(1/3))*x^(1/3))
```

IntegrateAlgebraic [A] time = 3.97, size = 54, normalized size = 0.79

$$-\frac{2\sqrt[3]{x}\left(a^2\left(-x^{2/3}\right)+4ab\sqrt[3]{x}+8b^2\right)}{a^3\sqrt{x^{2/3}\left(a\sqrt[3]{x}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-2*(8*b^2 + 4*a*b*x^(1/3) - a^2*x^(2/3))*x^(1/3))/(a^3*Sqrt[(b + a*x^(1/3)
)*x^(2/3)])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.22, size = 60, normalized size = 0.88

$$\frac{16\,b^{\frac{3}{2}}}{a^3}-\frac{6\,b^2}{\sqrt{ax^{\frac{1}{3}}+b}\,a^3}+\frac{2\left(\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^6-6\sqrt{ax^{\frac{1}{3}}+b}\,a^6b\right)}{a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 16*b^(3/2)/a^3 - 6*b^2/(sqrt(a*x^(1/3) + b)*a^3) + 2*((a*x^(1/3) + b)^(3/2)
*a^6 - 6*sqrt(a*x^(1/3) + b)*a^6*b)/a^9
```

maple [A] time = 0.05, size = 45, normalized size = 0.66

$$\frac{2\left(a\,x^{\frac{1}{3}}+b\right)\left(a^2x^{\frac{2}{3}}-4ab\,x^{\frac{1}{3}}-8b^2\right)x}{\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x+b*x^(2/3))^(3/2),x)
```

```
[Out] 2*x*(a*x^(1/3)+b)*(a^2*x^(2/3)-4*a*b*x^(1/3)-8*b^2)/(a*x+b*x^(2/3))^(3/2)/a
^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x/(a*x + b*x^(2/3))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(a\,x+b\,x^{2/3}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^(2/3))^(3/2),x)
[Out] int(x/(a*x + b*x^(2/3))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**(2/3)+a*x)**(3/2),x)
[Out] Integral(x/(a*x + b*x**(2/3))**(3/2), x)
```

3.119 $\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=60

$$\frac{6\sqrt[3]{x}}{b\sqrt{ax+bx^{2/3}}}-\frac{6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2006, 2029, 206}

$$\frac{6\sqrt[3]{x}}{b\sqrt{ax+bx^{2/3}}}-\frac{6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(-3/2),x]
```

```
[Out] (6*x^(1/3))/(b*Sqrt[b*x^(2/3) + a*x]) - (6*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b
*x^(2/3) + a*x]])/b^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2006

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx &= \frac{6\sqrt[3]{x}}{b\sqrt{bx^{2/3}+ax}}+\frac{\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{b}\\
&= \frac{6\sqrt[3]{x}}{b\sqrt{bx^{2/3}+ax}}-\frac{6\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b}\\
&= \frac{6\sqrt[3]{x}}{b\sqrt{bx^{2/3}+ax}}-\frac{6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 45, normalized size = 0.75

$$\frac{6\sqrt[3]{x}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{b\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(-3/2),x]
```

```
[Out] (6*x^(1/3)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (a*x^(1/3))/b])/(b*Sqrt[b*x^
(2/3) + a*x])
```

IntegrateAlgebraic [A] time = 2.65, size = 71, normalized size = 1.18

$$\frac{6\sqrt{ax+bx^{2/3}}}{b\sqrt[3]{x}\left(a\sqrt[3]{x}+b\right)}-\frac{6\tanh^{-1}\left(\frac{\sqrt{ax+bx^{2/3}}}{\sqrt{b}\sqrt[3]{x}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^(2/3) + a*x)^(-3/2),x]
```

```
[Out] (6*Sqrt[b*x^(2/3) + a*x])/(b*(b + a*x^(1/3))*x^(1/3)) - (6*ArcTanh[Sqrt[b*x
^(2/3) + a*x]/(Sqrt[b]*x^(1/3))])/b^(3/2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.21, size = 71, normalized size = 1.18

$$\frac{6\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b}-\frac{6\left(\sqrt{b}\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+\sqrt{-b}\right)}{\sqrt{-b}\,b^{\frac{3}{2}}}+\frac{6}{\sqrt{ax^{\frac{1}{3}}+b}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 6*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) - 6*(sqrt(b)*arctan(sqr
t(b)/sqrt(-b)) + sqrt(-b))/(sqrt(-b)*b^(3/2)) + 6/(sqrt(a*x^(1/3) + b)*b)
```

maple [A] time = 0.05, size = 56, normalized size = 0.93

$$-\frac{6\left(a\,x^{\frac{1}{3}}+b\right)\left(\sqrt{a\,x^{\frac{1}{3}}+b}\;b\,\mathrm{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)-b^{\frac{3}{2}}\right)x}{\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x^(2/3))^(3/2),x)
```

```
[Out] -6*x*(a*x^(1/3)+b)*(arctanh((a*x^(1/3)+b)^(1/2)/b^(1/2))*b*(a*x^(1/3)+b)^(1
/2)-b^(3/2))/(a*x+b*x^(2/3))^(3/2)/b^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(-3/2), x)
```

mupad [B] time = 5.36, size = 40, normalized size = 0.67

$$-\frac{2\,x\,\left(\frac{b}{a\,x^{1/3}}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{3}{2};\ \frac{5}{2};\ -\frac{b}{a\,x^{1/3}}\right)}{\left(a\,x+b\,x^{2/3}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^(2/3))^(3/2),x)
```

```
[Out] -(2*x*(b/(a*x^(1/3)) + 1)^(3/2)*hypergeom([3/2, 3/2], 5/2, -b/(a*x^(1/3))))
/(a*x + b*x^(2/3))^(3/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(-3/2), x)
```

3.120 $\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=146

$$\frac{105a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{8b^{9/2}}-\frac{105a^2\sqrt{ax+bx^{2/3}}}{8b^4x^{2/3}}+\frac{35a\sqrt{ax+bx^{2/3}}}{4b^3x}-\frac{7\sqrt{ax+bx^{2/3}}}{b^2x^{4/3}}+\frac{6}{bx^{2/3}\sqrt{ax+bx^{2/3}}}$$

Rubi [A] time = 0.24, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2023, 2025, 2029, 206}

$$-\frac{105a^2\sqrt{ax+bx^{2/3}}}{8b^4x^{2/3}}+\frac{105a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{8b^{9/2}}+\frac{35a\sqrt{ax+bx^{2/3}}}{4b^3x}-\frac{7\sqrt{ax+bx^{2/3}}}{b^2x^{4/3}}+\frac{6}{bx^{2/3}\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] 6/(b*x^(2/3)*Sqrt[b*x^(2/3) + a*x]) - (7*Sqrt[b*x^(2/3) + a*x])/(b^2*x^(4/3
)) + (35*a*Sqrt[b*x^(2/3) + a*x])/(4*b^3*x) - (105*a^2*Sqrt[b*x^(2/3) + a*x
])/(8*b^4*x^(2/3)) + (105*a^3*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*
x]])/(8*b^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}} + \frac{7\int \frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{b}$$
$$= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}} - \frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}} - \frac{(35a)\int \frac{1}{x^{4/3}\sqrt{bx^{2/3}+ax}}\,dx}{6b^2}$$
$$= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}} - \frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}} + \frac{35a\sqrt{bx^{2/3}+ax}}{4b^3x} + \frac{\left(35a^2\right)\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx}{8b^3}$$
$$= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}} - \frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}} + \frac{35a\sqrt{bx^{2/3}+ax}}{4b^3x} - \frac{105a^2\sqrt{bx^{2/3}+ax}}{8b^4x^{2/3}} - \frac{\left(35a^3\right)\int}{}$$
$$= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}} - \frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}} + \frac{35a\sqrt{bx^{2/3}+ax}}{4b^3x} - \frac{105a^2\sqrt{bx^{2/3}+ax}}{8b^4x^{2/3}} + \frac{\left(105a^3\right)S}{}$$
$$= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}} - \frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}} + \frac{35a\sqrt{bx^{2/3}+ax}}{4b^3x} - \frac{105a^2\sqrt{bx^{2/3}+ax}}{8b^4x^{2/3}} + \frac{105a^3\tan}{}$$

Mathematica [C] time = 0.09, size = 48, normalized size = 0.33

$$-\frac{6a^3\sqrt[3]{x}\,{}_2F_1\left(-\frac{1}{2},4;\frac{1}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{b^4\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] (-6*a^3*x^(1/3)*Hypergeometric2F1[-1/2, 4, 1/2, 1 + (a*x^(1/3))/b])/(b^4*Sq
rt[b*x^(2/3) + a*x])
```

IntegrateAlgebraic [A] time = 4.34, size = 128, normalized size = 0.88

$$\frac{\sqrt[3]{x}\sqrt{a\sqrt[3]{x}+b}\left(\frac{105a^3\tanh^{-1}\left(\frac{\sqrt{a\sqrt[3]{x}+b}}{\sqrt{b}}\right)}{8b^{9/2}}+\frac{-105a^3x-35a^2bx^{2/3}+14ab^2\sqrt[3]{x}-8b^3}{8b^4x\sqrt{a\sqrt[3]{x}+b}}\right)}{\sqrt{x^{2/3}\left(a\sqrt[3]{x}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] (Sqrt[b + a*x^(1/3)]*x^(1/3)*((-8*b^3 + 14*a*b^2*x^(1/3) - 35*a^2*b*x^(2/3)
 - 105*a^3*x)/(8*b^4*Sqrt[b + a*x^(1/3)]*x) + (105*a^3*ArcTanh[Sqrt[b + a*x
^(1/3)]/Sqrt[b]])/(8*b^(9/2))))/Sqrt[(b + a*x^(1/3))*x^(2/3)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.26, size = 105, normalized size = 0.72

$$-\frac{105\,a^3\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{8\,\sqrt{-b}\,b^4}-\frac{6\,a^3}{\sqrt{ax^{\frac{1}{3}}+b}\,b^4}-\frac{57\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^3-136\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^3b+87\sqrt{ax^{\frac{1}{3}}+b}\,a^3b^2}{8\,a^3b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -105/8*a^3*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) - 6*a^3/(sqr
t(a*x^(1/3) + b)*b^4) - 1/8*(57*(a*x^(1/3) + b)^(5/2)*a^3 - 136*(a*x^(1/3)
+ b)^(3/2)*a^3*b + 87*sqrt(a*x^(1/3) + b)*a^3*b^2)/(a^3*b^4*x)
```

maple [A] time = 0.06, size = 88, normalized size = 0.60

$$-\frac{\left(a\,x^{\frac{1}{3}}+b\right)\left(-105\sqrt{a\,x^{\frac{1}{3}}+b}\;a^3x\,\mathrm{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)+105a^3\sqrt{b}\;x+35a^2b^{\frac{3}{2}}x^{\frac{2}{3}}-14a\,b^{\frac{5}{2}}x^{\frac{1}{3}}+8b^{\frac{7}{2}}\right)}{8\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x+b*x^(2/3))^(3/2),x)
```

```
[Out] -1/8*(a*x^(1/3)+b)*(-14*b^(5/2)*x^(1/3)*a+35*b^(3/2)*x^(2/3)*a^2+105*x*a^3*
b^(1/2)+8*b^(7/2)-105*arctanh((a*x^(1/3)+b)^(1/2)/b^(1/2))*(a*x^(1/3)+b)^(1
/2)*x*a^3)/(a*x+b*x^(2/3))^(3/2)/b^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(2/3))^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{x\left(a\,x+b\,x^{2/3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^(2/3))^(3/2)),x)
```

```
[Out] int(1/(x*(a*x + b*x^(2/3))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x*(a*x + b*x**(2/3))**(3/2)), x)
```

3.121 $\displaystyle \int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=236

$$-\frac{9009a^6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{512b^{15/2}}+\frac{9009a^5\sqrt{ax+bx^{2/3}}}{512b^7x^{2/3}}-\frac{3003a^4\sqrt{ax+bx^{2/3}}}{256b^6x}+\frac{3003a^3\sqrt{ax+bx^{2/3}}}{320b^5x^{4/3}}-\frac{1287a^2\sqrt{ax+bx^{2/3}}}{160b^4x^{5/3}}$$

Rubi [A] time = 0.41, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2023, 2025, 2029, 206}

$$\frac{9009a^5\sqrt{ax+bx^{2/3}}}{512b^7x^{2/3}}-\frac{3003a^4\sqrt{ax+bx^{2/3}}}{256b^6x}+\frac{3003a^3\sqrt{ax+bx^{2/3}}}{320b^5x^{4/3}}-\frac{1287a^2\sqrt{ax+bx^{2/3}}}{160b^4x^{5/3}}-\frac{9009a^6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{512b^{15/2}}+\frac{143a\sqrt{ax+bx^{2/3}}}{20b^3x^2}-\frac{13\sqrt{ax+bx^{2/3}}}{2b^2x^{7/3}}+\frac{6}{bx^{5/3}\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] 6/(b*x^(5/3)*Sqrt[b*x^(2/3) + a*x]) - (13*Sqrt[b*x^(2/3) + a*x])/(2*b^2*x^(
7/3)) + (143*a*Sqrt[b*x^(2/3) + a*x])/(20*b^3*x^2) - (1287*a^2*Sqrt[b*x^(2/
3) + a*x])/(160*b^4*x^(5/3)) + (3003*a^3*Sqrt[b*x^(2/3) + a*x])/(320*b^5*x^
(4/3)) - (3003*a^4*Sqrt[b*x^(2/3) + a*x])/(256*b^6*x) + (9009*a^5*Sqrt[b*x^
(2/3) + a*x])/(512*b^7*x^(2/3)) - (9009*a^6*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[
b*x^(2/3) + a*x]])/(512*b^(15/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}} + \frac{13\int \frac{1}{x^{8/3}\sqrt{bx^{2/3}+ax}}\,dx}{b}$$

$$= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}} - \frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}} - \frac{(143a)\int \frac{1}{x^{7/3}\sqrt{bx^{2/3}+ax}}\,dx}{12b^2}$$

$$= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}} - \frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}} + \frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2} + \frac{\left(429a^2\right)\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx}{40b^3}$$

$$= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}} - \frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}} + \frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2} - \frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}} - (3$$

$$= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}} - \frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}} + \frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2} - \frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}} + 30$$

$$= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}} - \frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}} + \frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2} - \frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}} + 30$$

$$= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}} - \frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}} + \frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2} - \frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}} + 30$$

$$= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}} - \frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}} + \frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2} - \frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}} + 30$$

$$= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}} - \frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}} + \frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2} - \frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}} + 30$$

Mathematica [C] time = 0.08, size = 48, normalized size = 0.20

$$\frac{6a^6\sqrt[3]{x}\,{}_2F_1\left(-\frac{1}{2},7;\frac{1}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{b^7\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] (6*a^6*x^(1/3)*Hypergeometric2F1[-1/2, 7, 1/2, 1 + (a*x^(1/3))/b])/(b^7*Sqr
t[b*x^(2/3) + a*x])
```

IntegrateAlgebraic [A] time = 12.49, size = 165, normalized size = 0.70

$$\frac{\sqrt[3]{x}\sqrt{a\sqrt[3]{x}+b}\left(\frac{45045a^6x^2+15015a^5bx^{5/3}-6006a^4b^2x^{4/3}+3432a^3b^3x-2288a^2b^4x^{2/3}+1664ab^5\sqrt[3]{x}-1280b^6}{2560b^7x^2\sqrt{a\sqrt[3]{x}+b}} - \frac{9009a^6\tanh^{-1}\left(\frac{\sqrt{a\sqrt[3]{x}+b}}{\sqrt{b}}\right)}{512b^{15/2}}\right)}{\sqrt{x^{2/3}\left(a\sqrt[3]{x}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] (Sqrt[b + a*x^(1/3)]*x^(1/3)*((-1280*b^6 + 1664*a*b^5*x^(1/3) - 2288*a^2*b^
4*x^(2/3) + 3432*a^3*b^3*x - 6006*a^4*b^2*x^(4/3) + 15015*a^5*b*x^(5/3) + 4
5045*a^6*x^2)/(2560*b^7*Sqrt[b + a*x^(1/3)]*x^2) - (9009*a^6*ArcTanh[Sqrt[b
 + a*x^(1/3)]/Sqrt[b]])/(512*b^(15/2))))/Sqrt[(b + a*x^(1/3))*x^(2/3)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.39, size = 156, normalized size = 0.66

$$\frac{9009\, a^{6} \arctan\left(\frac{\sqrt{a x^{\frac{1}{3}} + b}}{\sqrt{-b}}\right)}{512 \sqrt{-b}\, b^{7}} + \frac{6\, a^{6}}{\sqrt{a x^{\frac{1}{3}} + b}\, b^{7}} + \frac{29685 \left(a x^{\frac{1}{3}} + b\right)^{\frac{11}{2}} a^{6} - 163095 \left(a x^{\frac{1}{3}} + b\right)^{\frac{9}{2}} a^{6} b + 364194 \left(a x^{\frac{1}{3}} + b\right)^{\frac{7}{2}} a^{6} b^{2} - 416094 \left(a x^{\frac{1}{3}} + b\right)^{\frac{5}{2}} a^{6} b^{3} + 246505 \left(a x^{\frac{1}{3}} + b\right)^{\frac{3}{2}} a^{6} b^{4} - 62475 \sqrt{a x^{\frac{1}{3}} + b}\, a^{6} b^{5}}{2560\, a^{6} b^{7} x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 9009/512*a^6*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7) + 6*a^6/(s
qrt(a*x^(1/3) + b)*b^7) + 1/2560*(29685*(a*x^(1/3) + b)^(11/2)*a^6 - 163095
*(a*x^(1/3) + b)^(9/2)*a^6*b + 364194*(a*x^(1/3) + b)^(7/2)*a^6*b^2 - 41609
4*(a*x^(1/3) + b)^(5/2)*a^6*b^3 + 246505*(a*x^(1/3) + b)^(3/2)*a^6*b^4 - 62
475*sqrt(a*x^(1/3) + b)*a^6*b^5)/(a^6*b^7*x^2)
```

maple [A] time = 0.06, size = 126, normalized size = 0.53

$$-\frac{\left(a x^{\frac{1}{3}} + b\right) \left(45045 \sqrt{a x^{\frac{1}{3}} + b}\, a^{6} x^{2} \operatorname{arctanh}\left(\frac{\sqrt{a x^{\frac{1}{3}} + b}}{\sqrt{b}}\right) - 45045 a^{6} \sqrt{b}\, x^{2} - 15015 a^{5} b^{\frac{3}{2}} x^{\frac{5}{3}} + 6006 a^{4} b^{\frac{5}{2}} x^{\frac{4}{3}} - 3432 a^{3} b^{\frac{7}{2}} x + 2288 a^{2} b^{\frac{9}{2}} x^{\frac{2}{3}} - 1664 a\, b^{\frac{11}{2}} x^{\frac{1}{3}} + 1280 b^{\frac{13}{2}}\right)}{2560 \left(a x + b x^{\frac{2}{3}}\right)^{\frac{3}{2}} b^{\frac{15}{2}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a*x+b*x^(2/3))^(3/2),x)
```

```
[Out] -1/2560*(a*x^(1/3)+b)*(1280*b^(13/2)+45045*(a*x^(1/3)+b)^(1/2)*arctanh((a*x
^(1/3)+b)^(1/2)/b^(1/2))*x^2*a^6+6006*x^(4/3)*b^(5/2)*a^4-3432*x*b^(7/2)*a^
3+2288*x^(2/3)*b^(9/2)*a^2-1664*x^(1/3)*b^(11/2)*a-45045*x^2*a^6*b^(1/2)-15
015*x^(5/3)*b^(3/2)*a^5)/x/(a*x+b*x^(2/3))^(3/2)/b^(15/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a x + b x^{\frac{2}{3}}\right)^{\frac{3}{2}} x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(2/3))^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{2} \left(a\, x + b\, x^{2/3}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^(2/3))^(3/2)),x)
```

```
[Out] int(1/(x^2*(a*x + b*x^(2/3))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a*x + b*x**(2/3))**(3/2)), x)
```

3.122 $\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=324

$$\frac{692835a^9\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{32768b^{21/2}}-\frac{692835a^8\sqrt{ax+bx^{2/3}}}{32768b^{10}x^{2/3}}+\frac{230945a^7\sqrt{ax+bx^{2/3}}}{16384b^9x}-\frac{46189a^6\sqrt{ax+bx^{2/3}}}{4096b^8x^{4/3}}+\frac{138567a^5\sqrt{\cdot}}{14336\cdots}$$

Rubi [A] time = 0.60, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2023, 2025, 2029, 206}

$$-\frac{692835a^8\sqrt{ax+bx^{2/3}}}{32768b^{10}x^{2/3}}+\frac{230945a^7\sqrt{ax+bx^{2/3}}}{16384b^9x}-\frac{46189a^6\sqrt{ax+bx^{2/3}}}{4096b^8x^{4/3}}+\frac{138567a^5\sqrt{ax+bx^{2/3}}}{14336b^7x^{5/3}}-\frac{46189a^4\sqrt{ax+bx^{2/3}}}{5376b^6x^2}+\frac{20995a^3\sqrt{ax+bx^{2/3}}}{2688b^5x^{7/3}}-\frac{1615a^2\sqrt{ax+bx^{2/3}}}{224b^4x^{8/3}}+\frac{692835a^9\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{32768b^{21/2}}+\frac{323a\sqrt{ax+bx^{2/3}}}{48b^3x^3}-\frac{19\sqrt{ax+bx^{2/3}}}{3b^2x^{10/3}}+\frac{6}{bx^{8/3}\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] 6/(b*x^(8/3)*Sqrt[b*x^(2/3) + a*x]) - (19*Sqrt[b*x^(2/3) + a*x])/(3*b^2*x^(
10/3)) + (323*a*Sqrt[b*x^(2/3) + a*x])/(48*b^3*x^3) - (1615*a^2*Sqrt[b*x^(2
/3) + a*x])/(224*b^4*x^(8/3)) + (20995*a^3*Sqrt[b*x^(2/3) + a*x])/(2688*b^5
*x^(7/3)) - (46189*a^4*Sqrt[b*x^(2/3) + a*x])/(5376*b^6*x^2) + (138567*a^5*
Sqrt[b*x^(2/3) + a*x])/(14336*b^7*x^(5/3)) - (46189*a^6*Sqrt[b*x^(2/3) + a*
x])/(4096*b^8*x^(4/3)) + (230945*a^7*Sqrt[b*x^(2/3) + a*x])/(16384*b^9*x) -
 (692835*a^8*Sqrt[b*x^(2/3) + a*x])/(32768*b^10*x^(2/3)) + (692835*a^9*ArcT
anh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(32768*b^(21/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} + \frac{19\int \frac{1}{x^{11/3}\sqrt{bx^{2/3}+ax}}\,dx}{b}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} - \frac{(323a)\int \frac{1}{x^{10/3}\sqrt{bx^{2/3}+ax}}\,dx}{18b^2}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} + \frac{\left(1615a^2\right)\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,d}{96b^3}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} - \frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}} -$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} - \frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}} +$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} - \frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}} +$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} - \frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}} +$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} - \frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}} +$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} - \frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}} +$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} - \frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}} +$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} - \frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}} +$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} - \frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}} +$$

Mathematica [C] time = 0.07, size = 48, normalized size = 0.15

$$-\frac{6a^9\sqrt[3]{x}\,{}_2F_1\left(-\frac{1}{2},10;\frac{1}{2};\frac{\sqrt[3]{x}\,a}{b}+1\right)}{b^{10}\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] (-6*a^9*x^(1/3)*Hypergeometric2F1[-1/2, 10, 1/2, 1 + (a*x^(1/3))/b])/(b^10*
Sqrt[b*x^(2/3) + a*x])
```

IntegrateAlgebraic [A] time = 17.21, size = 202, normalized size = 0.62

$$\frac{\sqrt[3]{x}\sqrt{a\sqrt[3]{x}+b}\left(\frac{692835a^9\tanh^{-1}\left(\frac{\sqrt{a\sqrt[3]{x}+b}}{\sqrt{b}}\right)}{32768b^{21/2}}+\frac{-14549535a^9x^3-4849845a^8bx^{8/3}+1939938a^7b^2x^{7/3}-1108536a^6b^3x^2+739024a^5b^4x^{5/3}-537472a^4b^5x^{4/3}+413440a^3b^6x-330752a^2b^7x^{2/3}+272384ab^8\sqrt[3]{x}-229376b^9}{688128b^{10}x^3\sqrt{a\sqrt[3]{x}+b}}\right)}{\sqrt{x^{2/3}\left(a\sqrt[3]{x}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] (Sqrt[b + a*x^(1/3)]*x^(1/3)*((-229376*b^9 + 272384*a*b^8*x^(1/3) - 330752*
a^2*b^7*x^(2/3) + 413440*a^3*b^6*x - 537472*a^4*b^5*x^(4/3) + 739024*a^5*b^
4*x^(5/3) - 1108536*a^6*b^3*x^2 + 1939938*a^7*b^2*x^(7/3) - 4849845*a^8*b*x
^(8/3) - 14549535*a^9*x^3)/(688128*b^10*Sqrt[b + a*x^(1/3)]*x^3) + (692835*
a^9*ArcTanh[Sqrt[b + a*x^(1/3)]/Sqrt[b]])/(32768*b^(21/2))))/Sqrt[(b + a*x^
(1/3))*x^(2/3)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.38, size = 207, normalized size = 0.64

$$-\frac{692835\,a^{9}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{32768\,\sqrt{-b}\,b^{10}}-\frac{6\,a^{9}}{\sqrt{ax^{\frac{1}{3}}+b}\,b^{10}}-\frac{10420767\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{9}-88937058\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{9}b+334408914\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^{9}b^{2}-724860666\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^{9}b^{3}+993296384\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{9}b^{4}-884769030\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^{9}b^{5}+503730990\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^{9}b^{6}-169799070\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^{9}b^{7}+26738145\,\sqrt{ax^{\frac{1}{3}}+b}\,a^{9}b^{8}}{688128\,a^{9}b^{10}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -692835/32768*a^9*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^10) - 6*
a^9/(sqrt(a*x^(1/3) + b)*b^10) - 1/688128*(10420767*(a*x^(1/3) + b)^(17/2)*
a^9 - 88937058*(a*x^(1/3) + b)^(15/2)*a^9*b + 334408914*(a*x^(1/3) + b)^(13
/2)*a^9*b^2 - 724860666*(a*x^(1/3) + b)^(11/2)*a^9*b^3 + 993296384*(a*x^(1/
3) + b)^(9/2)*a^9*b^4 - 884769030*(a*x^(1/3) + b)^(7/2)*a^9*b^5 + 503730990
*(a*x^(1/3) + b)^(5/2)*a^9*b^6 - 169799070*(a*x^(1/3) + b)^(3/2)*a^9*b^7 +
26738145*sqrt(a*x^(1/3) + b)*a^9*b^8)/(a^9*b^10*x^3)
```

maple [A] time = 0.07, size = 159, normalized size = 0.49

$$\frac{\left(a\,x^{\frac{1}{3}}+b\right)\left(14549535\sqrt{a\,x^{\frac{1}{3}}+b}\;a^{9}x^{3}\,\mathrm{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)-14549535a^{9}\sqrt{b}\;x^{3}-4849845a^{8}b^{\frac{3}{2}}x^{\frac{8}{3}}+1939938a^{7}b^{\frac{5}{2}}x^{\frac{7}{3}}-1108536a^{6}b^{\frac{7}{2}}x^{2}+739024a^{5}b^{\frac{9}{2}}x^{\frac{5}{3}}-537472a^{4}b^{\frac{11}{2}}x^{\frac{4}{3}}+413440a^{3}b^{\frac{13}{2}}x-330752a^{2}b^{\frac{15}{2}}x^{\frac{2}{3}}+272384a\,b^{\frac{17}{2}}x^{\frac{1}{3}}-229376b^{\frac{19}{2}}\right)}{688128\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}b^{\frac{21}{2}}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*x+b*x^(2/3))^(3/2),x)
```

```
[Out] 1/688128*(a*x^(1/3)+b)*(14549535*(a*x^(1/3)+b)^(1/2)*arctanh((a*x^(1/3)+b)^
(1/2)/b^(1/2))*x^3*a^9-229376*b^(19/2)-537472*b^(11/2)*x^(4/3)*a^4-4849845*
b^(3/2)*x^(8/3)*a^8+272384*b^(17/2)*x^(1/3)*a+413440*b^(13/2)*x*a^3-330752*
b^(15/2)*x^(2/3)*a^2+739024*b^(9/2)*x^(5/3)*a^5-14549535*x^3*a^9*b^(1/2)+19
39938*b^(5/2)*x^(7/3)*a^7-1108536*b^(7/2)*x^2*a^6)/x^2/(a*x+b*x^(2/3))^(3/2
)/b^(21/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}} x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(2/3))^(3/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3 \left(a\,x + b\,x^{2/3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x + b*x^(2/3))^(3/2)),x)
[Out] int(1/(x^3*(a*x + b*x^(2/3))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**(2/3)+a*x)**(3/2),x)
[Out] Integral(1/(x**3*(a*x + b*x**(2/3))**(3/2)), x)
```

3.123 $\int \frac{1}{x^4 \left(bx^{2/3}+ax\right)^{3/2}} \, dx$

Optimal. Leaf size=412

$$-\frac{50702925a^{12}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{2097152b^{27/2}}+\frac{50702925a^{11}\sqrt{ax+bx^{2/3}}}{2097152b^{13}x^{2/3}}-\frac{16900975a^{10}\sqrt{ax+bx^{2/3}}}{1048576b^{12}x}+\frac{3380195a^9\sqrt{ax+bx^{2/}}}{262144b^{11}x^{4/3}}$$

Rubi [A] time = 0.84, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2023, 2025, 2029, 206}

$$\frac{50702925a^{11}\sqrt{ax+bx^{2/3}}}{2097152b^{13}x^{2/3}}-\frac{16900975a^{10}\sqrt{ax+bx^{2/3}}}{1048576b^{12}x}+\frac{3380195a^9\sqrt{ax+bx^{2/3}}}{262144b^{11}x^{4/3}}-\frac{1448655a^8\sqrt{ax+bx^{2/3}}}{131072b^{10}x^{5/3}}+\frac{482885a^7\sqrt{ax+bx^{2/3}}}{49152b^9x^2}-\frac{2414425a^6\sqrt{ax+bx^{2/3}}}{270336b^8x^{7/3}}+\frac{185725a^5\sqrt{ax+bx^{2/3}}}{22528b^7x^{8/3}}-\frac{260015a^4\sqrt{ax+bx^{2/3}}}{33792b^6x^3}+\frac{15295a^3\sqrt{ax+bx^{2/3}}}{2112b^5x^{10/3}}-\frac{2415a^2\sqrt{ax+bx^{2/3}}}{352b^4x^{11/3}}-\frac{50702925a^{12}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{2097152b^{27/2}}+\frac{575a\sqrt{ax+bx^{2/3}}}{88b^3x^4}-\frac{25\sqrt{ax+bx^{2/3}}}{4b^2x^{13/3}}+\frac{6}{bx^{11/3}\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] 6/(b*x^(11/3)*Sqrt[b*x^(2/3) + a*x]) - (25*Sqrt[b*x^(2/3) + a*x])/(4*b^2*x^
(13/3)) + (575*a*Sqrt[b*x^(2/3) + a*x])/(88*b^3*x^4) - (2415*a^2*Sqrt[b*x^(
2/3) + a*x])/(352*b^4*x^(11/3)) + (15295*a^3*Sqrt[b*x^(2/3) + a*x])/(2112*b
^5*x^(10/3)) - (260015*a^4*Sqrt[b*x^(2/3) + a*x])/(33792*b^6*x^3) + (185725
*a^5*Sqrt[b*x^(2/3) + a*x])/(22528*b^7*x^(8/3)) - (2414425*a^6*Sqrt[b*x^(2/
3) + a*x])/(270336*b^8*x^(7/3)) + (482885*a^7*Sqrt[b*x^(2/3) + a*x])/(49152
*b^9*x^2) - (1448655*a^8*Sqrt[b*x^(2/3) + a*x])/(131072*b^10*x^(5/3)) + (33
80195*a^9*Sqrt[b*x^(2/3) + a*x])/(262144*b^11*x^(4/3)) - (16900975*a^10*Sqr
t[b*x^(2/3) + a*x])/(1048576*b^12*x) + (50702925*a^11*Sqrt[b*x^(2/3) + a*x]
)/(2097152*b^13*x^(2/3)) - (50702925*a^12*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*
x^(2/3) + a*x]])/(2097152*b^(27/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx &= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}+\frac{25\int \frac{1}{x^{14/3}\sqrt{bx^{2/3}+ax}}\,dx}{b}\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}-\frac{(575a)\int \frac{1}{x^{13/3}\sqrt{bx^{2/3}+ax}}\,dx}{24b^2}\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}+\frac{\left(4025a^2\right)\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}}{176b^3}\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}-\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+
\end{aligned}$$

Mathematica [C] time = 0.09, size = 48, normalized size = 0.12

$$\frac{6a^{12}\sqrt[3]{x}\,{}_2F_1\left(-\frac{1}{2},13;\frac{1}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{b^{13}\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] (6*a^12*x^(1/3)*Hypergeometric2F1[-1/2, 13, 1/2, 1 + (a*x^(1/3))/b])/(b^13*
Sqrt[b*x^(2/3) + a*x])
```

IntegrateAlgebraic [A] time = 18.97, size = 239, normalized size = 0.58

$$\frac{\sqrt[3]{x}\sqrt{a\sqrt[3]{x}+b}\left(\frac{1673196525a^{12}x^4+557732175a^{11}bx^{11/3}-223092870a^{10}b^2x^{10/3}+127481640a^9b^3x^3-84987760a^8b^4x^{8/3}+61809280a^7b^5x^{7/3}-47545600a^6b^6x^2+38036480a^5b^7x^{5/3}-31324160a^4b^8x^{4/3}+26378240a^3b^9x-22609920a^2b^{10}x^{2/3}+19660800ab^{11}\sqrt[3]{x}-17301504b^{12}}{69206016b^{13}x^4\sqrt{a\sqrt[3]{x}+b}}-\frac{50702925a^{12}\tanh^{-1}\left(\frac{\sqrt{a\sqrt[3]{x}+b}}{\sqrt{b}}\right)}{2097152b^{27/2}}\right)}{\sqrt{x^{2/3}\left(a\sqrt[3]{x}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] (Sqrt[b + a*x^(1/3)]*x^(1/3)*((-17301504*b^12 + 19660800*a*b^11*x^(1/3) - 2
2609920*a^2*b^10*x^(2/3) + 26378240*a^3*b^9*x - 31324160*a^4*b^8*x^(4/3) +
38036480*a^5*b^7*x^(5/3) - 47545600*a^6*b^6*x^2 + 61809280*a^7*b^5*x^(7/3)
- 84987760*a^8*b^4*x^(8/3) + 127481640*a^9*b^3*x^3 - 223092870*a^10*b^2*x^(
10/3) + 557732175*a^11*b*x^(11/3) + 1673196525*a^12*x^4)/(69206016*b^13*Sqr
t[b + a*x^(1/3)]*x^4) - (50702925*a^12*ArcTanh[Sqrt[b + a*x^(1/3)]/Sqrt[b]]
)/(2097152*b^(27/2))))/Sqrt[(b + a*x^(1/3))*x^(2/3)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.45, size = 258, normalized size = 0.63

$$\frac{50702925\,a^{12}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{2097152\,\sqrt{-b}\,b^{13}}+\frac{6\,a^{12}}{\sqrt{ax^{\frac{1}{3}}+b}\,b^{13}}+\frac{1257960429\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}}a^{12}-14537792973\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}a^{12}b+76667241519\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{12}b^2-243717614415\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{12}b^3+519393101810\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{12}b^4-780150847218\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^{12}b^5+844265343246\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^{12}b^6-659969685518\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{12}b^7+366679446705\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^{12}b^8-138840292305\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^{12}b^9+32660709939\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^{12}b^{10}-3724872723\sqrt{ax^{\frac{1}{3}}+b}\,a^{12}b^{11}}{69206016\,a^{12}b^{13}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 50702925/2097152*a^12*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^13)
+ 6*a^12/(sqrt(a*x^(1/3) + b)*b^13) + 1/69206016*(1257960429*(a*x^(1/3) + b
)^(23/2)*a^12 - 14537792973*(a*x^(1/3) + b)^(21/2)*a^12*b + 76667241519*(a*
x^(1/3) + b)^(19/2)*a^12*b^2 - 243717614415*(a*x^(1/3) + b)^(17/2)*a^12*b^3
 + 519393101810*(a*x^(1/3) + b)^(15/2)*a^12*b^4 - 780150847218*(a*x^(1/3) +
 b)^(13/2)*a^12*b^5 + 844265343246*(a*x^(1/3) + b)^(11/2)*a^12*b^6 - 659969
685518*(a*x^(1/3) + b)^(9/2)*a^12*b^7 + 366679446705*(a*x^(1/3) + b)^(7/2)*
a^12*b^8 - 138840292305*(a*x^(1/3) + b)^(5/2)*a^12*b^9 + 32660709939*(a*x^(
1/3) + b)^(3/2)*a^12*b^10 - 3724872723*sqrt(a*x^(1/3) + b)*a^12*b^11)/(a^12
*b^13*x^4)
```

maple [A] time = 0.08, size = 192, normalized size = 0.47

$$-\frac{\left(a\,x^{\frac{1}{3}}+b\right)\left(1673196525\sqrt{a\,x^{\frac{1}{3}}+b}\,a^{12}x^4\,\mathrm{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)-1673196525a^{12}\sqrt{b}\,x^4-557732175a^{11}b^{\frac{3}{2}}x^{\frac{11}{3}}+223092870a^{10}b^{\frac{5}{2}}x^{\frac{10}{3}}-127481640a^9b^{\frac{7}{2}}x^3+84987760a^8b^{\frac{9}{2}}x^{\frac{8}{3}}-61809280a^7b^{\frac{11}{2}}x^{\frac{7}{3}}+47545600a^6b^{\frac{13}{2}}x^2-38036480a^5b^{\frac{15}{2}}x^{\frac{5}{3}}+31324160a^4b^{\frac{17}{2}}x^{\frac{4}{3}}-26378240a^3b^{\frac{19}{2}}x+22609920a^2b^{\frac{21}{2}}x^{\frac{2}{3}}-19660800a\,b^{\frac{23}{2}}x^{\frac{1}{3}}+17301504b^{\frac{25}{2}}\right)}{69206016\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}b^{\frac{27}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a*x+b*x^(2/3))^(3/2),x)
```

```
[Out] -1/69206016*(a*x^(1/3)+b)*(17301504*b^(25/2)-1673196525*x^4*a^12*b^(1/2)-19
660800*b^(23/2)*x^(1/3)*a+22609920*b^(21/2)*x^(2/3)*a^2+1673196525*(a*x^(1/
3)+b)^(1/2)*arctanh((a*x^(1/3)+b)^(1/2)/b^(1/2))*x^4*a^12-26378240*b^(19/2)
```

```
*x*a^3+31324160*b^(17/2)*x^(4/3)*a^4-38036480*b^(15/2)*x^(5/3)*a^5+47545600
*b^(13/2)*x^2*a^6-61809280*b^(11/2)*x^(7/3)*a^7+84987760*b^(9/2)*x^(8/3)*a^
8-127481640*b^(7/2)*x^3*a^9+223092870*b^(5/2)*x^(10/3)*a^10-557732175*b^(3/
2)*x^(11/3)*a^11)/x^3/(a*x+b*x^(2/3))^(3/2)/b^(27/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*x + b*x^(2/3))^(3/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4 \left(a\,x + b\,x^{2/3}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a*x + b*x^(2/3))^(3/2)),x)
[Out] int(1/(x^4*(a*x + b*x^(2/3))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**(2/3)+a*x)**(3/2),x)
[Out] Integral(1/(x**4*(a*x + b*x**(2/3))**(3/2)), x)
```

3.124 $\int x^2 \left(a x^2+b x^3\right) \, dx$

Optimal. Leaf size=17

$$\frac{ax^5}{5}+\frac{bx^6}{6}$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$\frac{ax^5}{5}+\frac{bx^6}{6}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a*x^2 + b*x^3),x]

[Out] (a*x^5)/5 + (b*x^6)/6

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a x^2+b x^3\right) \, dx &= \int \left(a x^4+b x^5\right) \, dx\\ &= \frac{ax^5}{5}+\frac{bx^6}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^5}{5}+\frac{bx^6}{6}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a*x^2 + b*x^3),x]

[Out] (a*x^5)/5 + (b*x^6)/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a x^2+b x^3\right) \, dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(a*x^2 + b*x^3),x]

[Out] IntegrateAlgebraic[x^2*(a*x^2 + b*x^3), x]

fricas [A] time = 0.40, size = 13, normalized size = 0.76

$$\frac{1}{6}x^6b+\frac{1}{5}x^5a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*b + 1/5*x^5*a
```

giac [A] time = 0.19, size = 13, normalized size = 0.76

$$\frac{1}{6}\,bx^6 + \frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 1/6*b*x^6 + 1/5*a*x^5
```

maple [A] time = 0.04, size = 14, normalized size = 0.82

$$\frac{1}{6}b\,x^6 + \frac{1}{5}a\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x^2),x)
```

```
[Out] 1/5*a*x^5+1/6*b*x^6
```

maxima [A] time = 1.33, size = 13, normalized size = 0.76

$$\frac{1}{6}\,bx^6 + \frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] 1/6*b*x^6 + 1/5*a*x^5
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^5\,\left(6\,a+5\,b\,x\right)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^2 + b*x^3),x)
```

```
[Out] (x^5*(6*a + 5*b*x))/30
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^5}{5} + \frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x**2),x)
```

```
[Out] a*x**5/5 + b*x**6/6
```

3.125 $\int x\left(ax^2+bx^3\right)\,dx$

Optimal. Leaf size=17

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x^2 + b*x^3),x]
```

```
[Out] (a*x^4)/4 + (b*x^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(ax^2+bx^3\right)\,dx &= \int \left(ax^3+bx^4\right)\,dx\\ &= \frac{ax^4}{4}+\frac{bx^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x^2 + b*x^3),x]
```

```
[Out] (a*x^4)/4 + (b*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(ax^2+bx^3\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a*x^2 + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[x*(a*x^2 + b*x^3), x]
```

fricas [A] time = 0.44, size = 13, normalized size = 0.76

$$\frac{1}{5}x^5b+\frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2),x, algorithm="fricas")
[Out] 1/5*x^5*b + 1/4*x^4*a
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^5 + \frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2),x, algorithm="giac")
[Out] 1/5*b*x^5 + 1/4*a*x^4
```

maple [A] time = 0.04, size = 14, normalized size = 0.82

$$\frac{1}{5}b\,x^5 + \frac{1}{4}a\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x^2),x)
[Out] 1/4*a*x^4+1/5*b*x^5
```

maxima [A] time = 1.34, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^5 + \frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2),x, algorithm="maxima")
[Out] 1/5*b*x^5 + 1/4*a*x^4
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^4\,\left(5\,a+4\,b\,x\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x^2 + b*x^3),x)
[Out] (x^4*(5*a + 4*b*x))/20
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^4}{4} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x**2),x)
[Out] a*x**4/4 + b*x**5/5
```

3.126 $\int \left(ax^2+bx^3\right) dx$

Optimal. Leaf size=17

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[a*x^2 + b*x^3,x]
```

```
[Out] (a*x^3)/3 + (b*x^4)/4
```

Rubi steps

$$\int \left(ax^2+bx^3\right) dx=\frac{ax^3}{3}+\frac{bx^4}{4}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*x^2 + b*x^3,x]
```

```
[Out] (a*x^3)/3 + (b*x^4)/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^2+bx^3\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[a*x^2 + b*x^3,x]
```

```
[Out] IntegrateAlgebraic[a*x^2 + b*x^3, x]
```

fricas [A] time = 0.51, size = 13, normalized size = 0.76

$$\frac{1}{4}x^4b+\frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x^2,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*b + 1/3*x^3*a
```

giac [A] time = 0.16, size = 13, normalized size = 0.76

$$\frac{1}{4}bx^4+\frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x^2,x, algorithm="giac")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

maple [A] time = 0.04, size = 14, normalized size = 0.82

$$\frac{1}{4} b\, x^{4}+\frac{1}{3} a\, x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x^3+a*x^2,x)
```

```
[Out] 1/3*a*x^3+1/4*b*x^4
```

maxima [A] time = 1.34, size = 13, normalized size = 0.76

$$\frac{1}{4}\, b x^{4} + \frac{1}{3}\, a x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^3\,\left(4\,a+3\,b\,x\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*x^2 + b*x^3,x)
```

```
[Out] (x^3*(4*a + 3*b*x))/12
```

sympy [A] time = 0.07, size = 12, normalized size = 0.71

$$\frac{a x^{3}}{3} + \frac{b x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x**3+a*x**2,x)
```

```
[Out] a*x**3/3 + b*x**4/4
```

3.127 $\int \frac{ax^2+bx^3}{x} \, dx$

Optimal. Leaf size=17

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)/x,x]
```

```
[Out] (a*x^2)/2 + (b*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{ax^2+bx^3}{x} \, dx &= \int \left(ax+bx^2\right) \, dx \\ &= \frac{ax^2}{2}+\frac{bx^3}{3} \end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)/x,x]
```

```
[Out] (a*x^2)/2 + (b*x^3)/3
```

IntegrateAlgebraic [A] time = 0.02, size = 15, normalized size = 0.88

$$\frac{1}{6}x^2(3a+2bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x^3)/x,x]
```

```
[Out] (x^2*(3*a + 2*b*x))/6
```

fricas [A] time = 0.61, size = 13, normalized size = 0.76

$$\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/x,x, algorithm="fricas")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$\frac{1}{3}\,bx^3 + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/x,x, algorithm="giac")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

maple [A] time = 0.06, size = 14, normalized size = 0.82

$$\frac{1}{3}b\,x^3 + \frac{1}{2}a\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)/x,x)
```

```
[Out] 1/2*a*x^2+1/3*b*x^3
```

maxima [A] time = 1.36, size = 13, normalized size = 0.76

$$\frac{1}{3}\,bx^3 + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^2\,\left(3\,a+2\,b\,x\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)/x,x)
```

```
[Out] (x^2*(3*a + 2*b*x))/6
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^2}{2} + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)/x,x)
```

```
[Out] a*x**2/2 + b*x**3/3
```

3.128 $\int \frac{ax^2+bx^3}{x^2} dx$

Optimal. Leaf size=12

$$ax + \frac{bx^2}{2}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$ax + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)/x^2,x]
```

```
[Out] a*x + (b*x^2)/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{ax^2+bx^3}{x^2} dx &= \int (a+bx)\, dx \\ &= ax + \frac{bx^2}{2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$ax + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)/x^2,x]
```

```
[Out] a*x + (b*x^2)/2
```

IntegrateAlgebraic [A] time = 0.01, size = 12, normalized size = 1.00

$$ax + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x^3)/x^2,x]
```

```
[Out] a*x + (b*x^2)/2
```

fricas [A] time = 0.50, size = 10, normalized size = 0.83

$$\frac{1}{2}\, bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*x
```

giac [A] time = 0.15, size = 10, normalized size = 0.83

$$\frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x
```

maple [A] time = 0.05, size = 11, normalized size = 0.92

$$\frac{1}{2}b\,x^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)/x^2,x)
```

```
[Out] a*x+1/2*b*x^2
```

maxima [A] time = 1.34, size = 10, normalized size = 0.83

$$\frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x
```

mupad [B] time = 0.02, size = 10, normalized size = 0.83

$$\frac{b\,x^2}{2} + a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)/x^2,x)
```

```
[Out] a*x + (b*x^2)/2
```

sympy [A] time = 0.06, size = 8, normalized size = 0.67

$$ax + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)/x**2,x)
```

```
[Out] a*x + b*x**2/2
```

3.129 $\int x^2 \left(a x^2+b x^3\right)^2 \, dx$

Optimal. Leaf size=30

$$\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{b^2x^9}{9}$$

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + (b^2*x^9)/9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^2\left(ax^2+bx^3\right)^2\,dx &= \int x^6(a+bx)^2\,dx\\ &= \int \left(a^2x^6+2abx^7+b^2x^8\right)\,dx\\ &= \frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{b^2x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + (b^2*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(ax^2+bx^3\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a*x^2 + b*x^3)^2,x]
[Out] IntegrateAlgebraic[x^2*(a*x^2 + b*x^3)^2, x]
```

fricas [A] time = 0.34, size = 24, normalized size = 0.80

$$\frac{1}{9}x^9b^2 + \frac{1}{4}x^8ba + \frac{1}{7}x^7a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^2,x, algorithm="fricas")
[Out] 1/9*x^9*b^2 + 1/4*x^8*b*a + 1/7*x^7*a^2
```

giac [A] time = 0.15, size = 24, normalized size = 0.80

$$\frac{1}{9}b^2x^9 + \frac{1}{4}abx^8 + \frac{1}{7}a^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] 1/9*b^2*x^9 + 1/4*a*b*x^8 + 1/7*a^2*x^7
```

maple [A] time = 0.04, size = 25, normalized size = 0.83

$$\frac{1}{9}b^2x^9 + \frac{1}{4}ab\,x^8 + \frac{1}{7}a^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x^2)^2,x)
[Out] 1/7*a^2*x^7+1/4*a*b*x^8+1/9*b^2*x^9
```

maxima [A] time = 1.35, size = 24, normalized size = 0.80

$$\frac{1}{9}b^2x^9 + \frac{1}{4}abx^8 + \frac{1}{7}a^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] 1/9*b^2*x^9 + 1/4*a*b*x^8 + 1/7*a^2*x^7
```

mupad [B] time = 0.04, size = 24, normalized size = 0.80

$$\frac{a^2\,x^7}{7} + \frac{a\,b\,x^8}{4} + \frac{b^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^2 + b*x^3)^2,x)
[Out] (a^2*x^7)/7 + (b^2*x^9)/9 + (a*b*x^8)/4
```

sympy [A] time = 0.07, size = 24, normalized size = 0.80

$$\frac{a^2x^7}{7} + \frac{abx^8}{4} + \frac{b^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x**2)**2,x)
[Out] a**2*x**7/7 + a*b*x**8/4 + b**2*x**9/9
```

3.130 $\int x\left(ax^2+bx^3\right)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}$$

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 43}

$$\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + (b^2*x^8)/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x\left(ax^2+bx^3\right)^2\,dx &= \int x^5(a+bx)^2\,dx\\ &= \int \left(a^2x^5+2abx^6+b^2x^7\right)\,dx\\ &= \frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + (b^2*x^8)/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(ax^2+bx^3\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a*x^2 + b*x^3)^2,x]
[Out] IntegrateAlgebraic[x*(a*x^2 + b*x^3)^2, x]
```

fricas [A] time = 0.34, size = 24, normalized size = 0.80

$$\frac{1}{8}x^8b^2 + \frac{2}{7}x^7ba + \frac{1}{6}x^6a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^2,x, algorithm="fricas")
[Out] 1/8*x^8*b^2 + 2/7*x^7*b*a + 1/6*x^6*a^2
```

giac [A] time = 0.15, size = 24, normalized size = 0.80

$$\frac{1}{8}\,b^2x^8 + \frac{2}{7}\,abx^7 + \frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] 1/8*b^2*x^8 + 2/7*a*b*x^7 + 1/6*a^2*x^6
```

maple [A] time = 0.06, size = 25, normalized size = 0.83

$$\frac{1}{8}b^2x^8 + \frac{2}{7}ab\,x^7 + \frac{1}{6}a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x^2)^2,x)
[Out] 1/6*a^2*x^6+2/7*a*b*x^7+1/8*b^2*x^8
```

maxima [A] time = 1.20, size = 24, normalized size = 0.80

$$\frac{1}{8}\,b^2x^8 + \frac{2}{7}\,abx^7 + \frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] 1/8*b^2*x^8 + 2/7*a*b*x^7 + 1/6*a^2*x^6
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^6}{6} + \frac{2\,a\,b\,x^7}{7} + \frac{b^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x^2 + b*x^3)^2,x)
[Out] (a^2*x^6)/6 + (b^2*x^8)/8 + (2*a*b*x^7)/7
```

sympy [A] time = 0.07, size = 26, normalized size = 0.87

$$\frac{a^2x^6}{6} + \frac{2abx^7}{7} + \frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x**2)**2,x)
[Out] a**2*x**6/6 + 2*a*b*x**7/7 + b**2*x**8/8
```

3.131 $\int \left(ax^2+bx^3\right)^2 \, dx$

Optimal. Leaf size=30

$$\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 43}

$$\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + (b^2*x^7)/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(ax^2+bx^3\right)^2 \, dx &= \int x^4(a+bx)^2 \, dx\\ &= \int \left(a^2x^4+2abx^5+b^2x^6\right) dx\\ &= \frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + (b^2*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^2+bx^3\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x^3)^2,x]
[Out] IntegrateAlgebraic[(a*x^2 + b*x^3)^2, x]
```

fricas [A] time = 0.35, size = 24, normalized size = 0.80

$$\frac{1}{7}x^7b^2 + \frac{1}{3}x^6ba + \frac{1}{5}x^5a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2,x, algorithm="fricas")
[Out] 1/7*x^7*b^2 + 1/3*x^6*b*a + 1/5*x^5*a^2
```

giac [A] time = 0.16, size = 24, normalized size = 0.80

$$\frac{1}{7}b^2x^7 + \frac{1}{3}abx^6 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] 1/7*b^2*x^7 + 1/3*a*b*x^6 + 1/5*a^2*x^5
```

maple [A] time = 0.04, size = 25, normalized size = 0.83

$$\frac{1}{7}b^2x^7 + \frac{1}{3}ab\,x^6 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^2,x)
[Out] 1/5*a^2*x^5+1/3*a*b*x^6+1/7*b^2*x^7
```

maxima [A] time = 1.33, size = 24, normalized size = 0.80

$$\frac{1}{7}b^2x^7 + \frac{1}{3}abx^6 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] 1/7*b^2*x^7 + 1/3*a*b*x^6 + 1/5*a^2*x^5
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^5}{5} + \frac{a\,b\,x^6}{3} + \frac{b^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^2,x)
[Out] (a^2*x^5)/5 + (b^2*x^7)/7 + (a*b*x^6)/3
```

sympy [A] time = 0.07, size = 24, normalized size = 0.80

$$\frac{a^2x^5}{5} + \frac{abx^6}{3} + \frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**2,x)
[Out] a**2*x**5/5 + a*b*x**6/3 + b**2*x**7/7
```

3.132 $\int \frac{(ax^2+bx^3)^2}{x} \, dx$

Optimal. Leaf size=30

$$\frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{b^2x^6}{6}$$

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^2/x,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax^2+bx^3)^2}{x} \, dx &= \int x^3(a+bx)^2 \, dx \\
&= \int \left(a^2x^3 + 2abx^4 + b^2x^5\right) dx \\
&= \frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{b^2x^6}{6}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^2/x,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

IntegrateAlgebraic [A] time = 0.02, size = 30, normalized size = 1.00

$$\frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x^3)^2/x,x]
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

fricas [A] time = 0.39, size = 24, normalized size = 0.80

$$\frac{1}{6}b^2x^6 + \frac{2}{5}abx^5 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2/x,x, algorithm="fricas")
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

giac [A] time = 0.17, size = 24, normalized size = 0.80

$$\frac{1}{6}b^2x^6 + \frac{2}{5}abx^5 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2/x,x, algorithm="giac")
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

maple [A] time = 0.04, size = 25, normalized size = 0.83

$$\frac{1}{6}b^2x^6 + \frac{2}{5}ab\,x^5 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^2/x,x)
[Out] 1/4*a^2*x^4+2/5*a*b*x^5+1/6*b^2*x^6
```

maxima [A] time = 1.27, size = 24, normalized size = 0.80

$$\frac{1}{6}b^2x^6 + \frac{2}{5}abx^5 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2/x,x, algorithm="maxima")
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^4}{4} + \frac{2\,a\,b\,x^5}{5} + \frac{b^2\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^2/x,x)
[Out] (a^2*x^4)/4 + (b^2*x^6)/6 + (2*a*b*x^5)/5
```

sympy [A] time = 0.07, size = 26, normalized size = 0.87

$$\frac{a^2x^4}{4} + \frac{2abx^5}{5} + \frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**2/x,x)
[Out] a**2*x**4/4 + 2*a*b*x**5/5 + b**2*x**6/6
```

3.133 $\int \frac{\left(ax^2+bx^3\right)^2}{x^2} dx$

Optimal. Leaf size=30

$$\frac{a^2x^3}{3} + \frac{1}{2}abx^4 + \frac{b^2x^5}{5}$$

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{a^2x^3}{3} + \frac{1}{2}abx^4 + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^2/x^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ax^2+bx^3\right)^2}{x^2} dx &= \int x^2(a+bx)^2 dx \\
&= \int \left(a^2x^2 + 2abx^3 + b^2x^4\right) dx \\
&= \frac{a^2x^3}{3} + \frac{1}{2}abx^4 + \frac{b^2x^5}{5}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^3}{3} + \frac{1}{2}abx^4 + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^2/x^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

IntegrateAlgebraic [A] time = 0.02, size = 30, normalized size = 1.00

$$\frac{a^2x^3}{3} + \frac{1}{2}abx^4 + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x^3)^2/x^2,x]
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

fricas [A] time = 0.38, size = 24, normalized size = 0.80

$$\frac{1}{5}\,b^2x^5 + \frac{1}{2}\,abx^4 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2/x^2,x, algorithm="fricas")
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

giac [A] time = 0.15, size = 24, normalized size = 0.80

$$\frac{1}{5}\,b^2x^5 + \frac{1}{2}\,abx^4 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2/x^2,x, algorithm="giac")
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

maple [A] time = 0.05, size = 25, normalized size = 0.83

$$\frac{1}{5}b^2x^5 + \frac{1}{2}ab\,x^4 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^2/x^2,x)
[Out] 1/3*a^2*x^3+1/2*a*b*x^4+1/5*b^2*x^5
```

maxima [A] time = 1.33, size = 24, normalized size = 0.80

$$\frac{1}{5}\,b^2x^5 + \frac{1}{2}\,abx^4 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2/x^2,x, algorithm="maxima")
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^3}{3} + \frac{a\,b\,x^4}{2} + \frac{b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^2/x^2,x)
[Out] (a^2*x^3)/3 + (b^2*x^5)/5 + (a*b*x^4)/2
```

sympy [A] time = 0.07, size = 24, normalized size = 0.80

$$\frac{a^2x^3}{3} + \frac{abx^4}{2} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**2/x**2,x)
[Out] a**2*x**3/3 + a*b*x**4/2 + b**2*x**5/5
```

3.134 $\int \frac{x^6}{ax^2+bx^3} \, dx$

Optimal. Leaf size=57

$$\frac{a^4 \log(a+bx)}{b^5} - \frac{a^3 x}{b^4} + \frac{a^2 x^2}{2b^3} - \frac{a x^3}{3b^2} + \frac{x^4}{4b}$$

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{a^2 x^2}{2b^3} - \frac{a^3 x}{b^4} + \frac{a^4 \log(a+bx)}{b^5} - \frac{a x^3}{3b^2} + \frac{x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x^2 + b*x^3),x]
```

```
[Out] -((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Log
[a + b*x])/b^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{ax^2+bx^3} \, dx &= \int \frac{x^4}{a+bx} \, dx \\
&= \int \left(-\frac{a^3}{b^4} + \frac{a^2 x}{b^3} - \frac{a x^2}{b^2} + \frac{x^3}{b} + \frac{a^4}{b^4 (a+bx)} \right) dx \\
&= -\frac{a^3 x}{b^4} + \frac{a^2 x^2}{2b^3} - \frac{a x^3}{3b^2} + \frac{x^4}{4b} + \frac{a^4 \log(a+bx)}{b^5}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 1.00

$$\frac{a^4 \log(a+bx)}{b^5} - \frac{a^3 x}{b^4} + \frac{a^2 x^2}{2b^3} - \frac{a x^3}{3b^2} + \frac{x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x^2 + b*x^3),x]
```

```
[Out] -((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Log
[a + b*x])/b^5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{ax^2+bx^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(a*x^2 + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[x^6/(a*x^2 + b*x^3), x]
```

fricas [A] time = 0.39, size = 52, normalized size = 0.91

$$\frac{3\,b^4x^4 - 4\,ab^3x^3 + 6\,a^2b^2x^2 - 12\,a^3bx + 12\,a^4\log\left(bx + a\right)}{12\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*x^4 - 4*a*b^3*x^3 + 6*a^2*b^2*x^2 - 12*a^3*b*x + 12*a^4*log(b*x
 + a))/b^5
```

giac [A] time = 0.15, size = 53, normalized size = 0.93

$$\frac{a^4\log\left(|bx + a|\right)}{b^5} + \frac{3\,b^3x^4 - 4\,ab^2x^3 + 6\,a^2bx^2 - 12\,a^3x}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] a^4*log(abs(b*x + a))/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 1
2*a^3*x)/b^4
```

maple [A] time = 0.04, size = 52, normalized size = 0.91

$$\frac{x^4}{4b} - \frac{a\,x^3}{3b^2} + \frac{a^2x^2}{2b^3} + \frac{a^4\ln\left(bx + a\right)}{b^5} - \frac{a^3x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a*x^2),x)
```

```
[Out] -a^3/b^4*x+1/2*a^2/b^3*x^2-1/3*a/b^2*x^3+1/4*x^4/b+a^4*ln(b*x+a)/b^5
```

maxima [A] time = 1.27, size = 52, normalized size = 0.91

$$\frac{a^4\log\left(bx + a\right)}{b^5} + \frac{3\,b^3x^4 - 4\,ab^2x^3 + 6\,a^2bx^2 - 12\,a^3x}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] a^4*log(b*x + a)/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 12*a^3
*x)/b^4
```

mupad [B] time = 5.09, size = 51, normalized size = 0.89

$$\frac{x^4}{4\,b} + \frac{a^4\,\ln\left(a + b\,x\right)}{b^5} - \frac{a\,x^3}{3\,b^2} - \frac{a^3\,x}{b^4} + \frac{a^2\,x^2}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x^2 + b*x^3),x)
```

```
[Out] x^4/(4*b) + (a^4*log(a + b*x))/b^5 - (a*x^3)/(3*b^2) - (a^3*x)/b^4 + (a^2*x
^2)/(2*b^3)
```

<u>**sympy**</u> [**A**] time = 0.14, size = 49, normalized size = 0.86

$$\frac{a^4 \log{(a + bx)}}{b^5} - \frac{a^3 x}{b^4} + \frac{a^2 x^2}{2b^3} - \frac{a x^3}{3b^2} + \frac{x^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a*x**2),x)
```

```
[Out] a**4*log(a + b*x)/b**5 - a**3*x/b**4 + a**2*x**2/(2*b**3) - a*x**3/(3*b**2)
 + x**4/(4*b)
```

3.135 $\int \frac{x^5}{ax^2+bx^3}\,dx$

Optimal. Leaf size=44

$$-\frac{a^3\log(a+bx)}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}$$

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{a^2x}{b^3}-\frac{a^3\log(a+bx)}{b^4}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x^2 + b*x^3),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{ax^2+bx^3}\,dx &= \int \frac{x^3}{a+bx}\,dx\\ &= \int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx\\ &= \frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}-\frac{a^3\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 44, normalized size = 1.00

$$-\frac{a^3\log(a+bx)}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x^2 + b*x^3),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{ax^2+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a*x^2 + b*x^3),x]
[Out] IntegrateAlgebraic[x^5/(a*x^2 + b*x^3), x]
```

fricas [A] time = 0.40, size = 41, normalized size = 0.93

$$\frac{2\,b^3x^3 - 3\,ab^2x^2 + 6\,a^2bx - 6\,a^3\log\left(bx + a\right)}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2),x, algorithm="fricas")
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))/b^4
```

giac [A] time = 0.14, size = 43, normalized size = 0.98

$$-\frac{a^3\log\left(|bx + a|\right)}{b^4} + \frac{2\,b^2x^3 - 3\,abx^2 + 6\,a^2x}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2),x, algorithm="giac")
[Out] -a^3*log(abs(b*x + a))/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

maple [A] time = 0.04, size = 41, normalized size = 0.93

$$\frac{x^3}{3b} - \frac{a\,x^2}{2b^2} - \frac{a^3\ln\left(bx + a\right)}{b^4} + \frac{a^2x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a*x^2),x)
[Out] a^2/b^3*x-1/2*a/b^2*x^2+1/3/b*x^3-a^3*ln(b*x+a)/b^4
```

maxima [A] time = 1.31, size = 42, normalized size = 0.95

$$-\frac{a^3\log\left(bx + a\right)}{b^4} + \frac{2\,b^2x^3 - 3\,abx^2 + 6\,a^2x}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2),x, algorithm="maxima")
[Out] -a^3*log(b*x + a)/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

mupad [B] time = 0.04, size = 40, normalized size = 0.91

$$\frac{x^3}{3\,b} - \frac{a^3\,\ln\left(a + b\,x\right)}{b^4} - \frac{a\,x^2}{2\,b^2} + \frac{a^2\,x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^2 + b*x^3),x)
[Out] x^3/(3*b) - (a^3*log(a + b*x))/b^4 - (a*x^2)/(2*b^2) + (a^2*x)/b^3
```

sympy [A] time = 0.13, size = 37, normalized size = 0.84

$$-\frac{a^3\log\left(a + bx\right)}{b^4} + \frac{a^2x}{b^3} - \frac{ax^2}{2b^2} + \frac{x^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a*x**2),x)
```

```
[Out] -a**3*log(a + b*x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3*b)
```

3.136 $\int \frac{x^4}{ax^2+bx^3}\,dx$

Optimal. Leaf size=31

$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x^2 + b*x^3),x]
```

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{ax^2+bx^3}\,dx &= \int \frac{x^2}{a+bx}\,dx\\ &= \int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx\\ &= -\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 31, normalized size = 1.00

$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x^2 + b*x^3),x]
```

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{ax^2+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a*x^2 + b*x^3),x]
[Out] IntegrateAlgebraic[x^4/(a*x^2 + b*x^3), x]
```

fricas [A] time = 0.39, size = 29, normalized size = 0.94

$$\frac{b^2x^2 - 2\,abx + 2\,a^2\log\left(bx + a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2),x, algorithm="fricas")
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3
```

giac [A] time = 0.19, size = 30, normalized size = 0.97

$$\frac{a^2\log\left(|bx + a|\right)}{b^3} + \frac{bx^2 - 2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2),x, algorithm="giac")
[Out] a^2*log(abs(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

maple [A] time = 0.04, size = 30, normalized size = 0.97

$$\frac{x^2}{2b} + \frac{a^2\ln\left(bx + a\right)}{b^3} - \frac{ax}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x^2),x)
[Out] -a/b^2*x+1/2/b*x^2+a^2*ln(b*x+a)/b^3
```

maxima [A] time = 1.37, size = 29, normalized size = 0.94

$$\frac{a^2\log\left(bx + a\right)}{b^3} + \frac{bx^2 - 2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2),x, algorithm="maxima")
[Out] a^2*log(b*x + a)/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

mupad [B] time = 0.04, size = 29, normalized size = 0.94

$$\frac{2\,a^2\,\ln\left(a + b\,x\right) + b^2\,x^2 - 2\,a\,b\,x}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^2 + b*x^3),x)
[Out] (2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3)
```

sympy [A] time = 0.12, size = 26, normalized size = 0.84

$$\frac{a^2\log\left(a + bx\right)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x**2),x)
[Out] a**2*log(a + b*x)/b**3 - a*x/b**2 + x**2/(2*b)
```

3.137 $\int \frac{x^3}{ax^2+bx^3} \, dx$

Optimal. Leaf size=18

$$\frac{x}{b} - \frac{a \log(a+bx)}{b^2}$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{x}{b} - \frac{a \log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x^2 + b*x^3),x]
[Out] x/b - (a*Log[a + b*x])/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{x^3}{ax^2+bx^3} \, dx &= \int \frac{x}{a+bx} \, dx \\ &= \int \left(\frac{1}{b} - \frac{a}{b(a+bx)}\right) dx \\ &= \frac{x}{b} - \frac{a \log(a+bx)}{b^2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{x}{b} - \frac{a \log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x^2 + b*x^3),x]
[Out] x/b - (a*Log[a + b*x])/b^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{ax^2+bx^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a*x^2 + b*x^3),x]
[Out] IntegrateAlgebraic[x^3/(a*x^2 + b*x^3), x]
```

fricas [A] time = 0.38, size = 17, normalized size = 0.94

$$\frac{bx - a\log\left(bx + a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2),x, algorithm="fricas")
[Out] (b*x - a*log(b*x + a))/b^2
```

giac [A] time = 0.15, size = 19, normalized size = 1.06

$$\frac{x}{b} - \frac{a\log\left(|bx + a|\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2),x, algorithm="giac")
[Out] x/b - a*log(abs(b*x + a))/b^2
```

maple [A] time = 0.05, size = 19, normalized size = 1.06

$$-\frac{a\ln\left(bx + a\right)}{b^2} + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a*x^2),x)
[Out] 1/b*x-a*ln(b*x+a)/b^2
```

maxima [A] time = 1.30, size = 18, normalized size = 1.00

$$\frac{x}{b} - \frac{a\log\left(bx + a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2),x, algorithm="maxima")
[Out] x/b - a*log(b*x + a)/b^2
```

mupad [B] time = 0.04, size = 18, normalized size = 1.00

$$-\frac{a\,\ln\left(a + b\,x\right) - b\,x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^3),x)
[Out] -(a*log(a + b*x) - b*x)/b^2
```

sympy [A] time = 0.11, size = 14, normalized size = 0.78

$$-\frac{a\log\left(a + bx\right)}{b^2} + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x**2),x)
[Out] -a*log(a + b*x)/b**2 + x/b
```

3.138 $\int \frac{x^2}{ax^2+bx^3}\,dx$

Optimal. Leaf size=10

$$\frac{\log(a+bx)}{b}$$

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 31}

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x^2 + b*x^3),x]
```

```
[Out] Log[a + b*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{ax^2+bx^3}\,dx &= \int \frac{1}{a+bx}\,dx\\ &= \frac{\log(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x^2 + b*x^3),x]
```

```
[Out] Log[a + b*x]/b
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{ax^2+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a*x^2 + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[x^2/(a*x^2 + b*x^3), x]
```

fricas [**A**] time = 0.38, size = 10, normalized size = 1.00

$$\frac{\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2),x, algorithm="fricas")
[Out] log(b*x + a)/b
```

giac [**A**] time = 0.15, size = 11, normalized size = 1.10

$$\frac{\log\left(|bx+a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2),x, algorithm="giac")
[Out] log(abs(b*x + a))/b
```

maple [**A**] time = 0.04, size = 11, normalized size = 1.10

$$\frac{\ln\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x^2),x)
[Out] 1/b*ln(b*x+a)
```

maxima [**A**] time = 1.32, size = 10, normalized size = 1.00

$$\frac{\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2),x, algorithm="maxima")
[Out] log(b*x + a)/b
```

mupad [**B**] time = 0.02, size = 10, normalized size = 1.00

$$\frac{\ln\left(a+b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2 + b*x^3),x)
[Out] log(a + b*x)/b
```

sympy [**A**] time = 0.07, size = 7, normalized size = 0.70

$$\frac{\log\left(a+bx\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x**2),x)
[Out] log(a + b*x)/b
```

3.139 $\int \frac{x}{ax^2+bx^3}\,dx$

Optimal. Leaf size=18

$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1584, 36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^2 + b*x^3),x]
```

```
[Out] Log[x]/a - Log[a + b*x]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{ax^2+bx^3}\,dx &= \int \frac{1}{x(a+bx)}\,dx\\ &= \frac{\int \frac{1}{x}\,dx}{a}-\frac{b\int \frac{1}{a+bx}\,dx}{a}\\ &= \frac{\log(x)}{a}-\frac{\log(a+bx)}{a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^2 + b*x^3),x]
```

```
[Out] Log[x]/a - Log[a + b*x]/a
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{ax^2 + bx^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a*x^2 + b*x^3),x]
[Out] IntegrateAlgebraic[x/(a*x^2 + b*x^3), x]
```

fricas [A] time = 0.38, size = 16, normalized size = 0.89

$$-\frac{\log(bx + a) - \log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2),x, algorithm="fricas")
[Out] -(log(b*x + a) - log(x))/a
```

giac [A] time = 0.16, size = 20, normalized size = 1.11

$$-\frac{\log(|bx + a|)}{a} + \frac{\log(|x|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2),x, algorithm="giac")
[Out] -log(abs(b*x + a))/a + log(abs(x))/a
```

maple [A] time = 0.05, size = 19, normalized size = 1.06

$$\frac{\ln(x)}{a} - \frac{\ln(bx + a)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x^2),x)
[Out] 1/a*ln(x)-ln(b*x+a)/a
```

maxima [A] time = 1.30, size = 18, normalized size = 1.00

$$-\frac{\log(bx + a)}{a} + \frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2),x, algorithm="maxima")
[Out] -log(b*x + a)/a + log(x)/a
```

mupad [B] time = 5.12, size = 15, normalized size = 0.83

$$-\frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a} + 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^2 + b*x^3),x)
```

```
[Out] -(2*atanh((2*b*x)/a + 1))/a
```

sympy [A] time = 0.16, size = 10, normalized size = 0.56

$$\frac{\log(x) - \log\left(\frac{a}{b} + x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x**2),x)
```

```
[Out] (log(x) - log(a/b + x))/a
```

3.140 $\int \frac{1}{ax^2+bx^3} \, dx$

Optimal. Leaf size=28

$$-\frac{b \log(x)}{a^2} + \frac{b \log(a+bx)}{a^2} - \frac{1}{ax}$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 44}

$$-\frac{b \log(x)}{a^2} + \frac{b \log(a+bx)}{a^2} - \frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(-1),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{ax^2+bx^3} \, dx &= \int \frac{1}{x^2(a+bx)} \, dx \\
&= \int \left(\frac{1}{ax^2} - \frac{b}{a^2x} + \frac{b^2}{a^2(a+bx)}\right) dx \\
&= -\frac{1}{ax} - \frac{b \log(x)}{a^2} + \frac{b \log(a+bx)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$-\frac{b \log(x)}{a^2} + \frac{b \log(a+bx)}{a^2} - \frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(-1),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ax^2+bx^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x^3)^(-1),x]
[Out] IntegrateAlgebraic[(a*x^2 + b*x^3)^(-1), x]
```

fricas [A] time = 0.40, size = 26, normalized size = 0.93

$$\frac{bx\log(bx+a)-bx\log(x)-a}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2),x, algorithm="fricas")
[Out] (b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

giac [A] time = 0.15, size = 30, normalized size = 1.07

$$\frac{b\log(|bx+a|)}{a^2}-\frac{b\log(|x|)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2),x, algorithm="giac")
[Out] b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

maple [A] time = 0.05, size = 29, normalized size = 1.04

$$-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x^2),x)
[Out] -1/a/x-1/a^2*b*ln(x)+b*ln(b*x+a)/a^2
```

maxima [A] time = 1.36, size = 28, normalized size = 1.00

$$\frac{b\log(bx+a)}{a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2),x, algorithm="maxima")
[Out] b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

mupad [B] time = 0.05, size = 25, normalized size = 0.89

$$\frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2}-\frac{1}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^3),x)
[Out] (2*b*atanh((2*b*x)/a + 1))/a^2 - 1/(a*x)
```

sympy [A] time = 0.20, size = 19, normalized size = 0.68

$$-\frac{1}{ax}+\frac{b\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x**2),x)
[Out] -1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

3.141 $\int \frac{1}{x(ax^2+bx^3)} dx$

Optimal. Leaf size=42

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 44}

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x^2 + b*x^3)),x]
```

```
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(ax^2+bx^3\right)}\,dx &= \int \frac{1}{x^3(a+bx)}\,dx\\ &= \int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx\\ &= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 1.00

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x^2 + b*x^3)),x]
```

```
[Out] -1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(ax^2+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a*x^2 + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a*x^2 + b*x^3)), x]
```

fricas [A] time = 0.39, size = 41, normalized size = 0.98

$$-\frac{2\,b^2x^2\log\left(bx+a\right)-2\,b^2x^2\log(x)-2\,abx+a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

giac [A] time = 0.15, size = 45, normalized size = 1.07

$$-\frac{b^2\log\left(|bx+a|\right)}{a^3}+\frac{b^2\log\left(|x|\right)}{a^3}+\frac{2\,abx-a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] -b^2*log(abs(b*x + a))/a^3 + b^2*log(abs(x))/a^3 + 1/2*(2*a*b*x - a^2)/(a^3
*x^2)
```

maple [A] time = 0.05, size = 41, normalized size = 0.98

$$\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln\left(bx+a\right)}{a^3}+\frac{b}{a^2x}-\frac{1}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x^2),x)
```

```
[Out] -1/2/a/x^2+1/a^2*b/x+1/a^3*b^2*ln(x)-b^2*ln(b*x+a)/a^3
```

maxima [A] time = 1.37, size = 40, normalized size = 0.95

$$-\frac{b^2\log\left(bx+a\right)}{a^3}+\frac{b^2\log(x)}{a^3}+\frac{2\,bx-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] -b^2*log(b*x + a)/a^3 + b^2*log(x)/a^3 + 1/2*(2*b*x - a)/(a^2*x^2)
```

mupad [B] time = 0.06, size = 38, normalized size = 0.90

$$-\frac{\frac{a^2}{2}-a\,b\,x}{a^3\,x^2}-\frac{2\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^2 + b*x^3)),x)
```

```
[Out] - (a^2/2 - a*b*x)/(a^3*x^2) - (2*b^2*atanh((2*b*x)/a + 1))/a^3
```

sympy [A] time = 0.21, size = 31, normalized size = 0.74

$$\frac{-a+2bx}{2a^2x^2}+\frac{b^2\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x**2),x)
```

```
[Out] (-a + 2*b*x)/(2*a**2*x**2) + b**2*(log(x) - log(a/b + x))/a**3
```

3.142 $\int \frac{1}{x^2(ax^2+bx^3)}\,dx$

Optimal. Leaf size=56

$$-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}-\frac{b^2}{a^3x}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 44}

$$-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x^2 + b*x^3)),x]
```

```
[Out] -1/(3*a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log[a
+ b*x])/a^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(ax^2+bx^3\right)}\,dx&=\int \frac{1}{x^4(a+bx)}\,dx\\&=\int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx\\&=-\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 1.00

$$-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}-\frac{b^2}{a^3x}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x^2 + b*x^3)),x]
```

```
[Out] -1/3*1/(a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log[
a + b*x])/a^4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a*x^2 + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a*x^2 + b*x^3)), x]
```

fricas [A] time = 0.38, size = 54, normalized size = 0.96

$$\frac{6\,b^3x^3\log\left(bx+a\right)-6\,b^3x^3\log(x)-6\,ab^2x^2+3\,a^2bx-2\,a^3}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log(b*x + a) - 6*b^3*x^3*log(x) - 6*a*b^2*x^2 + 3*a^2*b*x -
2*a^3)/(a^4*x^3)
```

giac [A] time = 0.16, size = 56, normalized size = 1.00

$$\frac{b^3\log\left(|bx+a|\right)}{a^4}-\frac{b^3\log\left(|x|\right)}{a^4}-\frac{6\,ab^2x^2-3\,a^2bx+2\,a^3}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] b^3*log(abs(b*x + a))/a^4 - b^3*log(abs(x))/a^4 - 1/6*(6*a*b^2*x^2 - 3*a^2*
b*x + 2*a^3)/(a^4*x^3)
```

maple [A] time = 0.05, size = 53, normalized size = 0.95

$$-\frac{b^3\ln(x)}{a^4}+\frac{b^3\ln\left(bx+a\right)}{a^4}-\frac{b^2}{a^3x}+\frac{b}{2a^2x^2}-\frac{1}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x^2),x)
```

```
[Out] -1/3/a/x^3+1/2/a^2*b/x^2-1/a^3*b^2/x-1/a^4*b^3*ln(x)+b^3*ln(b*x+a)/a^4
```

maxima [A] time = 1.33, size = 51, normalized size = 0.91

$$\frac{b^3\log\left(bx+a\right)}{a^4}-\frac{b^3\log(x)}{a^4}-\frac{6\,b^2x^2-3\,abx+2\,a^2}{6\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] b^3*log(b*x + a)/a^4 - b^3*log(x)/a^4 - 1/6*(6*b^2*x^2 - 3*a*b*x + 2*a^2)/(
a^3*x^3)
```

mupad [B] time = 0.06, size = 48, normalized size = 0.86

$$\frac{2\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^4}-\frac{\frac{a^3}{3}-\frac{a^2\,b\,x}{2}+a\,b^2\,x^2}{a^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x^2 + b*x^3)),x)
```

```
[Out] (2*b^3*atanh((2*b*x)/a + 1))/a^4 - (a^3/3 + a*b^2*x^2 - (a^2*b*x)/2)/(a^4*x
^3)
```

sympy [A] time = 0.24, size = 44, normalized size = 0.79

$$\frac{-2a^2 + 3abx - 6b^2x^2}{6a^3x^3} + \frac{b^3\left(-\log\left(x\right) + \log\left(\frac{a}{b} + x\right)\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x**2),x)
```

```
[Out] (-2*a**2 + 3*a*b*x - 6*b**2*x**2)/(6*a**3*x**3) + b**3*(-log(x) + log(a/b +
 x))/a**4
```

3.143 $\int \frac{x^8}{\left(ax^2+bx^3\right)^2} dx$

Optimal. Leaf size=58

$$-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}+\frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}$$

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$-\frac{a^4}{b^5(a+bx)}+\frac{3a^2x}{b^4}-\frac{4a^3\log(a+bx)}{b^5}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (3*a^2*x)/b^4 - (a*x^2)/b^3 + x^3/(3*b^2) - a^4/(b^5*(a + b*x)) - (4*a^3*Lo
g[a + b*x])/b^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\left(ax^2+bx^3\right)^2} dx &= \int \frac{x^4}{(a+bx)^2} dx\\ &= \int \left(\frac{3a^2}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{b^2}+\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}\right) dx\\ &= \frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.93

$$\frac{-\frac{3a^4}{a+bx}-12a^3\log(a+bx)+9a^2bx-3ab^2x^2+b^3x^3}{3b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (9*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 - (3*a^4)/(a + b*x) - 12*a^3*Log[a + b*x
])/(3*b^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(ax^2 + bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(a*x^2 + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[x^8/(a*x^2 + b*x^3)^2, x]
```

fricas [A] time = 0.38, size = 73, normalized size = 1.26

$$\frac{b^4x^4 - 2\,ab^3x^3 + 6\,a^2b^2x^2 + 9\,a^3bx - 3\,a^4 - 12\left(a^3bx + a^4\right)\log\left(bx + a\right)}{3\left(b^6x + ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b*
x + a^4)*log(b*x + a))/(b^6*x + a*b^5)
```

giac [A] time = 0.17, size = 62, normalized size = 1.07

$$-\frac{4\,a^3\log\left(|bx + a|\right)}{b^5} - \frac{a^4}{(bx + a)b^5} + \frac{b^4x^3 - 3\,ab^3x^2 + 9\,a^2b^2x}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -4*a^3*log(abs(b*x + a))/b^5 - a^4/((b*x + a)*b^5) + 1/3*(b^4*x^3 - 3*a*b^3
*x^2 + 9*a^2*b^2*x)/b^6
```

maple [A] time = 0.05, size = 57, normalized size = 0.98

$$\frac{x^3}{3b^2} - \frac{a\,x^2}{b^3} - \frac{a^4}{(bx + a)\,b^5} - \frac{4a^3\ln\left(bx + a\right)}{b^5} + \frac{3a^2x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a*x^2)^2,x)
```

```
[Out] 3*a^2/b^4*x-a/b^3*x^2+1/3/b^2*x^3-a^4/b^5/(b*x+a)-4*a^3*ln(b*x+a)/b^5
```

maxima [A] time = 1.31, size = 59, normalized size = 1.02

$$-\frac{a^4}{b^6x + ab^5} - \frac{4\,a^3\log\left(bx + a\right)}{b^5} + \frac{b^2x^3 - 3\,abx^2 + 9\,a^2x}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] -a^4/(b^6*x + a*b^5) - 4*a^3*log(b*x + a)/b^5 + 1/3*(b^2*x^3 - 3*a*b*x^2 +
9*a^2*x)/b^4
```

mupad [B] time = 0.04, size = 62, normalized size = 1.07

$$\frac{x^3}{3\,b^2} - \frac{4\,a^3\,\ln\left(a + b\,x\right)}{b^5} - \frac{a\,x^2}{b^3} + \frac{3\,a^2\,x}{b^4} - \frac{a^4}{b\,\left(x\,b^5 + a\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a*x^2 + b*x^3)^2,x)
```

```
[Out] x^3/(3*b^2) - (4*a^3*log(a + b*x))/b^5 - (a*x^2)/b^3 + (3*a^2*x)/b^4 - a^4/
(b*(a*b^4 + b^5*x))
```

sympy [**A**] time = 0.21, size = 54, normalized size = 0.93

$$-\frac{a^4}{ab^5 + b^6x} - \frac{4a^3 \log{(a + bx)}}{b^5} + \frac{3a^2x}{b^4} - \frac{ax^2}{b^3} + \frac{x^3}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a*x**2)**2,x)
```

```
[Out] -a**4/(a*b**5 + b**6*x) - 4*a**3*log(a + b*x)/b**5 + 3*a**2*x/b**4 - a*x**2
/b**3 + x**3/(3*b**2)
```

3.144 $\int \frac{x^7}{\left(ax^2+bx^3\right)^2} \, dx$

Optimal. Leaf size=46

$$\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}$$

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (-2*a*x)/b^3 + x^2/(2*b^2) + a^3/(b^4*(a + b*x)) + (3*a^2*Log[a + b*x])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\left(ax^2+bx^3\right)^2} \, dx &= \int \frac{x^3}{(a+bx)^2} \, dx \\
&= \int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right) dx \\
&= -\frac{2ax}{b^3}+\frac{x^2}{2b^2}+\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.93

$$\frac{\frac{2a^3}{a+bx}+6a^2\log(a+bx)-4abx+b^2x^2}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (-4*a*b*x + b^2*x^2 + (2*a^3)/(a + b*x) + 6*a^2*Log[a + b*x])/(2*b^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(ax^2+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(a*x^2 + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[x^7/(a*x^2 + b*x^3)^2, x]
```

fricas [A] time = 0.39, size = 62, normalized size = 1.35

$$\frac{b^3x^3-3\,ab^2x^2-4\,a^2bx+2\,a^3+6\left(a^2bx+a^3\right)\log\left(bx+a\right)}{2\left(b^5x+ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
+ a))/(b^5*x + a*b^4)
```

giac [A] time = 0.16, size = 48, normalized size = 1.04

$$\frac{3\,a^2\log\left(|bx+a|\right)}{b^4}+\frac{a^3}{(bx+a)b^4}+\frac{b^2x^2-4\,abx}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 3*a^2*log(abs(b*x + a))/b^4 + a^3/((b*x + a)*b^4) + 1/2*(b^2*x^2 - 4*a*b*x)
/b^4
```

maple [A] time = 0.05, size = 45, normalized size = 0.98

$$\frac{x^2}{2b^2}+\frac{a^3}{\left(bx+a\right)b^4}+\frac{3a^2\ln\left(bx+a\right)}{b^4}-\frac{2ax}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a*x^2)^2,x)
```

```
[Out] -2*a/b^3*x+1/2/b^2*x^2+a^3/b^4/(b*x+a)+3*a^2*ln(b*x+a)/b^4
```

maxima [A] time = 1.32, size = 47, normalized size = 1.02

$$\frac{a^3}{b^5x+ab^4}+\frac{3\,a^2\log\left(bx+a\right)}{b^4}+\frac{bx^2-4\,ax}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] a^3/(b^5*x + a*b^4) + 3*a^2*log(b*x + a)/b^4 + 1/2*(b*x^2 - 4*a*x)/b^3
```

mupad [B] time = 0.05, size = 50, normalized size = 1.09

$$\frac{x^2}{2\,b^2}+\frac{3\,a^2\,\ln\left(a+b\,x\right)}{b^4}+\frac{a^3}{b\,\left(x\,b^4+a\,b^3\right)}-\frac{2\,a\,x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a*x^2 + b*x^3)^2,x)
```

```
[Out] x^2/(2*b^2) + (3*a^2*log(a + b*x))/b^4 + a^3/(b*(a*b^3 + b^4*x)) - (2*a*x)/
b^3
```

sympy [A] time = 0.21, size = 44, normalized size = 0.96

$$\frac{a^3}{ab^4 + b^5x} + \frac{3a^2 \log{(a + bx)}}{b^4} - \frac{2ax}{b^3} + \frac{x^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a*x**2)**2,x)
```

```
[Out] a**3/(a*b**4 + b**5*x) + 3*a**2*log(a + b*x)/b**4 - 2*a*x/b**3 + x**2/(2*b*
*2)
```

3.145 $\int \frac{x^6}{\left(ax^2+bx^3\right)^2}\,dx$

Optimal. Leaf size=33

$$-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}+\frac{x}{b^2}$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x^2 + b*x^3)^2,x]
```

```
[Out] x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*Log[a + b*x])/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{\left(ax^2+bx^3\right)^2}\,dx &= \int \frac{x^2}{(a+bx)^2}\,dx\\ &= \int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx\\ &= \frac{x}{b^2}-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.88

$$\frac{-\frac{a^2}{a+bx}-2a\log(a+bx)+bx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (b*x - a^2/(a + b*x) - 2*a*Log[a + b*x])/b^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(ax^2 + bx^3)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(a*x^2 + b*x^3)^2,x]
[Out] IntegrateAlgebraic[x^6/(a*x^2 + b*x^3)^2, x]
```

fricas [A] time = 0.39, size = 47, normalized size = 1.42

$$\frac{b^2x^2 + abx - a^2 - 2\left(abx + a^2\right)\log\left(bx + a\right)}{b^4x + ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2)^2,x, algorithm="fricas")
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))/(b^4*x + a*b^3)
```

giac [A] time = 0.15, size = 34, normalized size = 1.03

$$\frac{x}{b^2} - \frac{2\,a\log\left(|bx + a|\right)}{b^3} - \frac{a^2}{(bx + a)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] x/b^2 - 2*a*log(abs(b*x + a))/b^3 - a^2/((b*x + a)*b^3)
```

maple [A] time = 0.05, size = 34, normalized size = 1.03

$$-\frac{a^2}{(bx + a)\,b^3} - \frac{2a\ln\left(bx + a\right)}{b^3} + \frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a*x^2)^2,x)
[Out] 1/b^2*x-a^2/b^3/(b*x+a)-2*a*ln(b*x+a)/b^3
```

maxima [A] time = 1.28, size = 36, normalized size = 1.09

$$-\frac{a^2}{b^4x + ab^3} + \frac{x}{b^2} - \frac{2\,a\log\left(bx + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] -a^2/(b^4*x + a*b^3) + x/b^2 - 2*a*log(b*x + a)/b^3
```

mupad [B] time = 0.04, size = 36, normalized size = 1.09

$$\frac{x}{b^2} - \frac{a^2}{x\,b^4 + a\,b^3} - \frac{2\,a\,\ln\left(a + b\,x\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x^2 + b*x^3)^2,x)
```

```
[Out] x/b^2 - a^2/(a*b^3 + b^4*x) - (2*a*log(a + b*x))/b^3
```

sympy [A] time = 0.18, size = 31, normalized size = 0.94

$$-\frac{a^2}{ab^3 + b^4x} - \frac{2a\log(a + bx)}{b^3} + \frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a*x**2)**2,x)
```

```
[Out] -a**2/(a*b**3 + b**4*x) - 2*a*log(a + b*x)/b**3 + x/b**2
```

3.146 $\int \frac{x^5}{\left(ax^2+bx^3\right)^2} dx$

Optimal. Leaf size=23

$$\frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x^2 + b*x^3)^2,x]
```

```
[Out] a/(b^2*(a + b*x)) + Log[a + b*x]/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(ax^2+bx^3\right)^2} dx &= \int \frac{x}{(a+bx)^2} dx \\
&= \int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right) dx \\
&= \frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.87

$$\frac{\frac{a}{a+bx}+\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a/(a + b*x) + Log[a + b*x])/b^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(ax^2+bx^3\right)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a*x^2 + b*x^3)^2,x]
[Out] IntegrateAlgebraic[x^5/(a*x^2 + b*x^3)^2, x]
```

fricas [A] time = 0.38, size = 28, normalized size = 1.22

$$\frac{(bx+a)\log(bx+a)+a}{b^3x+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2)^2,x, algorithm="fricas")
[Out] ((b*x + a)*log(b*x + a) + a)/(b^3*x + a*b^2)
```

giac [A] time = 0.15, size = 24, normalized size = 1.04

$$\frac{\log(|bx+a|)}{b^2}+\frac{a}{(bx+a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] log(abs(b*x + a))/b^2 + a/((b*x + a)*b^2)
```

maple [A] time = 0.05, size = 24, normalized size = 1.04

$$\frac{a}{(bx+a)\,b^2}+\frac{\ln(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a*x^2)^2,x)
[Out] a/b^2/(b*x+a)+ln(b*x+a)/b^2
```

maxima [A] time = 1.33, size = 26, normalized size = 1.13

$$\frac{a}{b^3x+ab^2}+\frac{\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] a/(b^3*x + a*b^2) + log(b*x + a)/b^2
```

mupad [B] time = 0.04, size = 23, normalized size = 1.00

$$\frac{\ln(a+b\,x)}{b^2}+\frac{a}{b^2\,(a+b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^2 + b*x^3)^2,x)
[Out] log(a + b*x)/b^2 + a/(b^2*(a + b*x))
```

sympy [A] time = 0.14, size = 20, normalized size = 0.87

$$\frac{a}{ab^2+b^3x}+\frac{\log(a+bx)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a*x**2)**2,x)
[Out] a/(a*b**2 + b**3*x) + log(a + b*x)/b**2
```

3.147 $\int \frac{x^4}{(ax^2+bx^3)^2} \, dx$

Optimal. Leaf size=12

$$-\frac{1}{b(a+bx)}$$

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 32}

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x^2 + b*x^3)^2,x]
```

```
[Out] -(1/(b*(a + b*x)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{x^4}{(ax^2+bx^3)^2} \, dx &= \int \frac{1}{(a+bx)^2} \, dx \\ &= -\frac{1}{b(a+bx)} \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x^2 + b*x^3)^2,x]
```

```
[Out] -(1/(b*(a + b*x)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(ax^2+bx^3)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a*x^2 + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[x^4/(a*x^2 + b*x^3)^2, x]
```

fricas [A] time = 0.38, size = 13, normalized size = 1.08

$$-\frac{1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^2,x, algorithm="fricas")
[Out] -1/(b^2*x + a*b)
```

giac [A] time = 0.15, size = 12, normalized size = 1.00

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] -1/((b*x + a)*b)
```

maple [A] time = 0.05, size = 13, normalized size = 1.08

$$-\frac{1}{(bx+a)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x^2)^2,x)
[Out] -1/b/(b*x+a)
```

maxima [A] time = 1.30, size = 13, normalized size = 1.08

$$-\frac{1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] -1/(b^2*x + a*b)
```

mupad [B] time = 5.17, size = 12, normalized size = 1.00

$$-\frac{1}{b\,(a+b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^2 + b*x^3)^2,x)
[Out] -1/(b*(a + b*x))
```

sympy [A] time = 0.14, size = 10, normalized size = 0.83

$$-\frac{1}{ab+b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x**2)**2,x)
[Out] -1/(a*b + b**2*x)
```

3.148 $\int \frac{x^3}{\left(ax^2+bx^3\right)^2} \, dx$

Optimal. Leaf size=29

$$-\frac{\log(a+bx)}{a^2}+\frac{\log(x)}{a^2}+\frac{1}{a(a+bx)}$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 44}

$$-\frac{\log(a+bx)}{a^2}+\frac{\log(x)}{a^2}+\frac{1}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x^2 + b*x^3)^2,x]
```

```
[Out] 1/(a*(a + b*x)) + Log[x]/a^2 - Log[a + b*x]/a^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(ax^2+bx^3\right)^2} \, dx &= \int \frac{1}{x(a+bx)^2} \, dx \\
&= \int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right) dx \\
&= \frac{1}{a(a+bx)}+\frac{\log(x)}{a^2}-\frac{\log(a+bx)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.83

$$\frac{\frac{a}{a+bx}-\log(a+bx)+\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a/(a + b*x) + Log[x] - Log[a + b*x])/a^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(ax^2+bx^3\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a*x^2 + b*x^3)^2,x]
[Out] IntegrateAlgebraic[x^3/(a*x^2 + b*x^3)^2, x]
```

fricas [A] time = 0.38, size = 39, normalized size = 1.34

$$-\frac{(bx+a)\log(bx+a)-(bx+a)\log(x)-a}{a^2bx+a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^2,x, algorithm="fricas")
[Out] -((b*x + a)*log(b*x + a) - (b*x + a)*log(x) - a)/(a^2*b*x + a^3)
```

giac [A] time = 0.15, size = 31, normalized size = 1.07

$$-\frac{\log(|bx+a|)}{a^2}+\frac{\log(|x|)}{a^2}+\frac{1}{(bx+a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] -log(abs(b*x + a))/a^2 + log(abs(x))/a^2 + 1/((b*x + a)*a)
```

maple [A] time = 0.06, size = 30, normalized size = 1.03

$$\frac{1}{(bx+a)\,a}+\frac{\ln(x)}{a^2}-\frac{\ln(bx+a)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a*x^2)^2,x)
[Out] 1/a/(b*x+a)+1/a^2*ln(x)-ln(b*x+a)/a^2
```

maxima [A] time = 1.29, size = 28, normalized size = 0.97

$$\frac{1}{abx+a^2}-\frac{\log(bx+a)}{a^2}+\frac{\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] 1/(a*b*x + a^2) - log(b*x + a)/a^2 + log(x)/a^2
```

mupad [B] time = 0.04, size = 26, normalized size = 0.90

$$\frac{1}{a^2+b\,x\,a}-\frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^3)^2,x)
[Out] 1/(a^2 + a*b*x) - (2*atanh((2*b*x)/a + 1))/a^2
```

sympy [A] time = 0.22, size = 22, normalized size = 0.76

$$\frac{1}{a^2+abx}+\frac{\log(x)-\log\left(\frac{a}{b}+x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x**2)**2,x)
```

```
[Out] 1/(a**2 + a*b*x) + (log(x) - log(a/b + x))/a**2
```

3.149 $\int \frac{x^2}{\left(ax^2+bx^3\right)^2} \, dx$

Optimal. Leaf size=42

$$-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{b}{a^2(a+bx)}-\frac{1}{a^2x}$$

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 44}

$$-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x^2 + b*x^3)^2,x]
```

```
[Out] -(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a^
3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(ax^2+bx^3\right)^2} \, dx &= \int \frac{1}{x^2(a+bx)^2} \, dx \\
&= \int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right) \, dx \\
&= -\frac{1}{a^2x}-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 35, normalized size = 0.83

$$-\frac{a\left(\frac{b}{a+bx}+\frac{1}{x}\right)-2b\log(a+bx)+2b\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x^2 + b*x^3)^2,x]
```

```
[Out] -((a*(x^(-1) + b/(a + b*x)) + 2*b*Log[x] - 2*b*Log[a + b*x])/a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax^2+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a*x^2 + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[x^2/(a*x^2 + b*x^3)^2, x]
```

fricas [A] time = 0.40, size = 63, normalized size = 1.50

$$-\frac{2\,abx+a^2-2\left(b^2x^2+abx\right)\log\left(bx+a\right)+2\left(b^2x^2+abx\right)\log(x)}{a^3bx^2+a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log(b*x + a) + 2*(b^2*x^2 + a*b*x)*lo
g(x))/(a^3*b*x^2 + a^4*x)
```

giac [A] time = 0.15, size = 45, normalized size = 1.07

$$\frac{2\,b\log\left(|bx+a|\right)}{a^3}-\frac{2\,b\log\left(|x|\right)}{a^3}-\frac{2\,bx+a}{\left(bx^2+ax\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 2*b*log(abs(b*x + a))/a^3 - 2*b*log(abs(x))/a^3 - (2*b*x + a)/((b*x^2 + a*x
)*a^2)
```

maple [A] time = 0.05, size = 43, normalized size = 1.02

$$-\frac{b}{\left(bx+a\right)a^2}-\frac{2b\ln(x)}{a^3}+\frac{2b\ln\left(bx+a\right)}{a^3}-\frac{1}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x^2)^2,x)
```

```
[Out] -1/a^2/x-b/a^2/(b*x+a)-2/a^3*b*ln(x)+2*b*ln(b*x+a)/a^3
```

maxima [A] time = 1.31, size = 45, normalized size = 1.07

$$-\frac{2\,bx+a}{a^2bx^2+a^3x}+\frac{2\,b\log\left(bx+a\right)}{a^3}-\frac{2\,b\log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] -(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*log(b*x + a)/a^3 - 2*b*log(x)/a^3
```

mupad [B] time = 5.34, size = 41, normalized size = 0.98

$$\frac{4\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^3}-\frac{\frac{1}{a}+\frac{2\,b\,x}{a^2}}{b\,x^2+a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2 + b*x^3)^2,x)
```

```
[Out] (4*b*atanh((2*b*x)/a + 1))/a^3 - (1/a + (2*b*x)/a^2)/(a*x + b*x^2)
```

sympy [A] time = 0.27, size = 37, normalized size = 0.88

$$\frac{-a-2bx}{a^3x+a^2bx^2}+\frac{2b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x**2)**2,x)
```

```
[Out] (-a - 2*b*x)/(a**3*x + a**2*b*x**2) + 2*b*(-log(x) + log(a/b + x))/a**3
```

3.150 $\int \frac{x}{\left(ax^2+bx^3\right)^2}\,dx$

Optimal. Leaf size=58

$$\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}+\frac{b^2}{a^3(a+bx)}+\frac{2b}{a^3x}-\frac{1}{2a^2x^2}$$

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 44}

$$\frac{b^2}{a^3(a+bx)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}+\frac{2b}{a^3x}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^2 + b*x^3)^2,x]
```

```
[Out] -1/(2*a^2*x^2) + (2*b)/(a^3*x) + b^2/(a^3*(a + b*x)) + (3*b^2*Log[x])/a^4 -
 (3*b^2*Log[a + b*x])/a^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(ax^2+bx^3\right)^2}\,dx&=\int \frac{1}{x^3(a+bx)^2}\,dx\\&=\int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx\\&=-\frac{1}{2a^2x^2}+\frac{2b}{a^3x}+\frac{b^2}{a^3(a+bx)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.09, size = 53, normalized size = 0.91

$$\frac{a\left(\frac{2b^2}{a+bx}-\frac{a}{x^2}+\frac{4b}{x}\right)-6b^2\log(a+bx)+6b^2\log(x)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a*(-(a/x^2) + (4*b)/x + (2*b^2)/(a + b*x)) + 6*b^2*Log[x] - 6*b^2*Log[a +
b*x])/(2*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(ax^2 + bx^3\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a*x^2 + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[x/(a*x^2 + b*x^3)^2, x]
```

fricas [A] time = 0.40, size = 86, normalized size = 1.48

$$\frac{6\,ab^2x^2 + 3\,a^2bx - a^3 - 6\left(b^3x^3 + ab^2x^2\right)\log\left(bx + a\right) + 6\left(b^3x^3 + ab^2x^2\right)\log(x)}{2\left(a^4bx^3 + a^5x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 - 6*(b^3*x^3 + a*b^2*x^2)*log(b*x + a) +
 6*(b^3*x^3 + a*b^2*x^2)*log(x))/(a^4*b*x^3 + a^5*x^2)
```

giac [A] time = 0.15, size = 64, normalized size = 1.10

$$-\frac{3\,b^2\log\left(|bx + a|\right)}{a^4} + \frac{3\,b^2\log\left(|x|\right)}{a^4} + \frac{6\,ab^2x^2 + 3\,a^2bx - a^3}{2\left(bx + a\right)a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -3*b^2*log(abs(b*x + a))/a^4 + 3*b^2*log(abs(x))/a^4 + 1/2*(6*a*b^2*x^2 + 3
*a^2*b*x - a^3)/((b*x + a)*a^4*x^2)
```

maple [A] time = 0.06, size = 57, normalized size = 0.98

$$\frac{b^2}{\left(bx + a\right)a^3} + \frac{3b^2\ln(x)}{a^4} - \frac{3b^2\ln\left(bx + a\right)}{a^4} + \frac{2b}{a^3x} - \frac{1}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x^2)^2,x)
```

```
[Out] -1/2/a^2/x^2+2/a^3*b/x+b^2/a^3/(b*x+a)+3/a^4*b^2*ln(x)-3*b^2*ln(b*x+a)/a^4
```

maxima [A] time = 1.32, size = 64, normalized size = 1.10

$$\frac{6\,b^2x^2 + 3\,abx - a^2}{2\left(a^3bx^3 + a^4x^2\right)} - \frac{3\,b^2\log\left(bx + a\right)}{a^4} + \frac{3\,b^2\log(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(6*b^2*x^2 + 3*a*b*x - a^2)/(a^3*b*x^3 + a^4*x^2) - 3*b^2*log(b*x + a)/
a^4 + 3*b^2*log(x)/a^4
```

mupad [B] time = 5.31, size = 57, normalized size = 0.98

$$\frac{\frac{3\,b^2\,x^2}{a^3} - \frac{1}{2\,a} + \frac{3\,b\,x}{2\,a^2}}{b\,x^3 + a\,x^2} - \frac{6\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a} + 1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^2 + b*x^3)^2,x)
```

```
[Out] ((3*b^2*x^2)/a^3 - 1/(2*a) + (3*b*x)/(2*a^2))/(a*x^2 + b*x^3) - (6*b^2*atan
h((2*b*x)/a + 1))/a^4
```

sympy [A] time = 0.31, size = 54, normalized size = 0.93

$$\frac{-a^2 + 3abx + 6b^2x^2}{2a^4x^2 + 2a^3bx^3} + \frac{3b^2\left(\log\left(x\right) - \log\left(\frac{a}{b} + x\right)\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x**2)**2,x)
```

```
[Out] (-a**2 + 3*a*b*x + 6*b**2*x**2)/(2*a**4*x**2 + 2*a**3*b*x**3) + 3*b**2*(log
(x) - log(a/b + x))/a**4
```

3.151 $\int \frac{1}{\left(ax^2+bx^3\right)^2}\,dx$

Optimal. Leaf size=69

$$-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}-\frac{b^3}{a^4(a+bx)}-\frac{3b^2}{a^4x}+\frac{b}{a^3x^2}-\frac{1}{3a^2x^3}$$

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 44}

$$-\frac{b^3}{a^4(a+bx)}-\frac{3b^2}{a^4x}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}+\frac{b}{a^3x^2}-\frac{1}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(-2),x]
```

```
[Out] -1/(3*a^2*x^3) + b/(a^3*x^2) - (3*b^2)/(a^4*x) - b^3/(a^4*(a + b*x)) - (4*b
^3*Log[x])/a^5 + (4*b^3*Log[a + b*x])/a^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(ax^2+bx^3\right)^2}\,dx &= \int \frac{1}{x^4(a+bx)^2}\,dx\\
&= \int \left(\frac{1}{a^2x^4}-\frac{2b}{a^3x^3}+\frac{3b^2}{a^4x^2}-\frac{4b^3}{a^5x}+\frac{b^4}{a^4(a+bx)^2}+\frac{4b^4}{a^5(a+bx)}\right)dx\\
&= -\frac{1}{3a^2x^3}+\frac{b}{a^3x^2}-\frac{3b^2}{a^4x}-\frac{b^3}{a^4(a+bx)}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 66, normalized size = 0.96

$$-\frac{\frac{a\left(a^3-2a^2bx+6ab^2x^2+12b^3x^3\right)}{x^3(a+bx)}-12b^3\log(a+bx)+12b^3\log(x)}{3a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(-2),x]
```

```
[Out] -1/3*((a*(a^3 - 2*a^2*b*x + 6*a*b^2*x^2 + 12*b^3*x^3))/(x^3*(a + b*x)) + 12
*b^3*Log[x] - 12*b^3*Log[a + b*x])/a^5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^2+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x^3)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(a*x^2 + b*x^3)^(-2), x]
```

fricas [A] time = 0.39, size = 95, normalized size = 1.38

$$-\frac{12\,ab^3x^3+6\,a^2b^2x^2-2\,a^3bx+a^4-12\left(b^4x^4+ab^3x^3\right)\log\left(bx+a\right)+12\left(b^4x^4+ab^3x^3\right)\log(x)}{3\left(a^5bx^4+a^6x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(12*a*b^3*x^3 + 6*a^2*b^2*x^2 - 2*a^3*b*x + a^4 - 12*(b^4*x^4 + a*b^3*
x^3)*log(b*x + a) + 12*(b^4*x^4 + a*b^3*x^3)*log(x))/(a^5*b*x^4 + a^6*x^3)
```

giac [A] time = 0.15, size = 73, normalized size = 1.06

$$\frac{4\,b^3\log\left(|bx+a|\right)}{a^5}-\frac{4\,b^3\log\left(|x|\right)}{a^5}-\frac{12\,ab^3x^3+6\,a^2b^2x^2-2\,a^3bx+a^4}{3\,(bx+a)a^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 4*b^3*log(abs(b*x + a))/a^5 - 4*b^3*log(abs(x))/a^5 - 1/3*(12*a*b^3*x^3 + 6
*a^2*b^2*x^2 - 2*a^3*b*x + a^4)/((b*x + a)*a^5*x^3)
```

maple [A] time = 0.05, size = 68, normalized size = 0.99

$$-\frac{b^3}{(bx+a)\,a^4}-\frac{4b^3\ln(x)}{a^5}+\frac{4b^3\ln\left(bx+a\right)}{a^5}-\frac{3b^2}{a^4x}+\frac{b}{a^3x^2}-\frac{1}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x^2)^2,x)
```

```
[Out] -1/3/a^2/x^3+b/a^3/x^2-3/a^4*b^2/x-b^3/a^4/(b*x+a)-4/a^5*b^3*ln(x)+4*b^3*ln
(b*x+a)/a^5
```

maxima [A] time = 1.34, size = 73, normalized size = 1.06

$$-\frac{12\,b^3x^3+6\,ab^2x^2-2\,a^2bx+a^3}{3\left(a^4bx^4+a^5x^3\right)}+\frac{4\,b^3\log\left(bx+a\right)}{a^5}-\frac{4\,b^3\log(x)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(12*b^3*x^3 + 6*a*b^2*x^2 - 2*a^2*b*x + a^3)/(a^4*b*x^4 + a^5*x^3) + 4
*b^3*log(b*x + a)/a^5 - 4*b^3*log(x)/a^5
```

mupad [B] time = 0.07, size = 69, normalized size = 1.00

$$\frac{8\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5}-\frac{\frac{1}{3\,a}+\frac{2\,b^2\,x^2}{a^3}+\frac{4\,b^3\,x^3}{a^4}-\frac{2\,b\,x}{3\,a^2}}{b\,x^4+a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^3)^2,x)
```

```
[Out] (8*b^3*atanh((2*b*x)/a + 1))/a^5 - (1/(3*a) + (2*b^2*x^2)/a^3 + (4*b^3*x^3)
/a^4 - (2*b*x)/(3*a^2))/(a*x^3 + b*x^4)
```

sympy [A] time = 0.33, size = 66, normalized size = 0.96

$$\frac{-a^3 + 2a^2bx - 6ab^2x^2 - 12b^3x^3}{3a^5x^3 + 3a^4bx^4} + \frac{4b^3\left(-\log\left(x\right) + \log\left(\frac{a}{b} + x\right)\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x**2)**2,x)
```

```
[Out] (-a**3 + 2*a**2*b*x - 6*a*b**2*x**2 - 12*b**3*x**3)/(3*a**5*x**3 + 3*a**4*b
*x**4) + 4*b**3*(-log(x) + log(a/b + x))/a**5
```

3.152 $\int \frac{1}{x\left(ax^2+bx^3\right)^2} dx$

Optimal. Leaf size=84

$$\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}+\frac{b^4}{a^5(a+bx)}+\frac{4b^3}{a^5x}-\frac{3b^2}{2a^4x^2}+\frac{2b}{3a^3x^3}-\frac{1}{4a^2x^4}$$

Rubi [A] time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 44}

$$-\frac{3b^2}{2a^4x^2}+\frac{b^4}{a^5(a+bx)}+\frac{4b^3}{a^5x}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}+\frac{2b}{3a^3x^3}-\frac{1}{4a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x^2 + b*x^3)^2),x]
```

```
[Out] -1/(4*a^2*x^4) + (2*b)/(3*a^3*x^3) - (3*b^2)/(2*a^4*x^2) + (4*b^3)/(a^5*x)
+ b^4/(a^5*(a + b*x)) + (5*b^4*Log[x])/a^6 - (5*b^4*Log[a + b*x])/a^6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(ax^2+bx^3\right)^2} dx &= \int \frac{1}{x^5(a+bx)^2} dx \\
&= \int \left(\frac{1}{a^2x^5}-\frac{2b}{a^3x^4}+\frac{3b^2}{a^4x^3}-\frac{4b^3}{a^5x^2}+\frac{5b^4}{a^6x}-\frac{b^5}{a^5(a+bx)^2}-\frac{5b^5}{a^6(a+bx)}\right) dx \\
&= -\frac{1}{4a^2x^4}+\frac{2b}{3a^3x^3}-\frac{3b^2}{2a^4x^2}+\frac{4b^3}{a^5x}+\frac{b^4}{a^5(a+bx)}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 79, normalized size = 0.94

$$\frac{\frac{a\left(-3a^4+5a^3bx-10a^2b^2x^2+30ab^3x^3+60b^4x^4\right)}{x^4(a+bx)}-60b^4\log(a+bx)+60b^4\log(x)}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x^2 + b*x^3)^2),x]
```

```
[Out] ((a*(-3*a^4 + 5*a^3*b*x - 10*a^2*b^2*x^2 + 30*a*b^3*x^3 + 60*b^4*x^4))/(x^4
*(a + b*x)) + 60*b^4*Log[x] - 60*b^4*Log[a + b*x])/(12*a^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(ax^2+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a*x^2 + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a*x^2 + b*x^3)^2), x]
```

fricas [A] time = 0.40, size = 108, normalized size = 1.29

$$\frac{60\,ab^4x^4+30\,a^2b^3x^3-10\,a^3b^2x^2+5\,a^4bx-3\,a^5-60\left(b^5x^5+ab^4x^4\right)\log\left(bx+a\right)+60\left(b^5x^5+ab^4x^4\right)\log(x)}{12\left(a^6bx^5+a^7x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(60*a*b^4*x^4 + 30*a^2*b^3*x^3 - 10*a^3*b^2*x^2 + 5*a^4*b*x - 3*a^5 -
60*(b^5*x^5 + a*b^4*x^4)*log(b*x + a) + 60*(b^5*x^5 + a*b^4*x^4)*log(x))/(a
^6*b*x^5 + a^7*x^4)
```

giac [A] time = 0.15, size = 86, normalized size = 1.02

$$-\frac{5\,b^4\log\left(|bx+a|\right)}{a^6}+\frac{5\,b^4\log\left(|x|\right)}{a^6}+\frac{60\,ab^4x^4+30\,a^2b^3x^3-10\,a^3b^2x^2+5\,a^4bx-3\,a^5}{12\,(bx+a)a^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -5*b^4*log(abs(b*x + a))/a^6 + 5*b^4*log(abs(x))/a^6 + 1/12*(60*a*b^4*x^4 +
 30*a^2*b^3*x^3 - 10*a^3*b^2*x^2 + 5*a^4*b*x - 3*a^5)/((b*x + a)*a^6*x^4)
```

maple [A] time = 0.05, size = 79, normalized size = 0.94

$$\frac{b^4}{(bx+a)\,a^5}+\frac{5b^4\ln(x)}{a^6}-\frac{5b^4\ln\left(bx+a\right)}{a^6}+\frac{4b^3}{a^5x}-\frac{3b^2}{2a^4x^2}+\frac{2b}{3a^3x^3}-\frac{1}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x^2)^2,x)
```

```
[Out] -1/4/a^2/x^4+2/3*b/a^3/x^3-3/2/a^4*b^2/x^2+4/a^5*b^3/x+b^4/a^5/(b*x+a)+5/a^
6*b^4*ln(x)-5*b^4*ln(b*x+a)/a^6
```

maxima [A] time = 1.37, size = 86, normalized size = 1.02

$$\frac{60\,b^4x^4+30\,ab^3x^3-10\,a^2b^2x^2+5\,a^3bx-3\,a^4}{12\left(a^5bx^5+a^6x^4\right)}-\frac{5\,b^4\log\left(bx+a\right)}{a^6}+\frac{5\,b^4\log(x)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(60*b^4*x^4 + 30*a*b^3*x^3 - 10*a^2*b^2*x^2 + 5*a^3*b*x - 3*a^4)/(a^5*
b*x^5 + a^6*x^4) - 5*b^4*log(b*x + a)/a^6 + 5*b^4*log(x)/a^6
```

mupad [B] time = 0.08, size = 79, normalized size = 0.94

$$\frac{\frac{5\,b^3\,x^3}{2\,a^4}-\frac{5\,b^2\,x^2}{6\,a^3}-\frac{1}{4\,a}+\frac{5\,b^4\,x^4}{a^5}+\frac{5\,b\,x}{12\,a^2}}{b\,x^5+a\,x^4}-\frac{10\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^2 + b*x^3)^2),x)
```

```
[Out] ((5*b^3*x^3)/(2*a^4) - (5*b^2*x^2)/(6*a^3) - 1/(4*a) + (5*b^4*x^4)/a^5 + (5
*b*x)/(12*a^2))/(a*x^4 + b*x^5) - (10*b^4*atanh((2*b*x)/a + 1))/a^6
```

sympy [A] time = 0.36, size = 80, normalized size = 0.95

$$\frac{-3a^4 + 5a^3bx - 10a^2b^2x^2 + 30ab^3x^3 + 60b^4x^4}{12a^6x^4 + 12a^5bx^5} + \frac{5b^4\left(\log(x) - \log\left(\frac{a}{b} + x\right)\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x**2)**2,x)
```

```
[Out] (-3*a**4 + 5*a**3*b*x - 10*a**2*b**2*x**2 + 30*a*b**3*x**3 + 60*b**4*x**4)/
(12*a**6*x**4 + 12*a**5*b*x**5) + 5*b**4*(log(x) - log(a/b + x))/a**6
```

3.153 $\int x^2\sqrt{ax^2+bx^3}\,dx$

Optimal. Leaf size=105

$$-\frac{32a^3\left(ax^2+bx^3\right)^{3/2}}{315b^4x^3}+\frac{16a^2\left(ax^2+bx^3\right)^{3/2}}{105b^3x^2}-\frac{4a\left(ax^2+bx^3\right)^{3/2}}{21b^2x}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}$$

Rubi [A] time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$-\frac{32a^3\left(ax^2+bx^3\right)^{3/2}}{315b^4x^3}+\frac{16a^2\left(ax^2+bx^3\right)^{3/2}}{105b^3x^2}-\frac{4a\left(ax^2+bx^3\right)^{3/2}}{21b^2x}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*(a*x^2 + b*x^3)^(3/2))/(9*b) - (32*a^3*(a*x^2 + b*x^3)^(3/2))/(315*b^4*x
^3) + (16*a^2*(a*x^2 + b*x^3)^(3/2))/(105*b^3*x^2) - (4*a*(a*x^2 + b*x^3)^(
3/2))/(21*b^2*x)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2\sqrt{ax^2+bx^3}\,dx &= \frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}-\frac{(2a)\int x\sqrt{ax^2+bx^3}\,dx}{3b}\\
&= \frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}-\frac{4a\left(ax^2+bx^3\right)^{3/2}}{21b^2x}+\frac{\left(8a^2\right)\int\sqrt{ax^2+bx^3}\,dx}{21b^2}\\
&= \frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}+\frac{16a^2\left(ax^2+bx^3\right)^{3/2}}{105b^3x^2}-\frac{4a\left(ax^2+bx^3\right)^{3/2}}{21b^2x}-\frac{\left(16a^3\right)\int\frac{\sqrt{ax^2+bx^3}}{x}\,dx}{105b^3}\\
&= \frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}-\frac{32a^3\left(ax^2+bx^3\right)^{3/2}}{315b^4x^3}+\frac{16a^2\left(ax^2+bx^3\right)^{3/2}}{105b^3x^2}-\frac{4a\left(ax^2+bx^3\right)^{3/2}}{21b^2x}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 0.50

$$\frac{2\left(x^2(a+bx)\right)^{3/2}\left(-16a^3+24a^2bx-30ab^2x^2+35b^3x^3\right)}{315b^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*(x^2*(a + b*x))^(3/2)*(-16*a^3 + 24*a^2*b*x - 30*a*b^2*x^2 + 35*b^3*x^3)
)/(315*b^4*x^3)
```

IntegrateAlgebraic [A] time = 0.05, size = 66, normalized size = 0.63

$$\frac{2\sqrt{ax^2+bx^3}\left(-16a^4+8a^3bx-6a^2b^2x^2+5ab^3x^3+35b^4x^4\right)}{315b^4x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^3]*(-16*a^4 + 8*a^3*b*x - 6*a^2*b^2*x^2 + 5*a*b^3*x^3 +
 35*b^4*x^4))/(315*b^4*x)
```

fricas [A] time = 0.38, size = 62, normalized size = 0.59

$$\frac{2\left(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4\right)\sqrt{bx^3+ax^2}}{315\,b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt(
b*x^3 + a*x^2)/(b^4*x)
```

giac [A] time = 0.16, size = 131, normalized size = 1.25

$$\frac{32\,a^{\frac{9}{2}}\mathrm{sgn}(x)}{315\,b^4}+\frac{2\left(\frac{9\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)a\mathrm{sgn}(x)}{b^3}+\frac{\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}\,a^4\right)\mathrm{sgn}(x)}{b^3}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 32/315*a^(9/2)*sgn(x)/b^4 + 2/315*(9*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2
)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a*sgn(x)/b^3 + (35*(b*
x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x +
 a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*sgn(x)/b^3)/b
```

maple [A] time = 0.04, size = 57, normalized size = 0.54

$$-\frac{2\,(bx+a)\left(-35b^3x^3+30a\,b^2x^2-24a^2bx+16a^3\right)\sqrt{b\,x^3+a\,x^2}}{315b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -2/315*(b*x+a)*(-35*b^3*x^3+30*a*b^2*x^2-24*a^2*b*x+16*a^3)*(b*x^3+a*x^2)^(
1/2)/b^4/x
```

maxima [A] time = 1.46, size = 53, normalized size = 0.50

$$\frac{2\left(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4\right)\sqrt{bx+a}}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt(
b*x + a)/b^4
```

mupad [B] time = 5.49, size = 62, normalized size = 0.59

$$\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(-16\,a^4+8\,a^3\,b\,x-6\,a^2\,b^2\,x^2+5\,a\,b^3\,x^3+35\,b^4\,x^4\right)}{315\,b^4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(35*b^4*x^4 - 16*a^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2
 + 8*a^3*b*x))/(315*b^4*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{x^2\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x**2*(a + b*x)), x)
```

3.154 $\int x\sqrt{ax^2+bx^3}\,dx$

Optimal. Leaf size=80

$$\frac{16a^2\left(ax^2+bx^3\right)^{3/2}}{105b^3x^3}-\frac{8a\left(ax^2+bx^3\right)^{3/2}}{35b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}$$

Rubi [A] time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2016, 2002, 2014}

$$\frac{16a^2\left(ax^2+bx^3\right)^{3/2}}{105b^3x^3}-\frac{8a\left(ax^2+bx^3\right)^{3/2}}{35b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (16*a^2*(a*x^2 + b*x^3)^(3/2))/(105*b^3*x^3) - (8*a*(a*x^2 + b*x^3)^(3/2))/
(35*b^2*x^2) + (2*(a*x^2 + b*x^3)^(3/2))/(7*b*x)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{ax^2+bx^3}\,dx &= \frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}-\frac{(4a)\int\sqrt{ax^2+bx^3}\,dx}{7b}\\
&= -\frac{8a\left(ax^2+bx^3\right)^{3/2}}{35b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}+\frac{\left(8a^2\right)\int\frac{\sqrt{ax^2+bx^3}}{x}\,dx}{35b^2}\\
&= \frac{16a^2\left(ax^2+bx^3\right)^{3/2}}{105b^3x^3}-\frac{8a\left(ax^2+bx^3\right)^{3/2}}{35b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 0.52

$$\frac{2\left(x^2(a+bx)\right)^{3/2}\left(8a^2-12abx+15b^2x^2\right)}{105b^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*(x^2*(a + b*x))^(3/2)*(8*a^2 - 12*a*b*x + 15*b^2*x^2))/(105*b^3*x^3)
```

IntegrateAlgebraic [A] time = 0.04, size = 55, normalized size = 0.69

$$\frac{2\sqrt{ax^2+bx^3}\left(8a^3-4a^2bx+3ab^2x^2+15b^3x^3\right)}{105b^3x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^3]*(8*a^3 - 4*a^2*b*x + 3*a*b^2*x^2 + 15*b^3*x^3))/(105
*b^3*x)
```

fricas [A] time = 0.39, size = 51, normalized size = 0.64

$$\frac{2\left(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3\right)\sqrt{bx^3+ax^2}}{105\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2)/(b
^3*x)
```

giac [A] time = 0.16, size = 108, normalized size = 1.35

$$-\frac{16\,a^{\frac{7}{2}}\operatorname{sgn}(x)}{105\,b^3}+\frac{2\left(\frac{7\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a\operatorname{sgn}(x)}{b^2}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)\operatorname{sgn}(x)}{b^2}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -16/105*a^(7/2)*sgn(x)/b^3 + 2/105*(7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/
2)*a + 15*sqrt(b*x + a)*a^2)*a*sgn(x)/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x
+ a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*sgn(x)/b^2)/b
```

maple [A] time = 0.05, size = 46, normalized size = 0.58

$$\frac{2\,(bx+a)\left(15b^2x^2-12abx+8a^2\right)\sqrt{b\,x^3+a\,x^2}}{105b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 2/105*(b*x+a)*(15*b^2*x^2-12*a*b*x+8*a^2)*(b*x^3+a*x^2)^(1/2)/b^3/x
```

maxima [A] time = 1.44, size = 42, normalized size = 0.52

$$\frac{2\left(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3\right)\sqrt{bx+a}}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

mupad [B] time = 5.48, size = 51, normalized size = 0.64

$$\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(8\,a^3-4\,a^2\,b\,x+3\,a\,b^2\,x^2+15\,b^3\,x^3\right)}{105\,b^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(8*a^3 + 15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x))/(1
05*b^3*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x^2\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**2*(a + b*x)), x)
```

3.155 $\int \sqrt{ax^2+bx^3}\,dx$

Optimal. Leaf size=52

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{4a\left(ax^2+bx^3\right)^{3/2}}{15b^2x^3}$$

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2002, 2014}

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{4a\left(ax^2+bx^3\right)^{3/2}}{15b^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-4*a*(a*x^2 + b*x^3)^(3/2))/(15*b^2*x^3) + (2*(a*x^2 + b*x^3)^(3/2))/(5*b*
x^2)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{ax^2+bx^3}\,dx &= \frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{(2a)\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx}{5b}\\ &= -\frac{4a\left(ax^2+bx^3\right)^{3/2}}{15b^2x^3}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.60

$$\frac{2\left(x^2(a+bx)\right)^{3/2}(3bx-2a)}{15b^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*(x^2*(a + b*x))^(3/2)*(-2*a + 3*b*x))/(15*b^2*x^3)
```

IntegrateAlgebraic [A] time = 0.03, size = 43, normalized size = 0.83

$$\frac{2\left(-2a^2+abx+3b^2x^2\right)\sqrt{ax^2+bx^3}}{15b^2x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*(-2*a^2 + a*b*x + 3*b^2*x^2)*Sqrt[a*x^2 + b*x^3])/(15*b^2*x)
```

fricas [A] time = 0.38, size = 39, normalized size = 0.75

$$\frac{2\left(3\,b^2x^2+abx-2\,a^2\right)\sqrt{bx^3+ax^2}}{15\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x^3 + a*x^2)/(b^2*x)
```

giac [A] time = 0.18, size = 81, normalized size = 1.56

$$\frac{4\,a^{\frac{5}{2}}\operatorname{sgn}(x)}{15\,b^2}+\frac{2\left(\frac{5\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)a\operatorname{sgn}(x)}{b}+\frac{\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)\operatorname{sgn}(x)}{b}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*a^(5/2)*sgn(x)/b^2 + 2/15*(5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a*s
gn(x)/b + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)
*sgn(x)/b)/b
```

maple [A] time = 0.05, size = 35, normalized size = 0.67

$$-\frac{2\,(bx+a)\,(-3bx+2a)\,\sqrt{b\,x^3+a\,x^2}}{15b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -2/15*(b*x+a)*(-3*b*x+2*a)*(b*x^3+a*x^2)^(1/2)/b^2/x
```

maxima [A] time = 1.42, size = 30, normalized size = 0.58

$$\frac{2\left(3\,b^2x^2+abx-2\,a^2\right)\sqrt{bx+a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

mupad [B] time = 5.30, size = 39, normalized size = 0.75

$$\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(-2\,a^2+a\,b\,x+3\,b^2\,x^2\right)}{15\,b^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(3*b^2*x^2 - 2*a^2 + a*b*x))/(15*b^2*x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^2 + bx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**2 + b*x**3), x)
```

3.156 $\int \frac{\sqrt{ax^2+bx^3}}{x} \, dx$

Optimal. Leaf size=25

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{3bx^3}$$

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3]/x,x]
```

```
[Out] (2*(a*x^2 + b*x^3)^(3/2))/(3*b*x^3)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{ax^2+bx^3}}{x} \, dx = \frac{2\left(ax^2+bx^3\right)^{3/2}}{3bx^3}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.92

$$\frac{2\left(x^2(a+bx)\right)^{3/2}}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x,x]
```

```
[Out] (2*(x^2*(a + b*x))^(3/2))/(3*b*x^3)
```

IntegrateAlgebraic [A] time = 0.03, size = 25, normalized size = 1.00

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x^2 + b*x^3]/x,x]
```

```
[Out] (2*(a*x^2 + b*x^3)^(3/2))/(3*b*x^3)
```

fricas [A] time = 0.40, size = 26, normalized size = 1.04

$$\frac{2\sqrt{bx^3+ax^2}\,(bx+a)}{3\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x^3 + a*x^2)*(b*x + a)/(b*x)
```

giac [B] time = 0.15, size = 50, normalized size = 2.00

$$-\frac{2\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,b}+\frac{2\left(3\,\sqrt{bx+a}\,a\mathrm{sgn}(x)+\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)\mathrm{sgn}(x)\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -2/3*a^(3/2)*sgn(x)/b + 2/3*(3*sqrt(b*x + a)*a*sgn(x) + ((b*x + a)^(3/2) -
3*sqrt(b*x + a)*a)*sgn(x))/b
```

maple [A] time = 0.04, size = 27, normalized size = 1.08

$$\frac{2\,(bx+a)\,\sqrt{b\,x^{3}+a\,x^{2}}}{3bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2)/x,x)
```

```
[Out] 2/3*(b*x+a)*(b*x^3+a*x^2)^(1/2)/b/x
```

maxima [A] time = 1.43, size = 12, normalized size = 0.48

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{b\,x^{3}+a\,x^{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(1/2)/x,x)
```

```
[Out] int((a*x^2 + b*x^3)^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{2}\,(a+bx)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x))/x, x)
```

3.157 $\int \frac{\sqrt{ax^2+bx^3}}{x^2} \, dx$

Optimal. Leaf size=51

$$\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)$$

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2021, 2008, 206}

$$\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3]/x^2,x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^3])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^
3]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^2+bx^3}}{x^2} \, dx &= \frac{2\sqrt{ax^2+bx^3}}{x}+a\int \frac{1}{\sqrt{ax^2+bx^3}} \, dx\\ &= \frac{2\sqrt{ax^2+bx^3}}{x}-(2a)\,\text{Subst}\left(\int \frac{1}{1-ax^2} \, dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)\\ &= \frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 1.04

$$\frac{2x\left(-\sqrt{a}\sqrt{a+bx}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+a+bx\right)}{\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x^2,x]
```

```
[Out] (2*x*(a + b*x - Sqrt[a]*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/Sqrt
[x^2*(a + b*x)]
```

IntegrateAlgebraic [A] time = 0.07, size = 51, normalized size = 1.00

$$\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x^2 + b*x^3]/x^2,x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^3])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^
3]]
```

fricas [A] time = 0.41, size = 111, normalized size = 2.18

$$\left[\frac{\sqrt{a}\,x\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}}{x},\ \frac{2\left(\sqrt{-a}\,x\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*x*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*sq
rt(b*x^3 + a*x^2))/x, 2*(sqrt(-a)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*
x)) + sqrt(b*x^3 + a*x^2))/x]
```

giac [A] time = 0.16, size = 67, normalized size = 1.31

$$\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}}+2\,\sqrt{bx+a}\,\mathrm{sgn}(x)-\frac{2\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\,\sqrt{a}\right)\mathrm{sgn}(x)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*sqrt(b*x + a)*sgn(x)
 - 2*(a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)
```

maple [A] time = 0.04, size = 51, normalized size = 1.00

$$\frac{2\sqrt{b\,x^3+a\,x^2}\,\left(-\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\sqrt{bx+a}\right)}{\sqrt{bx+a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2)/x^2,x)
```

```
[Out] 2*(b*x^3+a*x^2)^(1/2)*(-a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+(b*x+a)^(1/2
))/x/(b*x+a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{bx^3+ax^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x^2)/x^2, x)
```

mupad [B] time = 5.36, size = 73, normalized size = 1.43

$$\frac{2\sqrt{b\,x^3+a\,x^2}}{x}+\frac{\sqrt{a}\,\mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{\frac{1}{x}}\,1\mathrm{i}}{\sqrt{b}}\right)\sqrt{b\,x^3+a\,x^2}\,{\left(\frac{1}{x}\right)}^{3/2}\,2\mathrm{i}}{\sqrt{b}\,\sqrt{\frac{a}{b\,x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(1/2)/x^2,x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2))/x + (a^(1/2)*asin((a^(1/2)*(1/x)^(1/2)*1i)/b^(1/2
))*(a*x^2 + b*x^3)^(1/2)*(1/x)^(3/2)*2i)/(b^(1/2)*(a/(b*x) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\,(a+bx)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x))/x**2, x)
```

3.158 $\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx$

Optimal. Leaf size=52

$$-\frac{\sqrt{ax^2+bx^3}}{x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2020, 2008, 206}

$$-\frac{\sqrt{ax^2+bx^3}}{x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3]/x^3,x]
```

```
[Out] -(Sqrt[a*x^2 + b*x^3]/x^2) - (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/S
qrt[a]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx &= -\frac{\sqrt{ax^2+bx^3}}{x^2}+\frac{1}{2}b\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx\\
&= -\frac{\sqrt{ax^2+bx^3}}{x^2}-b\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)\\
&= -\frac{\sqrt{ax^2+bx^3}}{x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 48, normalized size = 0.92

$$-\frac{bx\sqrt{\frac{bx}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+a+bx}{\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x^3,x]
```

```
[Out] -((a + b*x + b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])/Sqrt[x^2*(a
+ b*x)])
```

IntegrateAlgebraic [A] time = 0.07, size = 52, normalized size = 1.00

$$-\frac{\sqrt{ax^2+bx^3}}{x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x^2 + b*x^3]/x^3,x]
```

```
[Out] -(Sqrt[a*x^2 + b*x^3]/x^2) - (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/S
qrt[a]
```

fricas [A] time = 0.41, size = 127, normalized size = 2.44

$$\left[\frac{\sqrt{a}\,bx^2\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}\,a}{2\,ax^2},\ \frac{\sqrt{-a}\,bx^2\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)-\sqrt{bx^3+ax^2}\,a}{ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x^2*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2
) - 2*sqrt(b*x^3 + a*x^2)*a)/(a*x^2), (sqrt(-a)*b*x^2*arctan(sqrt(b*x^3 + a
*x^2)*sqrt(-a)/(a*x)) - sqrt(b*x^3 + a*x^2)*a)/(a*x^2)]
```

giac [A] time = 0.26, size = 45, normalized size = 0.87

$$\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}}-\frac{\sqrt{bx+a}\,b\mathrm{sgn}(x)}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] (b^2*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) - sqrt(b*x + a)*b*sgn(x
)/x)/b
```

maple [A] time = 0.05, size = 56, normalized size = 1.08

$$-\frac{\sqrt{b\,x^3+a\,x^2}\,\left(bx\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\sqrt{bx+a}\,\sqrt{a}\right)}{\sqrt{bx+a}\,\sqrt{a}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2)/x^3,x)
```

```
[Out] -(b*x^3+a*x^2)^(1/2)*(arctanh((b*x+a)^(1/2)/a^(1/2))*x*b+(b*x+a)^(1/2)*a^(1
/2))/x^2/(b*x+a)^(1/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{bx^3+ax^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate(sqrt(b*x^3 + a*x^2)/x^3, x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{b\,x^3+a\,x^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(1/2)/x^3,x)
[Out] int((a*x^2 + b*x^3)^(1/2)/x^3, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\,(a+bx)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(1/2)/x**3,x)
[Out] Integral(sqrt(x**2*(a + b*x))/x**3, x)
```

3.159 $\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx$

Optimal. Leaf size=84

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{3/2}}-\frac{b\sqrt{ax^2+bx^3}}{4ax^2}-\frac{\sqrt{ax^2+bx^3}}{2x^3}$$

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2008, 206}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{3/2}}-\frac{b\sqrt{ax^2+bx^3}}{4ax^2}-\frac{\sqrt{ax^2+bx^3}}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3]/x^4,x]
```

```
[Out] -Sqrt[a*x^2 + b*x^3]/(2*x^3) - (b*Sqrt[a*x^2 + b*x^3])/(4*a*x^2) + (b^2*Arc
Tanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(4*a^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx = -\frac{\sqrt{ax^2+bx^3}}{2x^3} + \frac{1}{4}b\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx$$
$$= -\frac{\sqrt{ax^2+bx^3}}{2x^3} - \frac{b\sqrt{ax^2+bx^3}}{4ax^2} - \frac{b^2\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{8a}$$
$$= -\frac{\sqrt{ax^2+bx^3}}{2x^3} - \frac{b\sqrt{ax^2+bx^3}}{4ax^2} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)}{4a}$$
$$= -\frac{\sqrt{ax^2+bx^3}}{2x^3} - \frac{b\sqrt{ax^2+bx^3}}{4ax^2} + \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{3/2}}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.50

$$-\frac{2b^2\left(x^2(a+bx)\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{bx}{a}+1\right)}{3a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x^4,x]
```

```
[Out] (-2*b^2*(x^2*(a + b*x))^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x)/a])/
(3*a^3*x^3)
```

IntegrateAlgebraic [A] time = 0.07, size = 69, normalized size = 0.82

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{3/2}} + \frac{(-2a-bx)\sqrt{ax^2+bx^3}}{4ax^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x^2 + b*x^3]/x^4,x]
```

```
[Out] ((-2*a - b*x)*Sqrt[a*x^2 + b*x^3])/(4*a*x^3) + (b^2*ArcTanh[(Sqrt[a]*x)/Sqr
t[a*x^2 + b*x^3]])/(4*a^(3/2))
```

fricas [A] time = 0.42, size = 149, normalized size = 1.77

$$\left[\frac{\sqrt{a}\,b^2x^3\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right) - 2\,\sqrt{bx^3+ax^2}\left(abx+2\,a^2\right)}{8\,a^2x^3}, -\frac{\sqrt{-a}\,b^2x^3\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right) + \sqrt{bx^3+ax^2}\left(abx+2\,a^2\right)}{4\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x
^2) - 2*sqrt(b*x^3 + a*x^2)*(a*b*x + 2*a^2))/(a^2*x^3), -1/4*(sqrt(-a)*b^2*
x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(a*b*x
 + 2*a^2))/(a^2*x^3)]
```

giac [A] time = 0.21, size = 72, normalized size = 0.86

$$-\frac{\frac{b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\text{sgn}(x)}{\sqrt{-a}\,a} + \frac{(bx+a)^{\frac{3}{2}}b^3\text{sgn}(x)+\sqrt{bx+a}\,ab^3\text{sgn}(x)}{ab^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/4*(b^3*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a) + ((b*x + a)^(
3/2)*b^3*sgn(x) + sqrt(b*x + a)*a*b^3*sgn(x))/(a*b^2*x^2))/b
```

maple [A] time = 0.06, size = 73, normalized size = 0.87

$$-\frac{\sqrt{b\,x^{3}+a\,x^{2}}\,\left(-a\,b^{2}x^{2}\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\sqrt{bx+a}\;a^{\frac{5}{2}}+\left(bx+a\right)^{\frac{3}{2}}a^{\frac{3}{2}}\right)}{4\sqrt{bx+a}\;a^{\frac{5}{2}}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2)/x^4,x)
```

```
[Out] -1/4*(b*x^3+a*x^2)^(1/2)*((b*x+a)^(3/2)*a^(3/2)-arctanh((b*x+a)^(1/2)/a^(1/
2))*a*x^2*b^2+(b*x+a)^(1/2)*a^(5/2))/x^3/(b*x+a)^(1/2)/a^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^{3}+ax^{2}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x^2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^{3}+a\,x^{2}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(1/2)/x^4,x)
```

```
[Out] int((a*x^2 + b*x^3)^(1/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{2}\left(a+bx\right)}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x))/x**4, x)
```

3.160 $\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx$

Optimal. Leaf size=112

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{5/2}}+\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}-\frac{\sqrt{ax^2+bx^3}}{3x^4}$$

Rubi [A] time = 0.14, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2008, 206}

$$\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{5/2}}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}-\frac{\sqrt{ax^2+bx^3}}{3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3]/x^5,x]
```

```
[Out] -Sqrt[a*x^2 + b*x^3]/(3*x^4) - (b*Sqrt[a*x^2 + b*x^3])/(12*a*x^3) + (b^2*Sq
rt[a*x^2 + b*x^3])/(8*a^2*x^2) - (b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^
3]])/(8*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx &= -\frac{\sqrt{ax^2+bx^3}}{3x^4}+\frac{1}{6}b\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx \\
&= -\frac{\sqrt{ax^2+bx^3}}{3x^4}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}-\frac{b^2\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{8a} \\
&= -\frac{\sqrt{ax^2+bx^3}}{3x^4}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}+\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}+\frac{b^3\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{16a^2} \\
&= -\frac{\sqrt{ax^2+bx^3}}{3x^4}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}+\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}-\frac{b^3\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{8a^2} \\
&= -\frac{\sqrt{ax^2+bx^3}}{3x^4}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}+\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.38

$$\frac{2b^3\left(x^2(a+bx)\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};\frac{bx}{a}+1\right)}{3a^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x^5,x]
```

```
[Out] (2*b^3*(x^2*(a + b*x))^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, 1 + (b*x)/a])/(
3*a^4*x^3)
```

IntegrateAlgebraic [A] time = 0.08, size = 80, normalized size = 0.71

$$\frac{\left(-8a^2-2abx+3b^2x^2\right)\sqrt{ax^2+bx^3}}{24a^2x^4}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x^2 + b*x^3]/x^5,x]
```

```
[Out] ((-8*a^2 - 2*a*b*x + 3*b^2*x^2)*Sqrt[a*x^2 + b*x^3])/(24*a^2*x^4) - (b^3*Ar
cTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(8*a^(5/2))
```

fricas [A] time = 0.41, size = 175, normalized size = 1.56

$$\left[\frac{3\sqrt{a}b^3x^4\log\left(\frac{bx^2+2ax-2\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\left(3ab^2x^2-2a^2bx-8a^3\right)\sqrt{bx^3+ax^2}}{48a^3x^4},\ \frac{3\sqrt{-a}b^3x^4\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)+\left(3ab^2x^2-2a^2bx-8a^3\right)\sqrt{bx^3+ax^2}}{24a^3x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a)
)/x^2) + 2*(3*a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^3*x^4)
, 1/24*(3*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (3*
a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^3*x^4)]
```

giac [A] time = 0.27, size = 92, normalized size = 0.82

$$\frac{\frac{3b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}\,a^2}+\frac{3(bx+a)^{\frac{5}{2}}b^4\mathrm{sgn}(x)-8(bx+a)^{\frac{3}{2}}ab^4\mathrm{sgn}(x)-3\sqrt{bx+a}\,a^2b^4\mathrm{sgn}(x)}{a^2b^3x^3}}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/24*(3*b^4*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^2) + (3*(b*x
+ a)^(5/2)*b^4*sgn(x) - 8*(b*x + a)^(3/2)*a*b^4*sgn(x) - 3*sqrt(b*x + a)*a^
2*b^4*sgn(x))/(a^2*b^3*x^3))/b
```

maple [A] time = 0.06, size = 89, normalized size = 0.79

$$\frac{\sqrt{b\,x^3+a\,x^2}\,\left(-3a^2b^3x^3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-3\sqrt{bx+a}\;a^{\frac{9}{2}}-8\,(bx+a)^{\frac{3}{2}}\,a^{\frac{7}{2}}+3\,(bx+a)^{\frac{5}{2}}\,a^{\frac{5}{2}}\right)}{24\sqrt{bx+a}\;a^{\frac{9}{2}}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2)/x^5,x)
```

```
[Out] 1/24*(b*x^3+a*x^2)^(1/2)*(3*(b*x+a)^(5/2)*a^(5/2)-8*(b*x+a)^(3/2)*a^(7/2)-3
*arctanh((b*x+a)^(1/2)/a^(1/2))*a^2*b^3*x^3-3*(b*x+a)^(1/2)*a^(9/2))/x^4/(b
*x+a)^(1/2)/a^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{bx^3+ax^2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x^2)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{b\,x^3+a\,x^2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(1/2)/x^5,x)
```

```
[Out] int((a*x^2 + b*x^3)^(1/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2\,(a+bx)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x))/x**5, x)
```

3.161 $\int x^2 \left(ax^2+bx^3\right)^{3/2} dx$

Optimal. Leaf size=161

$$-\frac{512a^5\left(ax^2+bx^3\right)^{5/2}}{45045b^6x^5}+\frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{9009b^5x^4}-\frac{64a^3\left(ax^2+bx^3\right)^{5/2}}{1287b^4x^3}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^2}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{39b^2x}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}$$

Rubi [A] time = 0.23, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$-\frac{512a^5\left(ax^2+bx^3\right)^{5/2}}{45045b^6x^5}+\frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{9009b^5x^4}-\frac{64a^3\left(ax^2+bx^3\right)^{5/2}}{1287b^4x^3}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^2}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{39b^2x}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*(a*x^2 + b*x^3)^(5/2))/(15*b) - (512*a^5*(a*x^2 + b*x^3)^(5/2))/(45045*b
^6*x^5) + (256*a^4*(a*x^2 + b*x^3)^(5/2))/(9009*b^5*x^4) - (64*a^3*(a*x^2 +
 b*x^3)^(5/2))/(1287*b^4*x^3) + (32*a^2*(a*x^2 + b*x^3)^(5/2))/(429*b^3*x^2
) - (4*a*(a*x^2 + b*x^3)^(5/2))/(39*b^2*x)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(ax^2+bx^3\right)^{3/2}dx &= \frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}-\frac{(2a)\int x\left(ax^2+bx^3\right)^{3/2}dx}{3b}\\
&= \frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{39b^2x}+\frac{\left(16a^2\right)\int\left(ax^2+bx^3\right)^{3/2}dx}{39b^2}\\
&= \frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^2}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{39b^2x}-\frac{\left(32a^3\right)\int\frac{\left(ax^2+bx^3\right)^{3/2}}{x}dx}{143b^3}\\
&= \frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}-\frac{64a^3\left(ax^2+bx^3\right)^{5/2}}{1287b^4x^3}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^2}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{39b^2x}+\\
&= \frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}+\frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{9009b^5x^4}-\frac{64a^3\left(ax^2+bx^3\right)^{5/2}}{1287b^4x^3}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^2}\\
&= \frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}-\frac{512a^5\left(ax^2+bx^3\right)^{5/2}}{45045b^6x^5}+\frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{9009b^5x^4}-\frac{64a^3\left(ax^2+bx^3\right)^{5/2}}{1287b^4x^3}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 80, normalized size = 0.50

$$
\frac{2x(a+bx)^3\left(-256a^5+640a^4bx-1120a^3b^2x^2+1680a^2b^3x^3-2310ab^4x^4+3003b^5x^5\right)}{45045b^6\sqrt{x^2(a+bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(a + b*x)^3*(-256*a^5 + 640*a^4*b*x - 1120*a^3*b^2*x^2 + 1680*a^2*b^3*
x^3 - 2310*a*b^4*x^4 + 3003*b^5*x^5))/(45045*b^6*Sqrt[x^2*(a + b*x)])
```

IntegrateAlgebraic [A] time = 4.70, size = 87, normalized size = 0.54

$$
\frac{2(a+bx)\left(x^2(a+bx)\right)^{3/2}\left(-9009a^5+32175a^4(a+bx)-50050a^3(a+bx)^2+40950a^2(a+bx)^3-17325a(a+bx)^4+3003(a+bx)^5\right)}{45045b^6x^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*(a + b*x)*(x^2*(a + b*x))^(3/2)*(-9009*a^5 + 32175*a^4*(a + b*x) - 50050
*a^3*(a + b*x)^2 + 40950*a^2*(a + b*x)^3 - 17325*a*(a + b*x)^4 + 3003*(a +
b*x)^5))/(45045*b^6*x^3)
```

fricas [A] time = 0.40, size = 95, normalized size = 0.59

$$
\frac{2\left(3003\,b^7x^7+3696\,ab^6x^6+63\,a^2b^5x^5-70\,a^3b^4x^4+80\,a^4b^3x^3-96\,a^5b^2x^2+128\,a^6bx-256\,a^7\right)\sqrt{bx^3+ax^2}}{45045\,b^6x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*b^7*x^7 + 3696*a*b^6*x^6 + 63*a^2*b^5*x^5 - 70*a^3*b^4*x^4 +
80*a^4*b^3*x^3 - 96*a^5*b^2*x^2 + 128*a^6*b*x - 256*a^7)*sqrt(b*x^3 + a*x^2
)/(b^6*x)
```

giac [B] time = 0.18, size = 282, normalized size = 1.75

$$
\frac{512\,a^{\frac{15}{2}}\operatorname{sgn}(x)}{45045\,b^6}+\frac{2\left(\frac{65\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^2-1386\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4-693\,\sqrt{bx+a}\,a^5\right)a^2\operatorname{sgn}(x)}{b^5}+\frac{30\left(231\,(bx+a)^{\frac{13}{2}}-1638\,(bx+a)^{\frac{11}{2}}a+5005\,(bx+a)^{\frac{9}{2}}a^2-8580\,(bx+a)^{\frac{7}{2}}a^3+9009\,(bx+a)^{\frac{5}{2}}a^4-6006\,(bx+a)^{\frac{3}{2}}a^5+3003\,\sqrt{bx+a}\,a^6\right)a\operatorname{sgn}(x)}{b^5}+\frac{7\left(429\,(bx+a)^{\frac{15}{2}}-3465\,(bx+a)^{\frac{13}{2}}a+12285\,(bx+a)^{\frac{11}{2}}a^2-25025\,(bx+a)^{\frac{9}{2}}a^3+32175\,(bx+a)^{\frac{7}{2}}a^4-27027\,(bx+a)^{\frac{5}{2}}a^5+15015\,(bx+a)^{\frac{3}{2}}a^6-6435\,\sqrt{bx+a}\,a^7\right)\operatorname{sgn}(x)}{b^5}\right)}{45045\,b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 512/45045*a^(15/2)*sgn(x)/b^6 + 2/45045*(65*(63*(b*x + a)^(11/2) - 385*(b*x
 + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(
b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a^2*sgn(x)/b^5 + 30*(231*(b*x +
 a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x
 + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 300
3*sqrt(b*x + a)*a^6)*a*sgn(x)/b^5 + 7*(429*(b*x + a)^(15/2) - 3465*(b*x + a
)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175
*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^
6 - 6435*sqrt(b*x + a)*a^7)*sgn(x)/b^5)/b
```

maple [A] time = 0.05, size = 79, normalized size = 0.49

$$-\frac{2\left(bx+a\right)\left(-3003x^{5}b^{5}+2310a\,b^{4}x^{4}-1680a^{2}b^{3}x^{3}+1120a^{3}b^{2}x^{2}-640a^{4}bx+256a^{5}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{45045b^{6}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -2/45045*(b*x+a)*(-3003*b^5*x^5+2310*a*b^4*x^4-1680*a^2*b^3*x^3+1120*a^3*b^
2*x^2-640*a^4*b*x+256*a^5)*(b*x^3+a*x^2)^(3/2)/b^6/x^3
```

maxima [A] time = 1.52, size = 86, normalized size = 0.53

$$\frac{2\left(3003\,b^{7}x^{7}+3696\,ab^{6}x^{6}+63\,a^{2}b^{5}x^{5}-70\,a^{3}b^{4}x^{4}+80\,a^{4}b^{3}x^{3}-96\,a^{5}b^{2}x^{2}+128\,a^{6}bx-256\,a^{7}\right)\sqrt{bx+a}}{45045\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*b^7*x^7 + 3696*a*b^6*x^6 + 63*a^2*b^5*x^5 - 70*a^3*b^4*x^4 +
80*a^4*b^3*x^3 - 96*a^5*b^2*x^2 + 128*a^6*b*x - 256*a^7)*sqrt(b*x + a)/b^6
```

mupad [B] time = 5.24, size = 80, normalized size = 0.50

$$-\frac{2\,\sqrt{b\,x^3+a\,x^2}\,{\left(a+b\,x\right)}^2\,\left(256\,a^5-640\,a^4\,b\,x+1120\,a^3\,b^2\,x^2-1680\,a^2\,b^3\,x^3+2310\,a\,b^4\,x^4-3003\,b^5\,x^5\right)}{45045\,b^6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] -(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(256*a^5 - 3003*b^5*x^5 + 2310*a*b^4*
x^4 + 1120*a^3*b^2*x^2 - 1680*a^2*b^3*x^3 - 640*a^4*b*x))/(45045*b^6*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\left(x^{2}\left(a+bx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(x**2*(a + b*x))**(3/2), x)
```

3.162 $\int x\left(ax^2+bx^3\right)^{3/2}\,dx$

Optimal. Leaf size=136

$$\frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{15015b^5x^5}-\frac{128a^3\left(ax^2+bx^3\right)^{5/2}}{3003b^4x^4}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^3}-\frac{16a\left(ax^2+bx^3\right)^{5/2}}{143b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}$$

Rubi [A] time = 0.17, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2016, 2002, 2014}

$$\frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{15015b^5x^5}-\frac{128a^3\left(ax^2+bx^3\right)^{5/2}}{3003b^4x^4}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^3}-\frac{16a\left(ax^2+bx^3\right)^{5/2}}{143b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (256*a^4*(a*x^2 + b*x^3)^(5/2))/(15015*b^5*x^5) - (128*a^3*(a*x^2 + b*x^3)^
(5/2))/(3003*b^4*x^4) + (32*a^2*(a*x^2 + b*x^3)^(5/2))/(429*b^3*x^3) - (16*
a*(a*x^2 + b*x^3)^(5/2))/(143*b^2*x^2) + (2*(a*x^2 + b*x^3)^(5/2))/(13*b*x)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int x\left(ax^2+bx^3\right)^{3/2}dx &= \frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{(8a)\int\left(ax^2+bx^3\right)^{3/2}dx}{13b}\\
&= -\frac{16a\left(ax^2+bx^3\right)^{5/2}}{143b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}+\frac{\left(48a^2\right)\int\frac{\left(ax^2+bx^3\right)^{3/2}}{x}dx}{143b^2}\\
&= \frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^3}-\frac{16a\left(ax^2+bx^3\right)^{5/2}}{143b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{\left(64a^3\right)\int\frac{\left(ax^2+bx^3\right)^{3/2}}{x^2}dx}{429b^3}\\
&= -\frac{128a^3\left(ax^2+bx^3\right)^{5/2}}{3003b^4x^4}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^3}-\frac{16a\left(ax^2+bx^3\right)^{5/2}}{143b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}+\\
&= \frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{15015b^5x^5}-\frac{128a^3\left(ax^2+bx^3\right)^{5/2}}{3003b^4x^4}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^3}-\frac{16a\left(ax^2+bx^3\right)^{5/2}}{143b^2x^2}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 69, normalized size = 0.51

$$\frac{2x(a+bx)^3\left(128a^4-320a^3bx+560a^2b^2x^2-840ab^3x^3+1155b^4x^4\right)}{15015b^5\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(a + b*x)^3*(128*a^4 - 320*a^3*b*x + 560*a^2*b^2*x^2 - 840*a*b^3*x^3 +
 1155*b^4*x^4))/(15015*b^5*Sqrt[x^2*(a + b*x)])
```

IntegrateAlgebraic [A] time = 4.56, size = 75, normalized size = 0.55

$$\frac{2(a+bx)\left(x^2(a+bx)\right)^{3/2}\left(3003a^4-8580a^3(a+bx)+10010a^2(a+bx)^2-5460a(a+bx)^3+1155(a+bx)^4\right)}{15015b^5x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*(a + b*x)*(x^2*(a + b*x))^(3/2)*(3003*a^4 - 8580*a^3*(a + b*x) + 10010*a
^2*(a + b*x)^2 - 5460*a*(a + b*x)^3 + 1155*(a + b*x)^4))/(15015*b^5*x^3)
```

fricas [A] time = 0.39, size = 84, normalized size = 0.62

$$\frac{2\left(1155\,b^6x^6+1470\,ab^5x^5+35\,a^2b^4x^4-40\,a^3b^3x^3+48\,a^4b^2x^2-64\,a^5bx+128\,a^6\right)\sqrt{bx^3+ax^2}}{15015\,b^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15015*(1155*b^6*x^6 + 1470*a*b^5*x^5 + 35*a^2*b^4*x^4 - 40*a^3*b^3*x^3 +
48*a^4*b^2*x^2 - 64*a^5*b*x + 128*a^6)*sqrt(b*x^3 + a*x^2)/(b^5*x)
```

giac [B] time = 0.19, size = 246, normalized size = 1.81

$$-\frac{256\,a^{\frac{13}{2}}\operatorname{sgn}(x)}{15015\,b^5}+\frac{2\left(\frac{143\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}\,a^4\right)a^2\operatorname{sgn}(x)}{b^4}+\frac{130\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^2-1386\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4-693\,\sqrt{bx+a}\,a^5\right)a\operatorname{sgn}(x)}{b^4}+\frac{15\left(231\,(bx+a)^{\frac{13}{2}}-1638\,(bx+a)^{\frac{11}{2}}a+5005\,(bx+a)^{\frac{9}{2}}a^2-8580\,(bx+a)^{\frac{7}{2}}a^3+9009\,(bx+a)^{\frac{5}{2}}a^4-6006\,(bx+a)^{\frac{3}{2}}a^5+3003\,\sqrt{bx+a}\,a^6\right)\operatorname{sgn}(x)}{b^4}\right)}{45045\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -256/15015*a^(13/2)*sgn(x)/b^5 + 2/45045*(143*(35*(b*x + a)^(9/2) - 180*(b*
x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sq
rt(b*x + a)*a^4)*a^2*sgn(x)/b^4 + 130*(63*(b*x + a)^(11/2) - 385*(b*x + a)^
(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x +
a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a*sgn(x)/b^4 + 15*(231*(b*x + a)^(13/
2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7
/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b
*x + a)*a^6)*sgn(x)/b^4)/b
```

maple [A] time = 0.05, size = 68, normalized size = 0.50

$$\frac{2\,(bx+a)\left(1155x^4b^4-840a\,b^3x^3+560a^2x^2b^2-320a^3xb+128a^4\right)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{15015b^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] 2/15015*(b*x+a)*(1155*b^4*x^4-840*a*b^3*x^3+560*a^2*b^2*x^2-320*a^3*b*x+128
*a^4)*(b*x^3+a*x^2)^(3/2)/b^5/x^3
```

maxima [A] time = 1.55, size = 75, normalized size = 0.55

$$\frac{2\left(1155\,b^6x^6+1470\,ab^5x^5+35\,a^2b^4x^4-40\,a^3b^3x^3+48\,a^4b^2x^2-64\,a^5bx+128\,a^6\right)\sqrt{bx+a}}{15015\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(1155*b^6*x^6 + 1470*a*b^5*x^5 + 35*a^2*b^4*x^4 - 40*a^3*b^3*x^3 +
48*a^4*b^2*x^2 - 64*a^5*b*x + 128*a^6)*sqrt(b*x + a)/b^5
```

mupad [B] time = 5.24, size = 69, normalized size = 0.51

$$\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(a+b\,x\right)^2\,\left(128\,a^4-320\,a^3\,b\,x+560\,a^2\,b^2\,x^2-840\,a\,b^3\,x^3+1155\,b^4\,x^4\right)}{15015\,b^5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(128*a^4 + 1155*b^4*x^4 - 840*a*b^3*x^
3 + 560*a^2*b^2*x^2 - 320*a^3*b*x))/(15015*b^5*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(x^2\,(a+bx)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x*(x**2*(a + b*x))**(3/2), x)
```

3.163 $\int \left(ax^2+bx^3\right)^{3/2} \, dx$

Optimal. Leaf size=108

$$-\frac{32a^3\left(ax^2+bx^3\right)^{5/2}}{1155b^4x^5}+\frac{16a^2\left(ax^2+bx^3\right)^{5/2}}{231b^3x^4}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{33b^2x^3}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}$$

Rubi [A] time = 0.14, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2002, 2016, 2014}

$$-\frac{32a^3\left(ax^2+bx^3\right)^{5/2}}{1155b^4x^5}+\frac{16a^2\left(ax^2+bx^3\right)^{5/2}}{231b^3x^4}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{33b^2x^3}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-32*a^3*(a*x^2 + b*x^3)^(5/2))/(1155*b^4*x^5) + (16*a^2*(a*x^2 + b*x^3)^(5
/2))/(231*b^3*x^4) - (4*a*(a*x^2 + b*x^3)^(5/2))/(33*b^2*x^3) + (2*(a*x^2 +
 b*x^3)^(5/2))/(11*b*x^2)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \left(ax^2+bx^3\right)^{3/2} dx = \frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2} - \frac{(6a)\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x} dx}{11b}$$
$$= -\frac{4a\left(ax^2+bx^3\right)^{5/2}}{33b^2x^3} + \frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2} + \frac{\left(8a^2\right)\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^2} dx}{33b^2}$$
$$= \frac{16a^2\left(ax^2+bx^3\right)^{5/2}}{231b^3x^4} - \frac{4a\left(ax^2+bx^3\right)^{5/2}}{33b^2x^3} + \frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2} - \frac{\left(16a^3\right)\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^3} dx}{231b^3}$$
$$= -\frac{32a^3\left(ax^2+bx^3\right)^{5/2}}{1155b^4x^5} + \frac{16a^2\left(ax^2+bx^3\right)^{5/2}}{231b^3x^4} - \frac{4a\left(ax^2+bx^3\right)^{5/2}}{33b^2x^3} + \frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}$$

Mathematica [A]   time = 0.03, size = 58, normalized size = 0.54

$$\frac{2x(a+bx)^3\left(-16a^3+40a^2bx-70ab^2x^2+105b^3x^3\right)}{1155b^4\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(a + b*x)^3*(-16*a^3 + 40*a^2*b*x - 70*a*b^2*x^2 + 105*b^3*x^3))/(1155
*b^4*Sqrt[x^2*(a + b*x)])
```

IntegrateAlgebraic [A]   time = 4.72, size = 84, normalized size = 0.78

$$-\frac{2\left(x^2(a+bx)\right)^{3/2}\left(231a^3(a+bx)^{5/2}-495a^2(a+bx)^{7/2}-105(a+bx)^{11/2}+385a(a+bx)^{9/2}\right)}{1155b^4x^3(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-2*(x^2*(a + b*x))^(3/2)*(231*a^3*(a + b*x)^(5/2) - 495*a^2*(a + b*x)^(7/2
) + 385*a*(a + b*x)^(9/2) - 105*(a + b*x)^(11/2)))/(1155*b^4*x^3*(a + b*x)^
(3/2))
```

fricas [A]   time = 0.40, size = 73, normalized size = 0.68

$$\frac{2\left(105\,b^5x^5+140\,ab^4x^4+5\,a^2b^3x^3-6\,a^3b^2x^2+8\,a^4bx-16\,a^5\right)\sqrt{bx^3+ax^2}}{1155\,b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^4
*b*x - 16*a^5)*sqrt(b*x^3 + a*x^2)/(b^4*x)
```

giac [B]   time = 0.23, size = 210, normalized size = 1.94

$$\frac{32\,a^{\frac{11}{2}}\operatorname{sgn}(x)}{1155\,b^4} + \frac{2\left(\frac{99\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)a^2\operatorname{sgn}(x)}{b^3} + \frac{22\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}\,a^4\right)a\operatorname{sgn}(x)}{b^3} + \frac{5\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^2-1386\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4-693\,\sqrt{bx+a}\,a^5\right)\operatorname{sgn}(x)}{b^3}\right)}{3465\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 32/1155*a^(11/2)*sgn(x)/b^4 + 2/3465*(99*(5*(b*x + a)^(7/2) - 21*(b*x + a)^
(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2*sgn(x)/b^3 + 2
2*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 4
20*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a*sgn(x)/b^3 + 5*(63*(b*x +
 a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x +
a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*sgn(x)/b^3
)/b
```

maple [A] time = 0.04, size = 57, normalized size = 0.53

$$-\frac{2\,(bx+a)\left(-105b^3x^3+70a\,b^2x^2-40a^2bx+16a^3\right)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{1155b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -2/1155*(b*x+a)*(-105*b^3*x^3+70*a*b^2*x^2-40*a^2*b*x+16*a^3)*(b*x^3+a*x^2)
^(3/2)/b^4/x^3
```

maxima [A] time = 1.44, size = 64, normalized size = 0.59

$$\frac{2\left(105\,b^5x^5+140\,ab^4x^4+5\,a^2b^3x^3-6\,a^3b^2x^2+8\,a^4bx-16\,a^5\right)\sqrt{bx+a}}{1155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^4
*b*x - 16*a^5)*sqrt(b*x + a)/b^4
```

mupad [B] time = 5.19, size = 58, normalized size = 0.54

$$-\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(a+b\,x\right)^2\,\left(16\,a^3-40\,a^2\,b\,x+70\,a\,b^2\,x^2-105\,b^3\,x^3\right)}{1155\,b^4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] -(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(16*a^3 - 105*b^3*x^3 + 70*a*b^2*x^2
- 40*a^2*b*x))/(1155*b^4*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ax^2+bx^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral((a*x**2 + b*x**3)**(3/2), x)
```

3.164 $\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x} \, dx$

Optimal. Leaf size=80

$$\frac{16a^2\left(ax^2+bx^3\right)^{5/2}}{315b^3x^5}-\frac{8a\left(ax^2+bx^3\right)^{5/2}}{63b^2x^4}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}$$

Rubi [A] time = 0.13, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{16a^2\left(ax^2+bx^3\right)^{5/2}}{315b^3x^5}-\frac{8a\left(ax^2+bx^3\right)^{5/2}}{63b^2x^4}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x,x]
```

```
[Out] (16*a^2*(a*x^2 + b*x^3)^(5/2))/(315*b^3*x^5) - (8*a*(a*x^2 + b*x^3)^(5/2))/
(63*b^2*x^4) + (2*(a*x^2 + b*x^3)^(5/2))/(9*b*x^3)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x} \, dx &= \frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{(4a)\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^2} \, dx}{9b}\\
&= -\frac{8a\left(ax^2+bx^3\right)^{5/2}}{63b^2x^4}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}+\frac{\left(8a^2\right)\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^3} \, dx}{63b^2}\\
&= \frac{16a^2\left(ax^2+bx^3\right)^{5/2}}{315b^3x^5}-\frac{8a\left(ax^2+bx^3\right)^{5/2}}{63b^2x^4}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 0.59

$$\frac{2x(a+bx)^3\left(8a^2-20abx+35b^2x^2\right)}{315b^3\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x,x]
```

```
[Out] (2*x*(a + b*x)^3*(8*a^2 - 20*a*b*x + 35*b^2*x^2))/(315*b^3*Sqrt[x^2*(a + b*
x)])
```

IntegrateAlgebraic [A] time = 4.66, size = 70, normalized size = 0.88

$$\frac{2\left(x^2(a+bx)\right)^{3/2}\left(63a^2(a+bx)^{5/2}+35(a+bx)^{9/2}-90a(a+bx)^{7/2}\right)}{315b^3x^3(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x^3)^(3/2)/x,x]
```

```
[Out] (2*(x^2*(a + b*x))^(3/2)*(63*a^2*(a + b*x)^(5/2) - 90*a*(a + b*x)^(7/2) + 3
5*(a + b*x)^(9/2)))/(315*b^3*x^3*(a + b*x)^(3/2))
```

fricas [A] time = 0.40, size = 62, normalized size = 0.78

$$\frac{2\left(35\,b^4x^4+50\,ab^3x^3+3\,a^2b^2x^2-4\,a^3bx+8\,a^4\right)\sqrt{bx^3+ax^2}}{315\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt(
b*x^3 + a*x^2)/(b^3*x)
```

giac [B] time = 0.24, size = 173, normalized size = 2.16

$$-\frac{16\,a^{\frac{9}{2}}\mathrm{sgn}(x)}{315\,b^3}+\frac{2\left(\frac{21\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a^2\mathrm{sgn}(x)}{b^2}+\frac{18\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)a\mathrm{sgn}(x)}{b^2}+\frac{\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}\,a^4\right)\mathrm{sgn}(x)}{b^2}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -16/315*a^(9/2)*sgn(x)/b^3 + 2/315*(21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3
/2)*a + 15*sqrt(b*x + a)*a^2)*a^2*sgn(x)/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(
b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a*sgn(x)/
b^2 + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2
 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*sgn(x)/b^2)/b
```

maple [A] time = 0.04, size = 46, normalized size = 0.58

$$\frac{2\,(bx+a)\left(35b^2x^2-20abx+8a^2\right)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{315b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x,x)
```

```
[Out] 2/315*(b*x+a)*(35*b^2*x^2-20*a*b*x+8*a^2)*(b*x^3+a*x^2)^(3/2)/b^3/x^3
```

maxima [A] time = 1.55, size = 53, normalized size = 0.66

$$\frac{2\left(35\,b^4x^4+50\,ab^3x^3+3\,a^2b^2x^2-4\,a^3bx+8\,a^4\right)\sqrt{bx+a}}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt(
b*x + a)/b^3
```

mupad [B] time = 5.18, size = 47, normalized size = 0.59

$$\frac{2\sqrt{b\,x^3+a\,x^2}\,\left(a+b\,x\right)^2\,\left(8\,a^2-20\,a\,b\,x+35\,b^2\,x^2\right)}{315\,b^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x,x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(8*a^2 + 35*b^2*x^2 - 20*a*b*x))/(315*
b^3*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\,(a+bx)\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x, x)
```

3.165 $\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^2} \, dx$

Optimal. Leaf size=52

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{35b^2x^5}$$

Rubi [A] time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{35b^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^2,x]
```

```
[Out] (-4*a*(a*x^2 + b*x^3)^(5/2))/(35*b^2*x^5) + (2*(a*x^2 + b*x^3)^(5/2))/(7*b*
x^4)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^2} \, dx &= \frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{(2a)\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^3} \, dx}{7b}\\ &= -\frac{4a\left(ax^2+bx^3\right)^{5/2}}{35b^2x^5}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 0.69

$$\frac{2x(a+bx)^3(5bx-2a)}{35b^2\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^2,x]
```

```
[Out] (2*x*(a + b*x)^3*(-2*a + 5*b*x))/(35*b^2*Sqrt[x^2*(a + b*x)])
```

IntegrateAlgebraic [A] time = 6.44, size = 60, normalized size = 1.15

$$-\frac{2\left(x^2(a+bx)\right)^{3/2}\left(2a^3-a^2bx-8ab^2x^2-5b^3x^3\right)}{35b^2x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x^3)^(3/2)/x^2,x]
```

```
[Out] (-2*(x^2*(a + b*x))^(3/2)*(2*a^3 - a^2*b*x - 8*a*b^2*x^2 - 5*b^3*x^3))/(35*
b^2*x^3*(a + b*x))
```

fricas [A] time = 0.39, size = 50, normalized size = 0.96

$$\frac{2\left(5\,b^3x^3+8\,ab^2x^2+a^2bx-2\,a^3\right)\sqrt{bx^3+ax^2}}{35\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x^3 + a*x^2)/(b^2*x
)
```

giac [B] time = 0.18, size = 136, normalized size = 2.62

$$\frac{4\,a^{\frac{7}{2}}\mathrm{sgn}(x)}{35\,b^2}+\frac{2\left(\frac{35\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)a^2\mathrm{sgn}(x)}{b}+\frac{14\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a\mathrm{sgn}(x)}{b}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)\mathrm{sgn}(x)}{b}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 4/35*a^(7/2)*sgn(x)/b^2 + 2/105*(35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a
^2*sgn(x)/b + 14*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x +
a)*a^2)*a*sgn(x)/b + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x
+ a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*sgn(x)/b)/b
```

maple [A] time = 0.04, size = 35, normalized size = 0.67

$$-\frac{2\,(bx+a)\,(-5bx+2a)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{35b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^2,x)
```

```
[Out] -2/35*(b*x+a)*(-5*b*x+2*a)*(b*x^3+a*x^2)^(3/2)/b^2/x^3
```

maxima [A] time = 1.47, size = 41, normalized size = 0.79

$$\frac{2\left(5\,b^3x^3+8\,ab^2x^2+a^2bx-2\,a^3\right)\sqrt{bx+a}}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x + a)/b^2
```

mupad [B] time = 5.17, size = 36, normalized size = 0.69

$$-\frac{2\,\left(2\,a-5\,b\,x\right)\,\sqrt{b\,x^3+a\,x^2}\,{\left(a+b\,x\right)}^2}{35\,b^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^2,x)
[Out] -(2*(2*a - 5*b*x)*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2)/(35*b^2*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^2\,(a+bx)\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**2,x)
[Out] Integral((x**2*(a + b*x))**(3/2)/x**2, x)
```

3.166 $\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^3} \, dx$

Optimal. Leaf size=25

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{5bx^5}$$

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^3,x]
```

```
[Out] (2*(a*x^2 + b*x^3)^(5/2))/(5*b*x^5)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^3} \, dx = \frac{2\left(ax^2+bx^3\right)^{5/2}}{5bx^5}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.92

$$\frac{2\left(x^2(a+bx)\right)^{5/2}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^3,x]
```

```
[Out] (2*(x^2*(a + b*x))^(5/2))/(5*b*x^5)
```

IntegrateAlgebraic [A] time = 8.59, size = 28, normalized size = 1.12

$$\frac{2(a+bx)\left(x^2(a+bx)\right)^{3/2}}{5bx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x^3)^(3/2)/x^3,x]
```

```
[Out] (2*(a + b*x)*(x^2*(a + b*x))^(3/2))/(5*b*x^3)
```

fricas [A] time = 0.40, size = 37, normalized size = 1.48

$$\frac{2\left(b^2x^2+2\,abx+a^2\right)\sqrt{bx^3+ax^2}}{5\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x^3 + a*x^2)/(b*x)
```

giac [B] time = 0.16, size = 89, normalized size = 3.56

$$-\frac{2\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{5\,b}+\frac{2\left(15\,\sqrt{bx+a}\,a^2\mathrm{sgn}(x)+10\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)a\mathrm{sgn}(x)+\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)\mathrm{sgn}(x)\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -2/5*a^(5/2)*sgn(x)/b + 2/15*(15*sqrt(b*x + a)*a^2*sgn(x) + 10*((b*x + a)^(
3/2) - 3*sqrt(b*x + a)*a)*a*sgn(x) + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2
)*a + 15*sqrt(b*x + a)*a^2)*sgn(x))/b
```

maple [A] time = 0.04, size = 27, normalized size = 1.08

$$\frac{2\,(bx+a)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{5b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^3,x)
```

```
[Out] 2/5*(b*x+a)*(b*x^3+a*x^2)^(3/2)/b/x^3
```

maxima [A] time = 1.39, size = 28, normalized size = 1.12

$$\frac{2\left(b^2x^2+2\,abx+a^2\right)\sqrt{bx+a}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/b
```

mupad [B] time = 5.62, size = 28, normalized size = 1.12

$$\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(a+b\,x\right)^2}{5\,b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^3,x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2)/(5*b*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^2\,(a+bx)\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**3,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**3, x)
```

3.167 $\int \frac{(ax^2+bx^3)^{3/2}}{x^4} dx$

Optimal. Leaf size=74

$$-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)+\frac{2a\sqrt{ax^2+bx^3}}{x}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}$$

Rubi [A] time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2021, 2008, 206}

$$-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)+\frac{2a\sqrt{ax^2+bx^3}}{x}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^4,x]
```

```
[Out] (2*a*Sqrt[a*x^2 + b*x^3])/x + (2*(a*x^2 + b*x^3)^(3/2))/(3*x^3) - 2*a^(3/2)
*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}\,dx &= \frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}+a\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx\\
&= \frac{2a\sqrt{ax^2+bx^3}}{x}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}+a^2\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx\\
&= \frac{2a\sqrt{ax^2+bx^3}}{x}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}-\left(2a^2\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)\\
&= \frac{2a\sqrt{ax^2+bx^3}}{x}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 68, normalized size = 0.92

$$\frac{2x\sqrt{a+bx}\left(\sqrt{a+bx}\,(4a+bx)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{3\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^4,x]
```

```
[Out] (2*x*Sqrt[a + b*x]*(Sqrt[a + b*x]*(4*a + b*x) - 3*a^(3/2)*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]]))/(3*Sqrt[x^2*(a + b*x)])
```

IntegrateAlgebraic [A] time = 10.83, size = 76, normalized size = 1.03

$$\frac{\left(x^2(a+bx)\right)^{3/2}\left(\frac{2}{3}\left((a+bx)^{3/2}+3a\sqrt{a+bx}\right)-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{x^3(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x^3)^(3/2)/x^4,x]
```

```
[Out] ((x^2*(a + b*x))^(3/2)*((2*(3*a*Sqrt[a + b*x] + (a + b*x)^(3/2)))/3 - 2*a^(
3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(x^3*(a + b*x)^(3/2))
```

fricas [A] time = 0.42, size = 130, normalized size = 1.76

$$\left[\frac{3\,a^{\frac{3}{2}}x\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}\,(bx+4\,a)}{3\,x},\ \frac{2\left(3\,\sqrt{-a}\,ax\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}\,(bx+4\,a)\right)}{3\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/3*(3*a^(3/2)*x*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2)
+ 2*sqrt(b*x^3 + a*x^2)*(b*x + 4*a))/x, 2/3*(3*sqrt(-a)*a*x*arctan(sqrt(b*x
^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(b*x + 4*a))/x]
```

giac [A] time = 0.17, size = 85, normalized size = 1.15

$$\frac{2\,a^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}}+\frac{2}{3}\,(bx+a)^{\frac{3}{2}}\mathrm{sgn}(x)+2\,\sqrt{bx+a}\,a\mathrm{sgn}(x)-\frac{2\left(3\,a^2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+4\,\sqrt{-a}\,a^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{3\,\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 2*a^2*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2/3*(b*x + a)^(3/2)*
sgn(x) + 2*sqrt(b*x + a)*a*sgn(x) - 2/3*(3*a^2*arctan(sqrt(a)/sqrt(-a)) + 4
*sqrt(-a)*a^(3/2))*sgn(x)/sqrt(-a)
```

maple [A] time = 0.05, size = 61, normalized size = 0.82

$$\frac{2\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(-3a^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+3\sqrt{bx+a}\,\,a+(bx+a)^{\frac{3}{2}}\right)}{3\,(bx+a)^{\frac{3}{2}}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^4,x)
```

```
[Out] 2/3*(b*x^3+a*x^2)^(3/2)*(-3*a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+(b*x+a)^
(3/2)+3*(b*x+a)^(1/2)*a)/x^3/(b*x+a)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + ax^2\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3 + a\,x^2\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^4,x)
```

```
[Out] int((a*x^2 + b*x^3)^(3/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(a + bx\right)\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**4,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**4, x)
```

3.168 $\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=73

$$\frac{3b\sqrt{ax^2+bx^3}}{x}-3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)-\frac{(ax^2+bx^3)^{3/2}}{x^4}$$

Rubi [A] time = 0.09, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2021, 2008, 206}

$$-\frac{(ax^2+bx^3)^{3/2}}{x^4}+\frac{3b\sqrt{ax^2+bx^3}}{x}-3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^5,x]
```

```
[Out] (3*b*Sqrt[a*x^2 + b*x^3])/x - (a*x^2 + b*x^3)^(3/2)/x^4 - 3*Sqrt[a]*b*ArcTa
nh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^5}\,dx &= -\frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}+\frac{1}{2}(3b)\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx\\ &= \frac{3b\sqrt{ax^2+bx^3}}{x}-\frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}+\frac{1}{2}(3ab)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx\\ &= \frac{3b\sqrt{ax^2+bx^3}}{x}-\frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}-(3ab)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)\\ &= \frac{3b\sqrt{ax^2+bx^3}}{x}-\frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}-3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.55

$$\frac{2b\left(x^2(a+bx)\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx}{a}+1\right)}{5a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^5,x]
```

```
[Out] (2*b*(x^2*(a + b*x))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x)/a])/(5*
a^2*x^5)
```

IntegrateAlgebraic [A] time = 13.19, size = 75, normalized size = 1.03

$$\frac{\left(x^2(a+bx)\right)^{3/2}\left(\frac{\sqrt{a+bx}\,(2(a+bx)-3a)}{x}-3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{x^3(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x^3)^(3/2)/x^5,x]
```

```
[Out] ((x^2*(a + b*x))^(3/2)*((Sqrt[a + b*x]*(-3*a + 2*(a + b*x)))/x - 3*Sqrt[a]*
b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(x^3*(a + b*x)^(3/2))
```

fricas [A] time = 0.43, size = 136, normalized size = 1.86

$$\left[\frac{3\sqrt{a}\,bx^2\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}\,(2\,bx-a)}{2\,x^2},\ \frac{3\,\sqrt{-a}\,bx^2\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}\,(2\,bx-a)}{x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/2*(3*sqrt(a)*b*x^2*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x
^2) + 2*sqrt(b*x^3 + a*x^2)*(2*b*x - a))/x^2, (3*sqrt(-a)*b*x^2*arctan(sqrt
(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(2*b*x - a))/x^2]
```

giac [A] time = 0.21, size = 62, normalized size = 0.85

$$\frac{\frac{3\,ab^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}}+2\,\sqrt{bx+a}\,b^2\mathrm{sgn}(x)-\frac{\sqrt{bx+a}\,ab\mathrm{sgn}(x)}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] (3*a*b^2*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*sqrt(b*x + a)*b
^2*sgn(x) - sqrt(b*x + a)*a*b*sgn(x)/x)/b
```

maple [A] time = 0.06, size = 72, normalized size = 0.99

$$-\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(3abx\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-2\sqrt{bx+a}\,\sqrt{a}\,bx+\sqrt{bx+a}\,a^{\frac{3}{2}}\right)}{\left(bx+a\right)^{\frac{3}{2}}\sqrt{a}\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^5,x)
```

```
[Out] -(b*x^3+a*x^2)^(3/2)*((b*x+a)^(1/2)*a^(3/2)-2*(b*x+a)^(1/2)*x*b*a^(1/2)+3*a
rctanh((b*x+a)^(1/2)/a^(1/2))*x*a*b)/x^4/(b*x+a)^(3/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+ax^{2}\right)^{\frac{3}{2}}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^3+a\,x^2\right)}^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^5,x)
```

```
[Out] int((a*x^2 + b*x^3)^(3/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}\left(a+bx\right)\right)^{\frac{3}{2}}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**5,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**5, x)
```

3.169 $\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=81

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4\sqrt{a}}-\frac{3b\sqrt{ax^2+bx^3}}{4x^2}-\frac{(ax^2+bx^3)^{3/2}}{2x^5}$$

Rubi [A] time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2020, 2008, 206}

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4\sqrt{a}}-\frac{3b\sqrt{ax^2+bx^3}}{4x^2}-\frac{(ax^2+bx^3)^{3/2}}{2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^6,x]
```

```
[Out] (-3*b*Sqrt[a*x^2 + b*x^3])/(4*x^2) - (a*x^2 + b*x^3)^(3/2)/(2*x^5) - (3*b^2
*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(4*Sqrt[a])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx &= -\frac{(ax^2+bx^3)^{3/2}}{2x^5}+\frac{1}{4}(3b)\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx\\
&= -\frac{3b\sqrt{ax^2+bx^3}}{4x^2}-\frac{(ax^2+bx^3)^{3/2}}{2x^5}+\frac{1}{8}\left(3b^2\right)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx\\
&= -\frac{3b\sqrt{ax^2+bx^3}}{4x^2}-\frac{(ax^2+bx^3)^{3/2}}{2x^5}-\frac{1}{4}\left(3b^2\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)\\
&= -\frac{3b\sqrt{ax^2+bx^3}}{4x^2}-\frac{(ax^2+bx^3)^{3/2}}{2x^5}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 72, normalized size = 0.89

$$-\frac{2a^2+3b^2x^2\sqrt{\frac{bx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+7abx+5b^2x^2}{4x\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^6,x]
```

```
[Out] -1/4*(2*a^2 + 7*a*b*x + 5*b^2*x^2 + 3*b^2*x^2*Sqrt[1 + (b*x)/a]*ArcTanh[Sqr
t[1 + (b*x)/a]])/(x*Sqrt[x^2*(a + b*x)])
```

IntegrateAlgebraic [A] time = 15.71, size = 82, normalized size = 1.01

$$\frac{\left(x^2(a+bx)\right)^{3/2}\left(-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{\sqrt{a+bx}\,(5(a+bx)-3a)}{4x^2}\right)}{x^3(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x^3)^(3/2)/x^6,x]
```

```
[Out] ((x^2*(a + b*x))^(3/2)*(-1/4*(Sqrt[a + b*x]*(-3*a + 5*(a + b*x)))/x^2 - (3*
b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*Sqrt[a])))/(x^3*(a + b*x)^(3/2))
```

fricas [A] time = 0.43, size = 154, normalized size = 1.90

$$\left[\frac{3\sqrt{a}\,b^2x^3\log\left(\frac{bx^2+2\,ax-2\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\sqrt{bx^3+ax^2}\left(5\,abx+2\,a^2\right)}{8\,ax^3},\ \frac{3\sqrt{-a}\,b^2x^3\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)-\sqrt{bx^3+ax^2}\left(5\,abx+2\,a^2\right)}{4\,ax^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))
/x^2) - 2*sqrt(b*x^3 + a*x^2)*(5*a*b*x + 2*a^2))/(a*x^3), 1/4*(3*sqrt(-a)*b
^2*x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) - sqrt(b*x^3 + a*x^2)*(5*
a*b*x + 2*a^2))/(a*x^3)]
```

giac [A] time = 0.26, size = 70, normalized size = 0.86

$$\frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}}-\frac{5\,(bx+a)^{\frac{3}{2}}b^3\mathrm{sgn}(x)-3\sqrt{bx+a}\,ab^3\mathrm{sgn}(x)}{b^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) - (5*(b*x + a)^(3
/2)*b^3*sgn(x) - 3*sqrt(b*x + a)*a*b^3*sgn(x))/(b^2*x^2))/b
```

maple [A] time = 0.06, size = 74, normalized size = 0.91

$$-\frac{\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(3b^2x^2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-3\sqrt{bx+a}\,a^{\frac{3}{2}}+5\,(bx+a)^{\frac{3}{2}}\sqrt{a}\right)}{4\,(bx+a)^{\frac{3}{2}}\sqrt{a}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^6,x)
```

```
[Out] -1/4*(b*x^3+a*x^2)^(3/2)*(3*arctanh((b*x+a)^(1/2)/a^(1/2))*x^2*b^2+5*(b*x+a
)^(3/2)*a^(1/2)-3*(b*x+a)^(1/2)*a^(3/2))/x^5/(b*x+a)^(3/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + ax^2\right)^{\frac{3}{2}}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3 + a\,x^2\right)^{3/2}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^6,x)
```

```
[Out] int((a*x^2 + b*x^3)^(3/2)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(a + bx\right)\right)^{\frac{3}{2}}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**6,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**6, x)
```

3.170 $\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=109

$$\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{3/2}}-\frac{b^2\sqrt{ax^2+bx^3}}{8ax^2}-\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}$$

Rubi [A] time = 0.13, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2008, 206}

$$\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{3/2}}-\frac{b^2\sqrt{ax^2+bx^3}}{8ax^2}-\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^7,x]
```

```
[Out] -(b*Sqrt[a*x^2 + b*x^3])/(4*x^3) - (b^2*Sqrt[a*x^2 + b*x^3])/(8*a*x^2) - (a
*x^2 + b*x^3)^(3/2)/(3*x^6) + (b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
)/(8*a^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^7}\,dx &= -\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}+\frac{1}{2}b\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx \\
&= -\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}+\frac{1}{8}b^2\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx \\
&= -\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{b^2\sqrt{ax^2+bx^3}}{8ax^2}-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}-\frac{b^3\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{16a} \\
&= -\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{b^2\sqrt{ax^2+bx^3}}{8ax^2}-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}+\frac{b^3\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{8a} \\
&= -\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{b^2\sqrt{ax^2+bx^3}}{8ax^2}-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}+\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{3/2}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 42, normalized size = 0.39

$$
\frac{2b^3\left(x^2(a+bx)\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{bx}{a}+1\right)}{5a^4x^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^7,x]
```

```
[Out] (2*b^3*(x^2*(a + b*x))^(5/2)*Hypergeometric2F1[5/2, 4, 7/2, 1 + (b*x)/a])/(
5*a^4*x^5)
```

IntegrateAlgebraic [A] time = 15.70, size = 97, normalized size = 0.89

$$
\frac{\left(x^2(a+bx)\right)^{3/2}\left(\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{3/2}}+\frac{\sqrt{a+bx}\left(3a^2-8a(a+bx)-3(a+bx)^2\right)}{24ax^3}\right)}{x^3(a+bx)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x^3)^(3/2)/x^7,x]
```

```
[Out] ((x^2*(a + b*x))^(3/2)*((Sqrt[a + b*x]*(3*a^2 - 8*a*(a + b*x) - 3*(a + b*x)
^2))/(24*a*x^3) + (b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(3/2))))/(x^3*(
a + b*x)^(3/2))
```

fricas [A] time = 0.43, size = 175, normalized size = 1.61

$$
\left[\frac{3\sqrt{a}\,b^3x^4\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)-2\left(3\,ab^2x^2+14\,a^2bx+8\,a^3\right)\sqrt{bx^3+ax^2}}{48\,a^2x^4},\ -\frac{3\sqrt{-a}\,b^3x^4\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+\left(3\,ab^2x^2+14\,a^2bx+8\,a^3\right)\sqrt{bx^3+ax^2}}{24\,a^2x^4}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a)
)/x^2) - 2*(3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^2*x^4
), -1/24*(3*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (
3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^2*x^4)]
```

giac [A] time = 0.23, size = 92, normalized size = 0.84

$$-\frac{\frac{3\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}\,a}+\frac{3\,(bx+a)^{\frac{5}{2}}b^4\mathrm{sgn}(x)+8\,(bx+a)^{\frac{3}{2}}ab^4\mathrm{sgn}(x)-3\,\sqrt{bx+a}\,a^2b^4\mathrm{sgn}(x)}{ab^3x^3}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/24*(3*b^4*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a) + (3*(b*x +
 a)^(5/2)*b^4*sgn(x) + 8*(b*x + a)^(3/2)*a*b^4*sgn(x) - 3*sqrt(b*x + a)*a^2
*b^4*sgn(x))/(a*b^3*x^3))/b
```

maple [A] time = 0.06, size = 87, normalized size = 0.80

$$-\frac{\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(-3a\,b^3x^3\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-3\sqrt{bx+a}\;a^{\frac{7}{2}}+8\,(bx+a)^{\frac{3}{2}}\,a^{\frac{5}{2}}+3\,(bx+a)^{\frac{5}{2}}\,a^{\frac{3}{2}}\right)}{24\,(bx+a)^{\frac{3}{2}}\,a^{\frac{5}{2}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^7,x)
```

```
[Out] -1/24*(b*x^3+a*x^2)^(3/2)*(3*(b*x+a)^(5/2)*a^(3/2)-3*arctanh((b*x+a)^(1/2)/
a^(1/2))*x^3*a*b^3+8*(b*x+a)^(3/2)*a^(5/2)-3*(b*x+a)^(1/2)*a^(7/2))/x^6/(b*
x+a)^(3/2)/a^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^3+ax^2\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(b\,x^3+a\,x^2\right)^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^7,x)
```

```
[Out] int((a*x^2 + b*x^3)^(3/2)/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^2\,(a+bx)\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**7,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**7, x)
```

3.171 $\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^8} dx$

Optimal. Leaf size=137

$$-\frac{3b^4 \tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{64a^{5/2}}+\frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}-\frac{b\sqrt{ax^2+bx^3}}{8x^4}$$

Rubi [A] time = 0.18, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2008, 206}

$$\frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2}-\frac{3b^4 \tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{64a^{5/2}}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}-\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^8,x]
```

```
[Out] -(b*Sqrt[a*x^2 + b*x^3])/(8*x^4) - (b^2*Sqrt[a*x^2 + b*x^3])/(32*a*x^3) + (
3*b^3*Sqrt[a*x^2 + b*x^3])/(64*a^2*x^2) - (a*x^2 + b*x^3)^(3/2)/(4*x^7) - (
3*b^4*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(64*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^8}\,dx &= -\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}+\frac{1}{8}(3b)\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx \\
&= -\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}+\frac{1}{16}b^2\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx \\
&= -\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}-\frac{\left(3b^3\right)\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{64a} \\
&= -\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}+\frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2}-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}+\frac{\left(3b^4\right)\int \frac{1}{\sqrt{ax^2+bx^3}}\,d}{128a^2} \\
&= -\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}+\frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2}-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}-\frac{\left(3b^4\right)\text{Subst}\left(\int \frac{1}{1-a}\right.}{64} \\
&= -\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}+\frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2}-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}-\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+b}}\right.}{64a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 42, normalized size = 0.31

$$-\frac{2b^4\left(x^2(a+bx)\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};\frac{bx}{a}+1\right)}{5a^5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^8,x]
```

```
[Out] (-2*b^4*(x^2*(a + b*x))^(5/2)*Hypergeometric2F1[5/2, 5, 7/2, 1 + (b*x)/a])/
(5*a^5*x^5)
```

IntegrateAlgebraic [A] time = 16.00, size = 109, normalized size = 0.80

$$\frac{\left(x^2(a+bx)\right)^{3/2}\left(\frac{\sqrt{a+bx}\left(3a^3-11a^2(a+bx)-11a(a+bx)^2+3(a+bx)^3\right)}{64a^2x^4}-\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{5/2}}\right)}{x^3(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x^3)^(3/2)/x^8,x]
```

```
[Out] ((x^2*(a + b*x))^(3/2)*((Sqrt[a + b*x]*(3*a^3 - 11*a^2*(a + b*x) - 11*a*(a
+ b*x)^2 + 3*(a + b*x)^3))/(64*a^2*x^4) - (3*b^4*ArcTanh[Sqrt[a + b*x]/Sqrt
[a]])/(64*a^(5/2))))/(x^3*(a + b*x)^(3/2))
```

fricas [A] time = 0.42, size = 197, normalized size = 1.44

$$\left[\frac{3\sqrt{a}\,b^4x^5\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)+2\left(3\,ab^3x^3-2\,a^2b^2x^2-24\,a^3bx-16\,a^4\right)\sqrt{bx^3+ax^2}}{128\,a^3x^5},\ \frac{3\sqrt{-a}\,b^4x^5\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+\left(3\,ab^3x^3-2\,a^2b^2x^2-24\,a^3bx-16\,a^4\right)\sqrt{bx^3+ax^2}}{64\,a^3x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/128*(3*sqrt(a)*b^4*x^5*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a
))/x^2) + 2*(3*a*b^3*x^3 - 2*a^2*b^2*x^2 - 24*a^3*b*x - 16*a^4)*sqrt(b*x^3
+ a*x^2))/(a^3*x^5), 1/64*(3*sqrt(-a)*b^4*x^5*arctan(sqrt(b*x^3 + a*x^2)*sq
```

```
rt(-a)/(a*x)) + (3*a*b^3*x^3 - 2*a^2*b^2*x^2 - 24*a^3*b*x - 16*a^4)*sqrt(b*
x^3 + a*x^2))/(a^3*x^5)]
```

giac [A] time = 0.24, size = 109, normalized size = 0.80

$$\frac{\frac{3\,b^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}\,a^2}+\frac{3\,(bx+a)^{\frac{7}{2}}b^5\mathrm{sgn}(x)-11\,(bx+a)^{\frac{5}{2}}ab^5\mathrm{sgn}(x)-11\,(bx+a)^{\frac{3}{2}}a^2b^5\mathrm{sgn}(x)+3\,\sqrt{bx+a}\,a^3b^5\mathrm{sgn}(x)}{a^2b^4x^4}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/64*(3*b^5*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^2) + (3*(b*x
+ a)^(7/2)*b^5*sgn(x) - 11*(b*x + a)^(5/2)*a*b^5*sgn(x) - 11*(b*x + a)^(3/2
)*a^2*b^5*sgn(x) + 3*sqrt(b*x + a)*a^3*b^5*sgn(x))/(a^2*b^4*x^4))/b
```

maple [A] time = 0.05, size = 101, normalized size = 0.74

$$\frac{\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(-3a^2b^4x^4\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+3\sqrt{bx+a}\,a^{\frac{11}{2}}-11\,(bx+a)^{\frac{3}{2}}a^{\frac{9}{2}}-11\,(bx+a)^{\frac{5}{2}}a^{\frac{7}{2}}+3\,(bx+a)^{\frac{7}{2}}a^{\frac{5}{2}}\right)}{64\,(bx+a)^{\frac{3}{2}}a^{\frac{9}{2}}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^8,x)
```

```
[Out] 1/64*(b*x^3+a*x^2)^(3/2)*(3*(b*x+a)^(7/2)*a^(5/2)-11*(b*x+a)^(5/2)*a^(7/2)-
3*arctanh((b*x+a)^(1/2)/a^(1/2))*a^2*x^4*b^4-11*(b*x+a)^(3/2)*a^(9/2)+3*(b*
x+a)^(1/2)*a^(11/2))/x^7/(b*x+a)^(3/2)/a^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^3+ax^2\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(b\,x^3+a\,x^2\right)^{3/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^8,x)
```

```
[Out] int((a*x^2 + b*x^3)^(3/2)/x^8, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^2\,(a+bx)\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**8,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**8, x)
```

3.172 $\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=165

$$\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{128a^{7/2}}-\frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2}+\frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3}-\frac{b^2\sqrt{ax^2+bx^3}}{80ax^4}-\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}-\frac{3b\sqrt{ax^2+bx^3}}{40x^5}$$

Rubi [A] time = 0.24, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2008, 206}

$$-\frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2}+\frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3}+\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{128a^{7/2}}-\frac{b^2\sqrt{ax^2+bx^3}}{80ax^4}-\frac{3b\sqrt{ax^2+bx^3}}{40x^5}-\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^9,x]
```

```
[Out] (-3*b*Sqrt[a*x^2 + b*x^3])/(40*x^5) - (b^2*Sqrt[a*x^2 + b*x^3])/(80*a*x^4)
+ (b^3*Sqrt[a*x^2 + b*x^3])/(64*a^2*x^3) - (3*b^4*Sqrt[a*x^2 + b*x^3])/(128
*a^3*x^2) - (a*x^2 + b*x^3)^(3/2)/(5*x^8) + (3*b^5*ArcTanh[(Sqrt[a]*x)/Sqrt
[a*x^2 + b*x^3]])/(128*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(ax^2+bx^3)^{3/2}}{x^9}\,dx &= -\frac{(ax^2+bx^3)^{3/2}}{5x^8} + \frac{1}{10}(3b)\int \frac{\sqrt{ax^2+bx^3}}{x^6}\,dx \\
&= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5} - \frac{(ax^2+bx^3)^{3/2}}{5x^8} + \frac{1}{80}\left(3b^2\right)\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx \\
&= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5} - \frac{b^2\sqrt{ax^2+bx^3}}{80ax^4} - \frac{(ax^2+bx^3)^{3/2}}{5x^8} - \frac{b^3\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx}{32a} \\
&= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5} - \frac{b^2\sqrt{ax^2+bx^3}}{80ax^4} + \frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3} - \frac{(ax^2+bx^3)^{3/2}}{5x^8} + \frac{\left(3b^4\right)\int \frac{1}{x\sqrt{ax^2+b}}}{128a^2} \\
&= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5} - \frac{b^2\sqrt{ax^2+bx^3}}{80ax^4} + \frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3} - \frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2} - \frac{(ax^2+bx^3)^{3/2}}{5x^8} \\
&= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5} - \frac{b^2\sqrt{ax^2+bx^3}}{80ax^4} + \frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3} - \frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2} - \frac{(ax^2+bx^3)^{3/2}}{5x^8} \\
&= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5} - \frac{b^2\sqrt{ax^2+bx^3}}{80ax^4} + \frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3} - \frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2} - \frac{(ax^2+bx^3)^{3/2}}{5x^8}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 42, normalized size = 0.25

$$\frac{2b^5\left(x^2(a+bx)\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},6;\frac{7}{2};\frac{bx}{a}+1\right)}{5a^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^9,x]
```

```
[Out] (2*b^5*(x^2*(a + b*x))^(5/2)*Hypergeometric2F1[5/2, 6, 7/2, 1 + (b*x)/a])/(
5*a^6*x^5)
```

IntegrateAlgebraic [A] time = 16.40, size = 121, normalized size = 0.73

$$\frac{\left(x^2(a+bx)\right)^{3/2}\left(\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{128a^{7/2}}+\frac{\sqrt{a+bx}\left(15a^4-70a^3(a+bx)-128a^2(a+bx)^2+70a(a+bx)^3-15(a+bx)^4\right)}{640a^3x^5}\right)}{x^3(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x^3)^(3/2)/x^9,x]
```

```
[Out] ((x^2*(a + b*x))^(3/2)*((Sqrt[a + b*x]*(15*a^4 - 70*a^3*(a + b*x) - 128*a^2
*(a + b*x)^2 + 70*a*(a + b*x)^3 - 15*(a + b*x)^4))/(640*a^3*x^5) + (3*b^5*A
rcTanh[Sqrt[a + b*x]/Sqrt[a]])/(128*a^(7/2))))/(x^3*(a + b*x)^(3/2))
```

fricas [A] time = 0.41, size = 219, normalized size = 1.33

$$\left[\frac{15\sqrt{a}\,b^5x^6\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)-2\left(15\,ab^4x^4-10\,a^2b^3x^3+8\,a^3b^2x^2+176\,a^4bx+128\,a^5\right)\sqrt{bx^3+ax^2}}{1280\,a^4x^6},-\frac{15\sqrt{-a}\,b^5x^6\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+\left(15\,ab^4x^4-10\,a^2b^3x^3+8\,a^3b^2x^2+176\,a^4bx+128\,a^5\right)\sqrt{bx^3+ax^2}}{640\,a^4x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/1280*(15*sqrt(a)*b^5*x^6*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt
(a))/x^2) - 2*(15*a*b^4*x^4 - 10*a^2*b^3*x^3 + 8*a^3*b^2*x^2 + 176*a^4*b*x
```

```
+ 128*a^5)*sqrt(b*x^3 + a*x^2))/(a^4*x^6), -1/640*(15*sqrt(-a)*b^5*x^6*arct
an(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (15*a*b^4*x^4 - 10*a^2*b^3*x^3 + 8
*a^3*b^2*x^2 + 176*a^4*b*x + 128*a^5)*sqrt(b*x^3 + a*x^2))/(a^4*x^6)]
```

giac [A] time = 0.27, size = 126, normalized size = 0.76

$$-\frac{\frac{15\,b^{6}\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}\,a^{3}}+\frac{15\,(bx+a)^{\frac{9}{2}}b^{6}\mathrm{sgn}(x)-70\,(bx+a)^{\frac{7}{2}}ab^{6}\mathrm{sgn}(x)+128\,(bx+a)^{\frac{5}{2}}a^{2}b^{6}\mathrm{sgn}(x)+70\,(bx+a)^{\frac{3}{2}}a^{3}b^{6}\mathrm{sgn}(x)-15\,\sqrt{bx+a}\,a^{4}b^{6}\mathrm{sgn}(x)}{a^{3}b^{5}x^{5}}}{640\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/640*(15*b^6*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^3) + (15*(
b*x + a)^(9/2)*b^6*sgn(x) - 70*(b*x + a)^(7/2)*a*b^6*sgn(x) + 128*(b*x + a)
^(5/2)*a^2*b^6*sgn(x) + 70*(b*x + a)^(3/2)*a^3*b^6*sgn(x) - 15*sqrt(b*x + a
)*a^4*b^6*sgn(x))/(a^3*b^5*x^5))/b
```

maple [A] time = 0.05, size = 113, normalized size = 0.68

$$-\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(-15a^{3}b^{5}x^{5}\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-15\sqrt{bx+a}\;a^{\frac{15}{2}}+70\,(bx+a)^{\frac{3}{2}}\,a^{\frac{13}{2}}+128\,(bx+a)^{\frac{5}{2}}\,a^{\frac{11}{2}}-70\,(bx+a)^{\frac{7}{2}}\,a^{\frac{9}{2}}+15\,(bx+a)^{\frac{9}{2}}\,a^{\frac{7}{2}}\right)}{640\,(bx+a)^{\frac{3}{2}}\,a^{\frac{13}{2}}\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^9,x)
```

```
[Out] -1/640*(b*x^3+a*x^2)^(3/2)*(15*(b*x+a)^(9/2)*a^(7/2)-70*(b*x+a)^(7/2)*a^(9/
2)+128*(b*x+a)^(5/2)*a^(11/2)-15*arctanh((b*x+a)^(1/2)/a^(1/2))*a^3*x^5*b^5
+70*(b*x+a)^(3/2)*a^(13/2)-15*(b*x+a)^(1/2)*a^(15/2))/x^8/(b*x+a)^(3/2)/a^(
13/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^{3}+ax^{2}\right)^{\frac{3}{2}}}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^9, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(b\,x^{3}+a\,x^{2}\right)^{3/2}}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^9,x)
```

```
[Out] int((a*x^2 + b*x^3)^(3/2)/x^9, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^{2}\left(a+bx\right)\right)^{\frac{3}{2}}}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**9,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**9, x)
```

3.173 $\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=103

$$-\frac{32a^3\sqrt{ax^2+bx^3}}{35b^4x}+\frac{16a^2\sqrt{ax^2+bx^3}}{35b^3}-\frac{12ax\sqrt{ax^2+bx^3}}{35b^2}+\frac{2x^2\sqrt{ax^2+bx^3}}{7b}$$

Rubi [A] time = 0.15, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 1588}

$$-\frac{32a^3\sqrt{ax^2+bx^3}}{35b^4x}+\frac{16a^2\sqrt{ax^2+bx^3}}{35b^3}-\frac{12ax\sqrt{ax^2+bx^3}}{35b^2}+\frac{2x^2\sqrt{ax^2+bx^3}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (16*a^2*Sqrt[a*x^2 + b*x^3])/(35*b^3) - (32*a^3*Sqrt[a*x^2 + b*x^3])/(35*b^
4*x) - (12*a*x*Sqrt[a*x^2 + b*x^3])/(35*b^2) + (2*x^2*Sqrt[a*x^2 + b*x^3])/
(7*b)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx &= \frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{(6a)\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx}{7b}\\ &= -\frac{12ax\sqrt{ax^2+bx^3}}{35b^2}+\frac{2x^2\sqrt{ax^2+bx^3}}{7b}+\frac{\left(24a^2\right)\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx}{35b^2}\\ &= \frac{16a^2\sqrt{ax^2+bx^3}}{35b^3}-\frac{12ax\sqrt{ax^2+bx^3}}{35b^2}+\frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{\left(16a^3\right)\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{35b^3}\\ &= \frac{16a^2\sqrt{ax^2+bx^3}}{35b^3}-\frac{32a^3\sqrt{ax^2+bx^3}}{35b^4x}-\frac{12ax\sqrt{ax^2+bx^3}}{35b^2}+\frac{2x^2\sqrt{ax^2+bx^3}}{7b}\end{aligned}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 0.51

$$\frac{2\sqrt{x^2(a+bx)}\left(-16a^3+8a^2bx-6ab^2x^2+5b^3x^3\right)}{35b^4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*Sqrt[x^2*(a + b*x)]*(-16*a^3 + 8*a^2*b*x - 6*a*b^2*x^2 + 5*b^3*x^3))/(35
*b^4*x)
```

IntegrateAlgebraic [A] time = 0.05, size = 55, normalized size = 0.53

$$\frac{2\sqrt{ax^2+bx^3}\left(-16a^3+8a^2bx-6ab^2x^2+5b^3x^3\right)}{35b^4x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^3]*(-16*a^3 + 8*a^2*b*x - 6*a*b^2*x^2 + 5*b^3*x^3))/(35
*b^4*x)
```

fricas [A] time = 0.39, size = 51, normalized size = 0.50

$$\frac{2\left(5\,b^3x^3-6\,ab^2x^2+8\,a^2bx-16\,a^3\right)\sqrt{bx^3+ax^2}}{35\,b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 - 6*a*b^2*x^2 + 8*a^2*b*x - 16*a^3)*sqrt(b*x^3 + a*x^2)/(b^
4*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(b*x^3 + a*x^2), x)
```

maple [A] time = 0.05, size = 55, normalized size = 0.53

$$-\frac{2\,(bx+a)\left(-5b^3x^3+6a\,b^2x^2-8a^2bx+16a^3\right)x}{35\sqrt{b\,x^3+a\,x^2}\;b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -2/35*(b*x+a)*(-5*b^3*x^3+6*a*b^2*x^2-8*a^2*b*x+16*a^3)*x/b^4/(b*x^3+a*x^2)
^(1/2)
```

maxima [A] time = 1.46, size = 53, normalized size = 0.51

$$\frac{2\left(5\,b^4x^4-ab^3x^3+2\,a^2b^2x^2-8\,a^3bx-16\,a^4\right)}{35\,\sqrt{bx+a}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*b^4*x^4 - a*b^3*x^3 + 2*a^2*b^2*x^2 - 8*a^3*b*x - 16*a^4)/(sqrt(b*x
 + a)*b^4)
```

mupad [B] time = 5.19, size = 51, normalized size = 0.50

$$-\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(16\,a^3-8\,a^2\,b\,x+6\,a\,b^2\,x^2-5\,b^3\,x^3\right)}{35\,b^4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] -(2*(a*x^2 + b*x^3)^(1/2)*(16*a^3 - 5*b^3*x^3 + 6*a*b^2*x^2 - 8*a^2*b*x))/(
35*b^4*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x**2*(a + b*x)), x)
```

3.174 $\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=75

$$\frac{16a^2\sqrt{ax^2+bx^3}}{15b^3x}-\frac{8a\sqrt{ax^2+bx^3}}{15b^2}+\frac{2x\sqrt{ax^2+bx^3}}{5b}$$

Rubi [A] time = 0.10, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 1588}

$$\frac{16a^2\sqrt{ax^2+bx^3}}{15b^3x}-\frac{8a\sqrt{ax^2+bx^3}}{15b^2}+\frac{2x\sqrt{ax^2+bx^3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-8*a*Sqrt[a*x^2 + b*x^3])/(15*b^2) + (16*a^2*Sqrt[a*x^2 + b*x^3])/(15*b^3*
x) + (2*x*Sqrt[a*x^2 + b*x^3])/(5*b)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx &= \frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{(4a)\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx}{5b}\\
&= -\frac{8a\sqrt{ax^2+bx^3}}{15b^2}+\frac{2x\sqrt{ax^2+bx^3}}{5b}+\frac{\left(8a^2\right)\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{15b^2}\\
&= -\frac{8a\sqrt{ax^2+bx^3}}{15b^2}+\frac{16a^2\sqrt{ax^2+bx^3}}{15b^3x}+\frac{2x\sqrt{ax^2+bx^3}}{5b}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.56

$$\frac{2\sqrt{x^2(a+bx)}\left(8a^2-4abx+3b^2x^2\right)}{15b^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*Sqrt[x^2*(a + b*x)]*(8*a^2 - 4*a*b*x + 3*b^2*x^2))/(15*b^3*x)
```

IntegrateAlgebraic [A] time = 0.04, size = 44, normalized size = 0.59

$$\frac{2\left(8a^2-4abx+3b^2x^2\right)\sqrt{ax^2+bx^3}}{15b^3x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[a*x^2 + b*x^3],x]
[Out] (2*(8*a^2 - 4*a*b*x + 3*b^2*x^2)*Sqrt[a*x^2 + b*x^3])/(15*b^3*x)
```

fricas [A] time = 0.39, size = 40, normalized size = 0.53

$$\frac{2\left(3\,b^2x^2-4\,abx+8\,a^2\right)\sqrt{bx^3+ax^2}}{15\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x^3 + a*x^2)/(b^3*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^3/sqrt(b*x^3 + a*x^2), x)
```

maple [A] time = 0.05, size = 44, normalized size = 0.59

$$\frac{2\,(bx+a)\left(3b^2x^2-4abx+8a^2\right)x}{15\sqrt{b\,x^3+a\,x^2}\;b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a*x^2)^(1/2),x)
[Out] 2/15*(b*x+a)*(3*b^2*x^2-4*a*b*x+8*a^2)*x/b^3/(b*x^3+a*x^2)^(1/2)
```

maxima [A] time = 1.46, size = 42, normalized size = 0.56

$$\frac{2\left(3\,b^3x^3-ab^2x^2+4\,a^2bx+8\,a^3\right)}{15\,\sqrt{bx+a}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/15*(3*b^3*x^3 - a*b^2*x^2 + 4*a^2*b*x + 8*a^3)/(sqrt(b*x + a)*b^3)
```

mupad [B] time = 5.20, size = 40, normalized size = 0.53

$$\frac{2\,\sqrt{b\,x^3+a\,x^2}\;\left(8\,a^2-4\,a\,b\,x+3\,b^2\,x^2\right)}{15\,b^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(8*a^2 + 3*b^2*x^2 - 4*a*b*x))/(15*b^3*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^2 (a + bx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(a + b*x)), x)
```

3.175 $\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=49

$$\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}$$

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 1588}

$$\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx &= \frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{(2a)\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{3b}\\ &= \frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.61

$$\frac{2(bx-2a)\sqrt{x^2(a+bx)}}{3b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*(-2*a + b*x)*Sqrt[x^2*(a + b*x)])/(3*b^2*x)
```

IntegrateAlgebraic [A] time = 0.03, size = 32, normalized size = 0.65

$$\frac{2(bx-2a)\sqrt{ax^2+bx^3}}{3b^2x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[a*x^2 + b*x^3],x]
[Out] (2*(-2*a + b*x)*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)
```

fricas [A] time = 0.39, size = 28, normalized size = 0.57

$$\frac{2\,\sqrt{bx^3+ax^2}\,(bx-2\,a)}{3\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(b*x^3 + a*x^2)*(b*x - 2*a)/(b^2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(b*x^3 + a*x^2), x)
```

maple [A] time = 0.04, size = 33, normalized size = 0.67

$$-\frac{2\,(bx+a)\,(-bx+2a)\,x}{3\sqrt{b\,x^3+a\,x^2}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x^2)^(1/2),x)
[Out] -2/3*(b*x+a)*(-b*x+2*a)*x/b^2/(b*x^3+a*x^2)^(1/2)
```

maxima [A] time = 1.43, size = 30, normalized size = 0.61

$$\frac{2\left(b^2x^2-abx-2\,a^2\right)}{3\,\sqrt{bx+a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/3*(b^2*x^2 - a*b*x - 2*a^2)/(sqrt(b*x + a)*b^2)
```

mupad [B] time = 5.16, size = 31, normalized size = 0.63

$$-\frac{\left(\frac{4\,a}{3\,b^2}-\frac{2\,x}{3\,b}\right)\,\sqrt{b\,x^3+a\,x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2 + b*x^3)^(1/2),x)
[Out] -(((4*a)/(3*b^2) - (2*x)/(3*b))*(a*x^2 + b*x^3)^(1/2))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x**2*(a + b*x)), x)
```

3.176 $\int \frac{x}{\sqrt{ax^2+bx^3}} \, dx$

Optimal. Leaf size=23

$$\frac{2\sqrt{ax^2+bx^3}}{bx}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1588}

$$\frac{2\sqrt{ax^2+bx^3}}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a*x^2 + b*x^3],x]
[Out] (2*Sqrt[a*x^2 + b*x^3])/(b*x)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{ax^2+bx^3}} \, dx = \frac{2\sqrt{ax^2+bx^3}}{bx}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.91

$$\frac{2\sqrt{x^2(a+bx)}}{bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a*x^2 + b*x^3],x]
[Out] (2*Sqrt[x^2*(a + b*x)])/(b*x)
```

IntegrateAlgebraic [A] time = 0.03, size = 23, normalized size = 1.00

$$\frac{2\sqrt{ax^2+bx^3}}{bx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[a*x^2 + b*x^3],x]
[Out] (2*Sqrt[a*x^2 + b*x^3])/(b*x)
```

fricas [A] time = 0.40, size = 21, normalized size = 0.91

$$\frac{2\sqrt{bx^3+ax^2}}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(b*x^3 + a*x^2)/(b*x)
```

giac [A] time = 0.20, size = 26, normalized size = 1.13

$$\frac{2}{\sqrt{\frac{b}{x} + \frac{a}{x^2}} - \frac{\sqrt{a}}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] 2/(sqrt(b/x + a/x^2) - sqrt(a)/x)
```

maple [A] time = 0.04, size = 25, normalized size = 1.09

$$\frac{2\,(bx + a)\,x}{\sqrt{b\,x^3 + a\,x^2}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x^2)^(1/2),x)
[Out] 2*x*(b*x+a)/b/(b*x^3+a*x^2)^(1/2)
```

maxima [A] time = 1.41, size = 12, normalized size = 0.52

$$\frac{2\,\sqrt{bx + a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(b*x + a)/b
```

mupad [B] time = 5.14, size = 17, normalized size = 0.74

$$\frac{2\,|x|\,\sqrt{a + b\,x}}{b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^2 + b*x^3)^(1/2),x)
[Out] (2*abs(x)*(a + b*x)^(1/2))/(b*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2\,(a + bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x**2)**(1/2),x)
[Out] Integral(x/sqrt(x**2*(a + b*x)), x)
```

3.177 $\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=30

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = $\{2008, 206\}$

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/Sqrt[a]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.53

$$-\frac{2x\sqrt{a+bx}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-2*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*Sqrt[x^2*(a +
b*x)])
```

IntegrateAlgebraic [A] time = 0.04, size = 30, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/Sqrt[a]
```

fricas [A] time = 0.41, size = 74, normalized size = 2.47

$$\left[\frac{\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)}{\sqrt{a}}, \frac{2\,\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2)/sqrt(a), 2*sqrt(-
a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x))/a]
```

giac [A] time = 0.16, size = 45, normalized size = 1.50

$$-\frac{2\,\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}} + \frac{2\,\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(sqrt(b*x + a)/sqrt(-
a))/(sqrt(-a)*sgn(x))
```

maple [A] time = 0.04, size = 39, normalized size = 1.30

$$-\frac{2\sqrt{bx+a}\;x\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{b\,x^3+a\,x^2}\;\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -2/(b*x^3+a*x^2)^(1/2)*x*(b*x+a)^(1/2)/a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*x^3 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] int(1/(a*x^2 + b*x^3)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2 + bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**2 + b*x**3), x)
```

3.178 $\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=54

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}$$

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2008, 206}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] -(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]
])/a^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx &= -\frac{\sqrt{ax^2+bx^3}}{ax^2}-\frac{b\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{2a}\\
&= -\frac{\sqrt{ax^2+bx^3}}{ax^2}+\frac{b\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{a}\\
&= -\frac{\sqrt{ax^2+bx^3}}{ax^2}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 66, normalized size = 1.22

$$\frac{2bx(a+bx)\left(\frac{\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)}{2\sqrt{\frac{bx}{a}+1}}-\frac{a}{2bx}\right)}{a^2\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (2*b*x*(a + b*x)*(-1/2*a/(b*x) + ArcTanh[Sqrt[1 + (b*x)/a]]/(2*Sqrt[1 + (b*
x)/a])))/(a^2*Sqrt[x^2*(a + b*x)])
```

IntegrateAlgebraic [A] time = 0.06, size = 54, normalized size = 1.00

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] -(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]
])/a^(3/2)
```

fricas [A] time = 0.40, size = 127, normalized size = 2.35

$$\left[\frac{\sqrt{a}\,bx^2\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}\,a}{2\,a^2x^2},-\frac{\sqrt{-a}\,bx^2\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}\,a}{a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x^2*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2
) - 2*sqrt(b*x^3 + a*x^2)*a)/(a^2*x^2), -(sqrt(-a)*b*x^2*arctan(sqrt(b*x^3
+ a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*a)/(a^2*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[0,
2,2]%%%}] at parameters values [62.4600259969,-13,46]-1/a*sqrt(a*(1/x)^2+b/
x)-2*b/4/a/sqrt(a)*ln(abs(2*sqrt(a)*(sqrt(a*(1/x)^2+b/x)-sqrt(a)/x)-b))
```

maple [A] time = 0.05, size = 55, normalized size = 1.02

$$-\frac{\sqrt{bx+a}\,\left(-abx\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\sqrt{bx+a}\,a^{\frac{3}{2}}\right)}{\sqrt{b\,x^3+a\,x^2}\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -(b*x+a)^(1/2)*((b*x+a)^(1/2)*a^(3/2)-arctanh((b*x+a)^(1/2)/a^(1/2))*x*a*b)
/(b*x^3+a*x^2)^(1/2)/a^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3 + ax^2}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\sqrt{b\,x^3 + a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^2 + b*x^3)^(1/2)),x)
```

```
[Out] int(1/(x*(a*x^2 + b*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x^2\,(a + bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**2*(a + b*x))), x)
```

3.179 $\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=87

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{5/2}}+\frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}$$

Rubi [A] time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2008, 206}

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{5/2}}+\frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] -Sqrt[a*x^2 + b*x^3]/(2*a*x^3) + (3*b*Sqrt[a*x^2 + b*x^3])/(4*a^2*x^2) - (3
*b^2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(4*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx &= -\frac{\sqrt{ax^2+bx^3}}{2ax^3}-\frac{(3b)\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{4a}\\
&= -\frac{\sqrt{ax^2+bx^3}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2}+\frac{\left(3b^2\right)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{8a^2}\\
&= -\frac{\sqrt{ax^2+bx^3}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2}-\frac{\left(3b^2\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{4a^2}\\
&= -\frac{\sqrt{ax^2+bx^3}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.46

$$-\frac{2b^2\sqrt{x^2(a+bx)}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{bx}{a}+1\right)}{a^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*b^2*Sqrt[x^2*(a + b*x)]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (b*x)/a])/(a
^3*x)
```

IntegrateAlgebraic [A] time = 0.07, size = 69, normalized size = 0.79

$$\frac{(3bx-2a)\sqrt{ax^2+bx^3}}{4a^2x^3}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] ((-2*a + 3*b*x)*Sqrt[a*x^2 + b*x^3])/(4*a^2*x^3) - (3*b^2*ArcTanh[(Sqrt[a]*
x)/Sqrt[a*x^2 + b*x^3]])/(4*a^(5/2))
```

fricas [A] time = 0.41, size = 153, normalized size = 1.76

$$\left[\frac{3\sqrt{a}\,b^2x^3\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}\left(3\,abx-2\,a^2\right)}{8\,a^3x^3},\ \frac{3\,\sqrt{-a}\,b^2x^3\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}\left(3\,abx-2\,a^2\right)}{4\,a^3x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))
/x^2) + 2*sqrt(b*x^3 + a*x^2)*(3*a*b*x - 2*a^2))/(a^3*x^3), 1/4*(3*sqrt(-a)
*b^2*x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(
3*a*b*x - 2*a^2))/(a^3*x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[0,
2,2]%%%}] at parameters values [62.4600259969,-13,46]2*(-4*a/16/a^2/x+6*b/1
6/a^2)*sqrt(a*(1/x)^2+b/x)+6*b^2/16/a^2/sqrt(a)*ln(abs(2*sqrt(a)*(sqrt(a*(1
/x)^2+b/x)-sqrt(a)/x)-b))
```

maple [A] time = 0.05, size = 77, normalized size = 0.89

$$-\frac{\sqrt{bx+a}\,\left(3a\,b^2x^2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-3\sqrt{bx+a}\,a^{\frac{3}{2}}bx+2\sqrt{bx+a}\,a^{\frac{5}{2}}\right)}{4\sqrt{b\,x^3+a\,x^2}\,a^{\frac{7}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -1/4*(b*x+a)^(1/2)*(2*(b*x+a)^(1/2)*a^(5/2)-3*(b*x+a)^(1/2)*a^(3/2)*x*b+3*a
rctanh((b*x+a)^(1/2)/a^(1/2))*a*x^2*b^2)/x/(b*x^3+a*x^2)^(1/2)/a^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3 + ax^2}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^2), x)
```

mupad [B] time = 5.41, size = 44, normalized size = 0.51

$$-\frac{2\sqrt{\frac{a}{b\,x}+1}\;{}_2F_1\left(\frac{1}{2},\frac{5}{2};\;\frac{7}{2};\;-\frac{a}{b\,x}\right)}{5\,x\,\sqrt{b\,x^3+a\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x^2 + b*x^3)^(1/2)),x)
```

```
[Out] -(2*(a/(b*x) + 1)^(1/2)*hypergeom([1/2, 5/2], 7/2, -a/(b*x)))/(5*x*(a*x^2 +
 b*x^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(a + b*x))), x)
```

3.180 $\int \frac{1}{x^3 \sqrt{ax^2+bx^3}} \, dx$

Optimal. Leaf size=115

$$\frac{5b^3 \tanh^{-1}\left(\frac{\sqrt{a}\, x}{\sqrt{ax^2+bx^3}}\right)}{8a^{7/2}} - \frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2} + \frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3} - \frac{\sqrt{ax^2+bx^3}}{3ax^4}$$

Rubi [A] time = 0.14, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2008, 206}

$$-\frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2} + \frac{5b^3 \tanh^{-1}\left(\frac{\sqrt{a}\, x}{\sqrt{ax^2+bx^3}}\right)}{8a^{7/2}} + \frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3} - \frac{\sqrt{ax^2+bx^3}}{3ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] -Sqrt[a*x^2 + b*x^3]/(3*a*x^4) + (5*b*Sqrt[a*x^2 + b*x^3])/(12*a^2*x^3) - (
5*b^2*Sqrt[a*x^2 + b*x^3])/(8*a^3*x^2) + (5*b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[a*
x^2 + b*x^3]])/(8*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx &= -\frac{\sqrt{ax^2+bx^3}}{3ax^4}-\frac{(5b)\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx}{6a} \\
&= -\frac{\sqrt{ax^2+bx^3}}{3ax^4}+\frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3}+\frac{\left(5b^2\right)\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{8a^2} \\
&= -\frac{\sqrt{ax^2+bx^3}}{3ax^4}+\frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3}-\frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2}-\frac{\left(5b^3\right)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{16a^3} \\
&= -\frac{\sqrt{ax^2+bx^3}}{3ax^4}+\frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3}-\frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2}+\frac{\left(5b^3\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{8a^3} \\
&= -\frac{\sqrt{ax^2+bx^3}}{3ax^4}+\frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3}-\frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2}+\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.35

$$\frac{2b^3\sqrt{x^2(a+bx)}\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};\frac{bx}{a}+1\right)}{a^4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (2*b^3*Sqrt[x^2*(a + b*x)]*Hypergeometric2F1[1/2, 4, 3/2, 1 + (b*x)/a])/(a^
4*x)
```

IntegrateAlgebraic [A] time = 0.07, size = 80, normalized size = 0.70

$$\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{7/2}}+\frac{\left(-8a^2+10abx-15b^2x^2\right)\sqrt{ax^2+bx^3}}{24a^3x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] ((-8*a^2 + 10*a*b*x - 15*b^2*x^2)*Sqrt[a*x^2 + b*x^3])/(24*a^3*x^4) + (5*b^
3*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(8*a^(7/2))
```

fricas [A] time = 0.42, size = 175, normalized size = 1.52

$$\left[\frac{15\sqrt{a}\,b^3x^4\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)-2\left(15\,ab^2x^2-10\,a^2bx+8\,a^3\right)\sqrt{bx^3+ax^2}}{48\,a^4x^4},\ -\frac{15\sqrt{-a}\,b^3x^4\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+\left(15\,ab^2x^2-10\,a^2bx+8\,a^3\right)\sqrt{bx^3+ax^2}}{24\,a^4x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a
))/x^2) - 2*(15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^4*x
^4), -1/24*(15*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x))
+ (15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^4*x^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[0,
2,2]%%%}] at parameters values [62.4600259969,-13,46]2*((-16*a^2/96/a^3/x+2
0*a*b/96/a^3)/x-30*b^2/96/a^3)*sqrt(a*(1/x)^2+b/x)-10*b^3/32/a^3/sqrt(a)*ln
(abs(2*sqrt(a)*(sqrt(a*(1/x)^2+b/x)-sqrt(a)/x)-b))
```

maple [A] time = 0.06, size = 95, normalized size = 0.83

$$-\frac{\sqrt{bx+a}\,\left(-15 a\,b^{3} x^{3} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+15 \sqrt{bx+a}\;a^{\frac{3}{2}} b^{2} x^{2}-10 \sqrt{bx+a}\;a^{\frac{5}{2}} bx+8 \sqrt{bx+a}\;a^{\frac{7}{2}}\right)}{24 \sqrt{b\,x^{3}+a\,x^{2}}\;a^{\frac{9}{2}} x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -1/24/x^2*(b*x+a)^(1/2)*(15*(b*x+a)^(1/2)*a^(3/2)*x^2*b^2-15*arctanh((b*x+a
)^(1/2)/a^(1/2))*x^3*a*b^3-10*(b*x+a)^(1/2)*a^(5/2)*x*b+8*(b*x+a)^(1/2)*a^(
7/2))/(b*x^3+a*x^2)^(1/2)/a^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^{3}+ax^{2}}\,x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3}\,\sqrt{b\,x^{3}+a\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x^2 + b*x^3)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a*x^2 + b*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\sqrt{x^{2}\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x**2*(a + b*x))), x)
```

3.181 $\int \frac{x^6}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{32a^2\sqrt{ax^2+bx^3}}{5b^4x}-\frac{16a\sqrt{ax^2+bx^3}}{5b^3}+\frac{12x\sqrt{ax^2+bx^3}}{5b^2}-\frac{2x^4}{b\sqrt{ax^2+bx^3}}$$

Rubi [A] time = 0.15, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2015, 2016, 1588}

$$\frac{32a^2\sqrt{ax^2+bx^3}}{5b^4x}+\frac{12x\sqrt{ax^2+bx^3}}{5b^2}-\frac{16a\sqrt{ax^2+bx^3}}{5b^3}-\frac{2x^4}{b\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-2*x^4)/(b*Sqrt[a*x^2 + b*x^3]) - (16*a*Sqrt[a*x^2 + b*x^3])/(5*b^3) + (32
*a^2*Sqrt[a*x^2 + b*x^3])/(5*b^4*x) + (12*x*Sqrt[a*x^2 + b*x^3])/(5*b^2)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^6}{\left(ax^2+bx^3\right)^{3/2}}\,dx = -\frac{2x^4}{b\sqrt{ax^2+bx^3}} + \frac{6\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx}{b}$$
$$= -\frac{2x^4}{b\sqrt{ax^2+bx^3}} + \frac{12x\sqrt{ax^2+bx^3}}{5b^2} - \frac{(24a)\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx}{5b^2}$$
$$= -\frac{2x^4}{b\sqrt{ax^2+bx^3}} - \frac{16a\sqrt{ax^2+bx^3}}{5b^3} + \frac{12x\sqrt{ax^2+bx^3}}{5b^2} + \frac{\left(16a^2\right)\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{5b^3}$$
$$= -\frac{2x^4}{b\sqrt{ax^2+bx^3}} - \frac{16a\sqrt{ax^2+bx^3}}{5b^3} + \frac{32a^2\sqrt{ax^2+bx^3}}{5b^4x} + \frac{12x\sqrt{ax^2+bx^3}}{5b^2}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.51

$$\frac{2x\left(16a^3+8a^2bx-2ab^2x^2+b^3x^3\right)}{5b^4\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(16*a^3 + 8*a^2*b*x - 2*a*b^2*x^2 + b^3*x^3))/(5*b^4*Sqrt[x^2*(a + b*x
)])
```

IntegrateAlgebraic [A] time = 3.87, size = 54, normalized size = 0.55

$$\frac{2x\left(5a^3+15a^2(a+bx)-5a(a+bx)^2+(a+bx)^3\right)}{5b^4\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(5*a^3 + 15*a^2*(a + b*x) - 5*a*(a + b*x)^2 + (a + b*x)^3))/(5*b^4*Sqr
t[x^2*(a + b*x)])
```

fricas [A] time = 0.40, size = 60, normalized size = 0.61

$$\frac{2\left(b^3x^3-2\,ab^2x^2+8\,a^2bx+16\,a^3\right)\sqrt{bx^3+ax^2}}{5\left(b^5x^2+ab^4x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)*sqrt(b*x^3 + a*x^2)/(b^5*x
^2 + a*b^4*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^3 + a*x^2)^(3/2), x)
```

maple [A] time = 0.05, size = 56, normalized size = 0.57

$$\frac{2\left(bx+a\right)\left(b^{3}x^{3}-2a\,b^{2}x^{2}+8a^{2}bx+16a^{3}\right)x^{3}}{5\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] 2/5*(b*x+a)*(b^3*x^3-2*a*b^2*x^2+8*a^2*b*x+16*a^3)*x^3/b^4/(b*x^3+a*x^2)^(3
/2)
```

maxima [A] time = 1.48, size = 41, normalized size = 0.42

$$\frac{2\left(b^{3}x^{3}-2\,ab^{2}x^{2}+8\,a^{2}bx+16\,a^{3}\right)}{5\,\sqrt{bx+a}\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)/(sqrt(b*x + a)*b^4)
```

mupad [B] time = 5.28, size = 57, normalized size = 0.58

$$\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(16\,a^3+8\,a^2\,b\,x-2\,a\,b^2\,x^2+b^3\,x^3\right)}{5\,b^4\,x\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(16*a^3 + b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x))/(5*b
^4*x*(a + b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(x^{2}\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**6/(x**2*(a + b*x))**(3/2), x)
```

3.182 $\int \frac{x^5}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=72

$$-\frac{16a\sqrt{ax^2+bx^3}}{3b^3x}+\frac{8\sqrt{ax^2+bx^3}}{3b^2}-\frac{2x^3}{b\sqrt{ax^2+bx^3}}$$

Rubi [A] time = 0.11, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 1588}

$$\frac{8\sqrt{ax^2+bx^3}}{3b^2}-\frac{16a\sqrt{ax^2+bx^3}}{3b^3x}-\frac{2x^3}{b\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-2*x^3)/(b*Sqrt[a*x^2 + b*x^3]) + (8*Sqrt[a*x^2 + b*x^3])/(3*b^2) - (16*a*
Sqrt[a*x^2 + b*x^3])/(3*b^3*x)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^5}{\left(ax^2+bx^3\right)^{3/2}}\,dx = -\frac{2x^3}{b\sqrt{ax^2+bx^3}} + \frac{4\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx}{b}$$
$$= -\frac{2x^3}{b\sqrt{ax^2+bx^3}} + \frac{8\sqrt{ax^2+bx^3}}{3b^2} - \frac{(8a)\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{3b^2}$$
$$= -\frac{2x^3}{b\sqrt{ax^2+bx^3}} + \frac{8\sqrt{ax^2+bx^3}}{3b^2} - \frac{16a\sqrt{ax^2+bx^3}}{3b^3x}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.54

$$\frac{2x\left(-8a^2-4abx+b^2x^2\right)}{3b^3\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x^2 + b*x^3)^(3/2),x]
[Out] (2*x*(-8*a^2 - 4*a*b*x + b^2*x^2))/(3*b^3*Sqrt[x^2*(a + b*x)])
```

IntegrateAlgebraic [A] time = 3.92, size = 42, normalized size = 0.58

$$\frac{2x\left(-3a^2-6a(a+bx)+(a+bx)^2\right)}{3b^3\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(a*x^2 + b*x^3)^(3/2),x]
[Out] (2*x*(-3*a^2 - 6*a*(a + b*x) + (a + b*x)^2))/(3*b^3*Sqrt[x^2*(a + b*x)])
```

fricas [A] time = 0.38, size = 49, normalized size = 0.68

$$\frac{2\left(b^2x^2-4\,abx-8\,a^2\right)\sqrt{bx^3+ax^2}}{3\left(b^4x^2+ab^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
[Out] 2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)*sqrt(b*x^3 + a*x^2)/(b^4*x^2 + a*b^3*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
[Out] integrate(x^5/(b*x^3 + a*x^2)^(3/2), x)
```

maple [A] time = 0.05, size = 46, normalized size = 0.64

$$-\frac{2\,(bx+a)\left(-b^2x^2+4abx+8a^2\right)x^3}{3\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -2/3*(b*x+a)*(-b^2*x^2+4*a*b*x+8*a^2)*x^3/b^3/(b*x^3+a*x^2)^(3/2)
```

maxima [A] time = 1.44, size = 30, normalized size = 0.42

$$\frac{2\left(b^2x^2 - 4\,abx - 8\,a^2\right)}{3\,\sqrt{bx + a}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)/(sqrt(b*x + a)*b^3)
```

mupad [B] time = 5.22, size = 47, normalized size = 0.65

$$-\frac{2\,\sqrt{b\,x^3 + a\,x^2}\,\left(8\,a^2 + 4\,a\,b\,x - b^2\,x^2\right)}{3\,b^3\,x\,\left(a + b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] -(2*(a*x^2 + b*x^3)^(1/2)*(8*a^2 - b^2*x^2 + 4*a*b*x))/(3*b^3*x*(a + b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(x^2\,(a + bx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**5/(x**2*(a + b*x))**(3/2), x)
```

3.183 $\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/2}} \, dx$

Optimal. Leaf size=47

$$\frac{4\sqrt{ax^2+bx^3}}{b^2x}-\frac{2x^2}{b\sqrt{ax^2+bx^3}}$$

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 1588}

$$\frac{4\sqrt{ax^2+bx^3}}{b^2x}-\frac{2x^2}{b\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x^2 + b*x^3)^(3/2),x]
[Out] (-2*x^2)/(b*Sqrt[a*x^2 + b*x^3]) + (4*Sqrt[a*x^2 + b*x^3])/(b^2*x)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/2}} \, dx &= -\frac{2x^2}{b\sqrt{ax^2+bx^3}}+\frac{2\int \frac{x}{\sqrt{ax^2+bx^3}} \, dx}{b}\\ &= -\frac{2x^2}{b\sqrt{ax^2+bx^3}}+\frac{4\sqrt{ax^2+bx^3}}{b^2x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.55

$$\frac{2x(2a+bx)}{b^2\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x^2 + b*x^3)^(3/2),x]
[Out] (2*x*(2*a + b*x))/(b^2*Sqrt[x^2*(a + b*x)])
```

IntegrateAlgebraic [A] time = 2.41, size = 37, normalized size = 0.79

$$\frac{2(2a+bx)\sqrt{ax^2+bx^3}}{b^2x(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/(a*x^2 + b*x^3)^(3/2),x]
[Out] (2*(2*a + b*x)*Sqrt[a*x^2 + b*x^3])/(b^2*x*(a + b*x))
```

fricas [A] time = 0.39, size = 38, normalized size = 0.81

$$\frac{2\sqrt{bx^3+ax^2}\,(bx+2a)}{b^3x^2+ab^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
[Out] 2*sqrt(b*x^3 + a*x^2)*(b*x + 2*a)/(b^3*x^2 + a*b^2*x)
```

giac [A] time = 0.28, size = 28, normalized size = 0.60

$$\frac{2\left(\frac{1}{b}+\frac{2a}{b^2x}\right)}{\sqrt{\frac{b}{x}+\frac{a}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
[Out] 2*(1/b + 2*a/(b^2*x))/sqrt(b/x + a/x^2)
```

maple [A] time = 0.05, size = 34, normalized size = 0.72

$$\frac{2\,(bx+a)\,(bx+2a)\,x^3}{\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x^2)^(3/2),x)
[Out] 2*(b*x+a)*(b*x+2*a)*x^3/b^2/(b*x^3+a*x^2)^(3/2)
```

maxima [A] time = 1.54, size = 19, normalized size = 0.40

$$\frac{2\,(bx+2\,a)}{\sqrt{bx+a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] 2*(b*x + 2*a)/(sqrt(b*x + a)*b^2)
```

mupad [B] time = 5.17, size = 35, normalized size = 0.74

$$\frac{2\,\left(2\,a+b\,x\right)\,\sqrt{b\,x^3+a\,x^2}}{b^2\,x\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] (2*(2*a + b*x)*(a*x^2 + b*x^3)^(1/2))/(b^2*x*(a + b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**4/(x**2*(a + b*x))**(3/2), x)
```

3.184 $\int \frac{x^3}{(ax^2+bx^3)^{3/2}} dx$

Optimal. Leaf size=21

$$-\frac{2x}{b\sqrt{ax^2+bx^3}}$$

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1588}

$$-\frac{2x}{b\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-2*x)/(b*Sqrt[a*x^2 + b*x^3])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}} dx = -\frac{2x}{b\sqrt{ax^2+bx^3}}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.90

$$-\frac{2x}{b\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-2*x)/(b*Sqrt[x^2*(a + b*x)])
```

IntegrateAlgebraic [A] time = 2.19, size = 30, normalized size = 1.43

$$-\frac{2\sqrt{ax^2+bx^3}}{bx(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^3])/(b*x*(a + b*x))
```

fricas [A] time = 0.38, size = 29, normalized size = 1.38

$$-\frac{2\sqrt{bx^3+ax^2}}{b^2x^2+abx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*x^3 + a*x^2)/(b^2*x^2 + a*b*x)
```

giac [A] time = 0.23, size = 37, normalized size = 1.76

$$\frac{2}{\left(\sqrt{a}\left(\sqrt{\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}\right)-b\right)\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/((sqrt(a)*(sqrt(b/x + a/x^2) - sqrt(a)/x) - b)*sqrt(a))
```

maple [A] time = 0.04, size = 27, normalized size = 1.29

$$-\frac{2\left(bx+a\right)x^3}{\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -2*(b*x+a)*x^3/b/(b*x^3+a*x^2)^(3/2)
```

maxima [A] time = 1.48, size = 12, normalized size = 0.57

$$-\frac{2}{\sqrt{bx+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(b*x + a)*b)
```

mupad [B] time = 5.07, size = 28, normalized size = 1.33

$$-\frac{2\sqrt{b\,x^3+a\,x^2}}{b\,x\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] -(2*(a*x^2 + b*x^3)^(1/2))/(b*x*(a + b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**3/(x**2*(a + b*x))**(3/2), x)
```

3.185 $\int \frac{x^2}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=52

$$\frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2023, 2008, 206}

$$\frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x)/(a*Sqrt[a*x^2 + b*x^3]) - (2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
)/a^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(ax^2+bx^3\right)^{3/2}}\,dx &= \frac{2x}{a\sqrt{ax^2+bx^3}}+\frac{\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{a}\\
&= \frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{a}\\
&= \frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 35, normalized size = 0.67

$$\frac{2x\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx}{a}+1\right)}{a\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x)/a])/(a*Sqrt[x^2*(a + b*x)])
```

IntegrateAlgebraic [A] time = 2.41, size = 63, normalized size = 1.21

$$\frac{2\sqrt{ax^2+bx^3}}{ax(a+bx)}-\frac{2\tanh^{-1}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}\,x}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^3])/(a*x*(a + b*x)) - (2*ArcTanh[Sqrt[a*x^2 + b*x^3]/(S
qrt[a]*x)])/a^(3/2)
```

fricas [A] time = 0.41, size = 156, normalized size = 3.00

$$\left[\frac{\left(bx^2+ax\right)\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}\,a}{a^2bx^2+a^3x},\frac{2\left(\left(bx^2+ax\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}\,a\right)}{a^2bx^2+a^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [((b*x^2 + a*x)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))
/x^2) + 2*sqrt(b*x^3 + a*x^2)*a)/(a^2*b*x^2 + a^3*x), 2*((b*x^2 + a*x)*sqrt
(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*a)/(a
^2*b*x^2 + a^3*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Unable to divide, perhaps due to rounding error%%%{%%%{1,[1]%%%},[2,
2]%%%}+%%%{%%{[-2,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,3]%%%}+%%%{1,[0,4]%%%} / %%
%{%%%{1,[2]%%%},[2,0]%%%}+%%%{%%{[%%%{-2,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%}
,[1,1]%%%}+%%%{%%%{1,[1]%%%},[0,2]%%%} Error: Bad Argument Value
```

maple [A] time = 0.05, size = 54, normalized size = 1.04

$$-\frac{2\,(bx+a)\left(\sqrt{bx+a}\;a\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-a^{\frac{3}{2}}\right)x^3}{\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -2*x^3*(b*x+a)*(arctanh((b*x+a)^(1/2)/a^(1/2))*a*(b*x+a)^(1/2)-a^(3/2))/(b*
x^3+a*x^2)^(3/2)/a^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^3 + ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^3 + a*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\left(b\,x^3 + a\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] int(x^2/(a*x^2 + b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2\left(a + bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**2/(x**2*(a + b*x))**(3/2), x)
```

3.186 $\int \frac{x}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=75

$$\frac{3b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{5/2}}-\frac{3\sqrt{ax^2+bx^3}}{a^2x^2}+\frac{2}{a\sqrt{ax^2+bx^3}}$$

Rubi [A] time = 0.09, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2023, 2025, 2008, 206}

$$-\frac{3\sqrt{ax^2+bx^3}}{a^2x^2}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{5/2}}+\frac{2}{a\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] 2/(a*Sqrt[a*x^2 + b*x^3]) - (3*Sqrt[a*x^2 + b*x^3])/(a^2*x^2) + (3*b*ArcTan
h[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{x}{\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{2}{a\sqrt{ax^2+bx^3}} + \frac{3\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{a}$$
$$= \frac{2}{a\sqrt{ax^2+bx^3}} - \frac{3\sqrt{ax^2+bx^3}}{a^2x^2} - \frac{(3b)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{2a^2}$$
$$= \frac{2}{a\sqrt{ax^2+bx^3}} - \frac{3\sqrt{ax^2+bx^3}}{a^2x^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)}{a^2}$$
$$= \frac{2}{a\sqrt{ax^2+bx^3}} - \frac{3\sqrt{ax^2+bx^3}}{a^2x^2} + \frac{3b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{5/2}}$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.48

$$-\frac{2bx\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{bx}{a}+1\right)}{a^2\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-2*b*x*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (b*x)/a])/(a^2*Sqrt[x^2*(a + b*
x)])
```

IntegrateAlgebraic [A] time = 3.79, size = 76, normalized size = 1.01

$$\frac{x\sqrt{a+bx}\left(\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2a-3(a+bx)}{a^2x\sqrt{a+bx}}\right)}{\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (x*Sqrt[a + b*x]*((2*a - 3*(a + b*x))/(a^2*x*Sqrt[a + b*x]) + (3*b*ArcTanh[
Sqrt[a + b*x]/Sqrt[a]])/a^(5/2)))/Sqrt[x^2*(a + b*x)]
```

fricas [A] time = 0.41, size = 189, normalized size = 2.52

$$\left[\frac{3\left(b^2x^3+abx^2\right)\sqrt{a}\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}\left(3\,abx+a^2\right)}{2\left(a^3bx^3+a^4x^2\right)},\,-\frac{3\left(b^2x^3+abx^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}\left(3\,abx+a^2\right)}{a^3bx^3+a^4x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b^2*x^3 + a*b*x^2)*sqrt(a)*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x
^2)*sqrt(a))/x^2) - 2*sqrt(b*x^3 + a*x^2)*(3*a*b*x + a^2))/(a^3*b*x^3 + a^4
*x^2), -(3*(b^2*x^3 + a*b*x^2)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)
/(a*x)) + sqrt(b*x^3 + a*x^2)*(3*a*b*x + a^2))/(a^3*b*x^3 + a^4*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[0,
2,2]%%%}] at parameters values [62.4600259969,-13,46]-4*a^2/4/a^4*sqrt(a*(1
/x)^2+b/x)+2*(-b^2/a^2/(-a*(sqrt(a*(1/x)^2+b/x)-sqrt(a)/x)+sqrt(a)*b)-3*b/4
/a^2/sqrt(a)*ln(abs(2*sqrt(a)*(sqrt(a*(1/x)^2+b/x)-sqrt(a)/x)-b)))
```

maple [A] time = 0.06, size = 62, normalized size = 0.83

$$\frac{(bx+a)\left(3\sqrt{bx+a}\; bx\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-3\sqrt{a}\; bx-a^{\frac{3}{2}}\right)x^{2}}{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] x^2*(b*x+a)*(3*(b*x+a)^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))*x*b-3*x*b*a^(1/
2)-a^(3/2))/(b*x^3+a*x^2)^(3/2)/a^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^{3}+ax^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*x^3 + a*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(b\,x^3+a\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] int(x/(a*x^2 + b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^{2} \left(a + bx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x/(x**2*(a + b*x))**(3/2), x)
```

3.187 $\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{7/2}}+\frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2}-\frac{5\sqrt{ax^2+bx^3}}{2a^2x^3}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

Rubi [A] time = 0.11, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2006, 2025, 2008, 206}

$$-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{7/2}}+\frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2}-\frac{5\sqrt{ax^2+bx^3}}{2a^2x^3}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(-3/2),x]
```

```
[Out] 2/(a*x*Sqrt[a*x^2 + b*x^3]) - (5*Sqrt[a*x^2 + b*x^3])/(2*a^2*x^3) + (15*b*S
qrt[a*x^2 + b*x^3])/(4*a^3*x^2) - (15*b^2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 +
b*x^3]])/(4*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2006

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(ax^2+bx^3\right)^{3/2}}\,dx &= \frac{2}{ax\sqrt{ax^2+bx^3}}+\frac{5\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx}{a}\\
&= \frac{2}{ax\sqrt{ax^2+bx^3}}-\frac{5\sqrt{ax^2+bx^3}}{2a^2x^3}-\frac{(15b)\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{4a^2}\\
&= \frac{2}{ax\sqrt{ax^2+bx^3}}-\frac{5\sqrt{ax^2+bx^3}}{2a^2x^3}+\frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2}+\frac{\left(15b^2\right)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{8a^3}\\
&= \frac{2}{ax\sqrt{ax^2+bx^3}}-\frac{5\sqrt{ax^2+bx^3}}{2a^2x^3}+\frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2}-\frac{\left(15b^2\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{4a^3}\\
&= \frac{2}{ax\sqrt{ax^2+bx^3}}-\frac{5\sqrt{ax^2+bx^3}}{2a^2x^3}+\frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2}-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 38, normalized size = 0.35

$$
\frac{2b^2x\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{bx}{a}+1\right)}{a^3\sqrt{x^2(a+bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(-3/2),x]
```

```
[Out] (2*b^2*x*Hypergeometric2F1[-1/2, 3, 1/2, 1 + (b*x)/a])/(a^3*Sqrt[x^2*(a + b
*x)])
```

IntegrateAlgebraic [A] time = 3.72, size = 95, normalized size = 0.86

$$
\frac{x\sqrt{a+bx}\left(\frac{8a^2-25a(a+bx)+15(a+bx)^2}{4a^3x^2\sqrt{a+bx}}-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}\right)}{\sqrt{x^2(a+bx)}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x^3)^(-3/2),x]
```

```
[Out] (x*Sqrt[a + b*x]*((8*a^2 - 25*a*(a + b*x) + 15*(a + b*x)^2)/(4*a^3*x^2*Sqrt
[a + b*x]) - (15*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(7/2))))/Sqrt[x^2
*(a + b*x)]
```

fricas [A] time = 0.41, size = 219, normalized size = 1.99

$$
\left[\frac{15\left(b^3x^4+ab^2x^3\right)\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)+2\left(15\,ab^2x^2+5\,a^2bx-2\,a^3\right)\sqrt{bx^3+ax^2}}{8\left(a^4bx^4+a^5x^3\right)},\ \frac{15\left(b^3x^4+ab^2x^3\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+\left(15\,ab^2x^2+5\,a^2bx-2\,a^3\right)\sqrt{bx^3+ax^2}}{4\left(a^4bx^4+a^5x^3\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^3*x^4 + a*b^2*x^3)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 +
a*x^2)*sqrt(a))/x^2) + 2*(15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x^3 + a*
x^2))/(a^4*b*x^4 + a^5*x^3), 1/4*(15*(b^3*x^4 + a*b^2*x^3)*sqrt(-a)*arctan(
```

```
sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sq
rt(b*x^3 + a*x^2))/(a^4*b*x^4 + a^5*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.06, size = 76, normalized size = 0.69

$$-\frac{(bx+a)\left(15\sqrt{bx+a}\,b^2x^2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-15\sqrt{a}\,b^2x^2-5a^{\frac{3}{2}}bx+2a^{\frac{5}{2}}\right)x}{4\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x^2)^(3/2),x)
[Out] -1/4*x*(b*x+a)*(15*arctanh((b*x+a)^(1/2)/a^(1/2))*(b*x+a)^(1/2)*x^2*b^2-5*a
^(3/2)*x*b-15*x^2*b^2*a^(1/2)+2*a^(5/2))/(b*x^3+a*x^2)^(3/2)/a^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^3 + a*x^2)^(-3/2), x)
```

mupad [B] time = 5.43, size = 42, normalized size = 0.38

$$-\frac{2\,x\,\left(\frac{a}{b\,x}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{a}{b\,x}\right)}{7\,\left(b\,x^3+a\,x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^3)^(3/2),x)
[Out] -(2*x*(a/(b*x) + 1)^(3/2)*hypergeom([3/2, 7/2], 9/2, -a/(b*x)))/(7*(a*x^2 +
 b*x^3)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(ax^2+bx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x**2)**(3/2),x)
[Out] Integral((a*x**2 + b*x**3)**(-3/2), x)
```

3.188 $\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=138

$$\frac{35b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{9/2}}-\frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2}+\frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

Rubi [A] time = 0.19, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2023, 2025, 2008, 206}

$$-\frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2}+\frac{35b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{9/2}}+\frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x^2 + b*x^3)^(3/2)),x]
```

```
[Out] 2/(a*x^2*Sqrt[a*x^2 + b*x^3]) - (7*Sqrt[a*x^2 + b*x^3])/(3*a^2*x^4) + (35*b
*Sqrt[a*x^2 + b*x^3])/(12*a^3*x^3) - (35*b^2*Sqrt[a*x^2 + b*x^3])/(8*a^4*x^
2) + (35*b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(8*a^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{2}{ax^2\sqrt{ax^2+bx^3}} + \frac{7\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx}{a}$$

$$= \frac{2}{ax^2\sqrt{ax^2+bx^3}} - \frac{7\sqrt{ax^2+bx^3}}{3a^2x^4} - \frac{(35b)\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx}{6a^2}$$

$$= \frac{2}{ax^2\sqrt{ax^2+bx^3}} - \frac{7\sqrt{ax^2+bx^3}}{3a^2x^4} + \frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3} + \frac{\left(35b^2\right)\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{8a^3}$$

$$= \frac{2}{ax^2\sqrt{ax^2+bx^3}} - \frac{7\sqrt{ax^2+bx^3}}{3a^2x^4} + \frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3} - \frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2} - \frac{\left(35b^3\right)\int \frac{}{\sqrt{}}}{16}$$

$$= \frac{2}{ax^2\sqrt{ax^2+bx^3}} - \frac{7\sqrt{ax^2+bx^3}}{3a^2x^4} + \frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3} - \frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2} + \frac{\left(35b^3\right)\text{Sub}}{}$$

$$= \frac{2}{ax^2\sqrt{ax^2+bx^3}} - \frac{7\sqrt{ax^2+bx^3}}{3a^2x^4} + \frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3} - \frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2} + \frac{35b^3\tanh^{-}}{8}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.28

$$-\frac{2b^3x\,{}_2F_1\left(-\frac{1}{2},4;\frac{1}{2};\frac{bx}{a}+1\right)}{a^4\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x^2 + b*x^3)^(3/2)),x]
```

```
[Out] (-2*b^3*x*Hypergeometric2F1[-1/2, 4, 1/2, 1 + (b*x)/a])/(a^4*Sqrt[x^2*(a +
b*x)])
```

IntegrateAlgebraic [A] time = 4.04, size = 107, normalized size = 0.78

$$\frac{x\sqrt{a+bx}\left(\frac{35b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{9/2}} + \frac{48a^3-231a^2(a+bx)+280a(a+bx)^2-105(a+bx)^3}{24a^4x^3\sqrt{a+bx}}\right)}{\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a*x^2 + b*x^3)^(3/2)),x]
```

```
[Out] (x*Sqrt[a + b*x]*((48*a^3 - 231*a^2*(a + b*x) + 280*a*(a + b*x)^2 - 105*(a
+ b*x)^3)/(24*a^4*x^3*Sqrt[a + b*x]) + (35*b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a
]])/(8*a^(9/2))))/Sqrt[x^2*(a + b*x)]
```

fricas [A] time = 0.41, size = 241, normalized size = 1.75

$$\left[\frac{105\left(b^4x^5+ab^3x^4\right)\sqrt{a}\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)-2\left(105\,ab^3x^3+35\,a^2b^2x^2-14\,a^3bx+8\,a^4\right)\sqrt{bx^3+ax^2}}{48\left(a^5bx^5+a^6x^4\right)}, -\frac{105\left(b^4x^5+ab^3x^4\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+\left(105\,ab^3x^3+35\,a^2b^2x^2-14\,a^3bx+8\,a^4\right)\sqrt{bx^3+ax^2}}{24\left(a^5bx^5+a^6x^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(105*(b^4*x^5 + a*b^3*x^4)*sqrt(a)*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3
+ a*x^2)*sqrt(a))/x^2) - 2*(105*a*b^3*x^3 + 35*a^2*b^2*x^2 - 14*a^3*b*x + 8
```

```
*a^4)*sqrt(b*x^3 + a*x^2))/(a^5*b*x^5 + a^6*x^4), -1/24*(105*(b^4*x^5 + a*b
^3*x^4)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (105*a*b^3*x^
3 + 35*a^2*b^2*x^2 - 14*a^3*b*x + 8*a^4)*sqrt(b*x^3 + a*x^2))/(a^5*b*x^5 +
a^6*x^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+ax^2\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a*x^2)^(3/2)*x), x)
```

maple [A] time = 0.06, size = 86, normalized size = 0.62

$$-\frac{(bx+a)\left(-105\sqrt{bx+a}\;b^3x^3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+105\sqrt{a}\;b^3x^3+35a^{\frac{3}{2}}b^2x^2-14a^{\frac{5}{2}}bx+8a^{\frac{7}{2}}\right)}{24\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -1/24*(b*x+a)*(-105*arctanh((b*x+a)^(1/2)/a^(1/2))*(b*x+a)^(1/2)*x^3*b^3-14
*a^(5/2)*x*b+35*a^(3/2)*x^2*b^2+105*b^3*x^3*a^(1/2)+8*a^(7/2))/(b*x^3+a*x^2
)^(3/2)/a^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+ax^2\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x^2)^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(b\,x^3+a\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^2 + b*x^3)^(3/2)),x)
```

```
[Out] int(1/(x*(a*x^2 + b*x^3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x*(x**2*(a + b*x))**(3/2)), x)
```

3.189 $\int \frac{1}{x^2(ax^2+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=166

$$-\frac{315b^4\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{64a^{11/2}}+\frac{315b^3\sqrt{ax^2+bx^3}}{64a^5x^2}-\frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3}+\frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4}-\frac{9\sqrt{ax^2+bx^3}}{4a^2x^5}+\frac{2}{ax^3\sqrt{ax^2}}$$

Rubi [A] time = 0.23, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2023, 2025, 2008, 206}

$$\frac{315b^3\sqrt{ax^2+bx^3}}{64a^5x^2}-\frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3}-\frac{315b^4\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{64a^{11/2}}+\frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4}-\frac{9\sqrt{ax^2+bx^3}}{4a^2x^5}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x^2 + b*x^3)^(3/2)),x]
```

```
[Out] 2/(a*x^3*Sqrt[a*x^2 + b*x^3]) - (9*Sqrt[a*x^2 + b*x^3])/(4*a^2*x^5) + (21*b
*Sqrt[a*x^2 + b*x^3])/(8*a^3*x^4) - (105*b^2*Sqrt[a*x^2 + b*x^3])/(32*a^4*x
^3) + (315*b^3*Sqrt[a*x^2 + b*x^3])/(64*a^5*x^2) - (315*b^4*ArcTanh[(Sqrt[a
]*x)/Sqrt[a*x^2 + b*x^3]])/(64*a^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{2}{ax^3\sqrt{ax^2+bx^3}} + \frac{9\int \frac{1}{x^4\sqrt{ax^2+bx^3}}\,dx}{a}$$

$$= \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} - \frac{(63b)\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx}{8a^2}$$

$$= \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4} + \frac{\left(105b^2\right)\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx}{16a^3}$$

$$= \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4} - \frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3} - \frac{\left(315b^3\right)\int \ldots}{64\ldots}$$

$$= \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4} - \frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3} + \frac{315b^3\sqrt{ax^2\ldots}}{64a^5x^2\ldots}$$

$$= \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4} - \frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3} + \frac{315b^3\sqrt{ax^2\ldots}}{64a^5x^2\ldots}$$

$$= \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4} - \frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3} + \frac{315b^3\sqrt{ax^2\ldots}}{64a^5x^2\ldots}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.23

$$\frac{2b^4x\,{}_2F_1\left(-\frac{1}{2},5;\frac{1}{2};\frac{bx}{a}+1\right)}{a^5\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x^2 + b*x^3)^(3/2)),x]
```

```
[Out] (2*b^4*x*Hypergeometric2F1[-1/2, 5, 1/2, 1 + (b*x)/a])/(a^5*Sqrt[x^2*(a + b
*x)])
```

IntegrateAlgebraic [A] time = 6.31, size = 119, normalized size = 0.72

$$\frac{x\sqrt{a+bx}\left(\frac{128a^4-837a^3(a+bx)+1533a^2(a+bx)^2-1155a(a+bx)^3+315(a+bx)^4}{64a^5x^4\sqrt{a+bx}} - \frac{315b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{11/2}}\right)}{\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a*x^2 + b*x^3)^(3/2)),x]
```

```
[Out] (x*Sqrt[a + b*x]*((128*a^4 - 837*a^3*(a + b*x) + 1533*a^2*(a + b*x)^2 - 115
5*a*(a + b*x)^3 + 315*(a + b*x)^4)/(64*a^5*x^4*Sqrt[a + b*x]) - (315*b^4*Ar
cTanh[Sqrt[a + b*x]/Sqrt[a]])/(64*a^(11/2))))/Sqrt[x^2*(a + b*x)]
```

fricas [A] time = 0.43, size = 263, normalized size = 1.58

$$\left[\frac{315\left(b^5x^6+ab^4x^5\right)\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)+2\left(315\,ab^4x^4+105\,a^2b^3x^3-42\,a^3b^2x^2+24\,a^4bx-16\,a^5\right)\sqrt{bx^3+ax^2}}{128\left(a^6bx^6+a^7x^5\right)}, \frac{315\left(b^5x^6+ab^4x^5\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+\left(315\,ab^4x^4+105\,a^2b^3x^3-42\,a^3b^2x^2+24\,a^4bx-16\,a^5\right)\sqrt{bx^3+ax^2}}{64\left(a^6bx^6+a^7x^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/128*(315*(b^5*x^6 + a*b^4*x^5)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3
 + a*x^2)*sqrt(a))/x^2) + 2*(315*a*b^4*x^4 + 105*a^2*b^3*x^3 - 42*a^3*b^2*x
^2 + 24*a^4*b*x - 16*a^5)*sqrt(b*x^3 + a*x^2))/(a^6*b*x^6 + a^7*x^5), 1/64*
(315*(b^5*x^6 + a*b^4*x^5)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*
x)) + (315*a*b^4*x^4 + 105*a^2*b^3*x^3 - 42*a^3*b^2*x^2 + 24*a^4*b*x - 16*a
^5)*sqrt(b*x^3 + a*x^2))/(a^6*b*x^6 + a^7*x^5)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3 + ax^2\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a*x^2)^(3/2)*x^2), x)
```

maple [A] time = 0.06, size = 100, normalized size = 0.60

$$-\frac{(bx+a)\left(315\sqrt{bx+a}\, b^4x^4 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) - 315\sqrt{a}\, b^4x^4 - 105a^{\frac{3}{2}}b^3x^3 + 42a^{\frac{5}{2}}b^2x^2 - 24a^{\frac{7}{2}}bx + 16a^{\frac{9}{2}}\right)}{64\left(b\,x^3 + a\,x^2\right)^{\frac{3}{2}} a^{\frac{11}{2}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -1/64*(b*x+a)*(315*arctanh((b*x+a)^(1/2)/a^(1/2))*(b*x+a)^(1/2)*x^4*b^4-24*
a^(7/2)*x*b+42*a^(5/2)*x^2*b^2-105*a^(3/2)*x^3*b^3-315*x^4*b^4*a^(1/2)+16*a
^(9/2))/x/(b*x^3+a*x^2)^(3/2)/a^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3 + ax^2\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x^2)^(3/2)*x^2), x)
```

mupad [B] time = 5.68, size = 44, normalized size = 0.27

$$-\frac{2\left(\frac{a}{b\,x}+1\right)^{3/2} {}_2\mathrm{F}_1\left(\frac{3}{2},\frac{11}{2};\ \frac{13}{2};\ -\frac{a}{b\,x}\right)}{11\,x\left(b\,x^3 + a\,x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x^2 + b*x^3)^(3/2)),x)
```

```
[Out] -(2*(a/(b*x) + 1)^(3/2)*hypergeom([3/2, 11/2], 13/2, -a/(b*x)))/(11*x*(a*x^
2 + b*x^3)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(x^2\left(a + bx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(x**2*(a + b*x))**(3/2)), x)
```

3.190 $\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=125

$$-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{8b^{7/2}}+\frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}}-\frac{5a\sqrt{x}\,\sqrt{ax^2+bx^3}}{12b^2}+\frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b}$$

Rubi [A] time = 0.17, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2024, 2029, 206}

$$\frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{8b^{7/2}}-\frac{5a\sqrt{x}\,\sqrt{ax^2+bx^3}}{12b^2}+\frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (5*a^2*Sqrt[a*x^2 + b*x^3])/(8*b^3*Sqrt[x]) - (5*a*Sqrt[x]*Sqrt[a*x^2 + b*x
^3])/(12*b^2) + (x^(3/2)*Sqrt[a*x^2 + b*x^3])/(3*b) - (5*a^3*ArcTanh[(Sqrt[
b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(8*b^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b} - \frac{(5a)\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx}{6b}$$

$$= -\frac{5a\sqrt{x}\sqrt{ax^2+bx^3}}{12b^2} + \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b} + \frac{\left(5a^2\right)\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx}{8b^2}$$

$$= \frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}} - \frac{5a\sqrt{x}\sqrt{ax^2+bx^3}}{12b^2} + \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b} - \frac{\left(5a^3\right)\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx}{16b^3}$$

$$= \frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}} - \frac{5a\sqrt{x}\sqrt{ax^2+bx^3}}{12b^2} + \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b} - \frac{\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{3/2}}{\sqrt{ax^2+b}}\right.}{8b^3}$$

$$= \frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}} - \frac{5a\sqrt{x}\sqrt{ax^2+bx^3}}{12b^2} + \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b} - \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{8b^{7/2}}$$

Mathematica [A] time = 0.13, size = 104, normalized size = 0.83

$$\frac{\sqrt{x^2(a+bx)}\left(\sqrt{b}\sqrt{x}\sqrt{\frac{bx}{a}+1}\left(15a^2-10abx+8b^2x^2\right)-15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\right)}{24b^{7/2}x\sqrt{\frac{bx}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (Sqrt[x^2*(a + b*x)]*(Sqrt[b]*Sqrt[x]*Sqrt[1 + (b*x)/a]*(15*a^2 - 10*a*b*x
+ 8*b^2*x^2) - 15*a^(5/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]))/(24*b^(7/2)*
x*Sqrt[1 + (b*x)/a])
```

IntegrateAlgebraic [A] time = 0.18, size = 106, normalized size = 0.85

$$\frac{5a^3\log\left(\sqrt{ax^2+bx^3}-\sqrt{b}\,x^{3/2}\right)}{8b^{7/2}} - \frac{5a^3\log\left(\sqrt{x}\right)}{4b^{7/2}} + \frac{\left(15a^2-10abx+8b^2x^2\right)\sqrt{ax^2+bx^3}}{24b^3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] ((15*a^2 - 10*a*b*x + 8*b^2*x^2)*Sqrt[a*x^2 + b*x^3])/(24*b^3*Sqrt[x]) - (5
*a^3*Log[Sqrt[x]])/(4*b^(7/2)) + (5*a^3*Log[-(Sqrt[b]*x^(3/2)) + Sqrt[a*x^2
 + b*x^3]])/(8*b^(7/2))
```

fricas [A] time = 0.42, size = 180, normalized size = 1.44

$$\left[\frac{15\,a^3\sqrt{b}\,x\log\left(\frac{2\,bx^2+ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{b}\,\sqrt{x}}{x}\right)+2\left(8\,b^3x^2-10\,ab^2x+15\,a^2b\right)\sqrt{bx^3+ax^2}\,\sqrt{x}}{48\,b^4x}, \frac{15\,a^3\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-b}}{bx^{\frac{3}{2}}}\right)+\left(8\,b^3x^2-10\,ab^2x+15\,a^2b\right)\sqrt{bx^3+ax^2}\,\sqrt{x}}{24\,b^4x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*
sqrt(x))/x) + 2*(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^3 + a*x^2)*sqr
t(x))/(b^4*x), 1/24*(15*a^3*sqrt(-b)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b)/
```

```
(b*x^(3/2))) + (8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^3 + a*x^2)*sqrt
(x))/(b^4*x)]
```

giac [A] time = 0.20, size = 64, normalized size = 0.51

$$\frac{1}{24}\sqrt{bx+a}\left(2x\left(\frac{4x}{b}-\frac{5a}{b^2}\right)+\frac{15a^2}{b^3}\right)\sqrt{x}+\frac{5a^3\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{8b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(b*x + a)*(2*x*(4*x/b - 5*a/b^2) + 15*a^2/b^3)*sqrt(x) + 5/8*a^3*l
og(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2)
```

maple [A] time = 0.07, size = 103, normalized size = 0.82

$$-\frac{\left(-16b^{\frac{9}{2}}x^4+4a\,b^{\frac{7}{2}}x^3-10a^2b^{\frac{5}{2}}x^2-30a^3b^{\frac{3}{2}}x+15\sqrt{(bx+a)\,x}\;a^3b\ln\left(\frac{2bx+a+2\sqrt{b\,x^2+ax}\;\sqrt{b}}{2\sqrt{b}}\right)\right)\sqrt{x}}{48\sqrt{b\,x^3+a\,x^2}\;b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -1/48*x^(1/2)*(-16*b^(9/2)*x^4+4*b^(7/2)*x^3*a-10*b^(5/2)*x^2*a^2-30*b^(3/2
)*x*a^3+15*(x*(b*x+a))^(1/2)*ln(1/2*(2*(b*x^2+a*x)^(1/2)*b^(1/2)+2*b*x+a)/b
^(1/2))*a^3*b)/(b*x^3+a*x^2)^(1/2)/b^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{7}{2}}}{\sqrt{bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/sqrt(b*x^3 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{7/2}}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] int(x^(7/2)/(a*x^2 + b*x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{7}{2}}}{\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(7/2)/sqrt(x**2*(a + b*x)), x)
```

3.191 $\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}} \, dx$

Optimal. Leaf size=95

$$\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{4b^{5/2}} - \frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}} + \frac{\sqrt{x}\,\sqrt{ax^2+bx^3}}{2b}$$

Rubi [A] time = 0.13, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2024, 2029, 206}

$$\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{4b^{5/2}} - \frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}} + \frac{\sqrt{x}\,\sqrt{ax^2+bx^3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-3*a*Sqrt[a*x^2 + b*x^3])/(4*b^2*Sqrt[x]) + (Sqrt[x]*Sqrt[a*x^2 + b*x^3])/
(2*b) + (3*a^2*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(4*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}} \, dx &= \frac{\sqrt{x}\,\sqrt{ax^2+bx^3}}{2b} - \frac{(3a)\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}} \, dx}{4b} \\
&= -\frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}} + \frac{\sqrt{x}\,\sqrt{ax^2+bx^3}}{2b} + \frac{\left(3a^2\right)\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}} \, dx}{8b^2} \\
&= -\frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}} + \frac{\sqrt{x}\,\sqrt{ax^2+bx^3}}{2b} + \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2} \, dx, x, \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{4b^2} \\
&= -\frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}} + \frac{\sqrt{x}\,\sqrt{ax^2+bx^3}}{2b} + \frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{4b^{5/2}}
\end{aligned}$$

Mathematica [A]   time = 0.05, size = 90, normalized size = 0.95

$$\frac{3a^{5/2}x\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+\sqrt{b}\,x^{3/2}\left(-3a^2-abx+2b^2x^2\right)}{4b^{5/2}\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (Sqrt[b]*x^(3/2)*(-3*a^2 - a*b*x + 2*b^2*x^2) + 3*a^(5/2)*x*Sqrt[1 + (b*x)/
a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(5/2)*Sqrt[x^2*(a + b*x)])
```

IntegrateAlgebraic [A]   time = 0.16, size = 95, normalized size = 1.00

$$-\frac{3a^2\log\left(\sqrt{ax^2+bx^3}-\sqrt{b}\,x^{3/2}\right)}{4b^{5/2}}+\frac{3a^2\log\left(\sqrt{x}\right)}{2b^{5/2}}+\frac{(2bx-3a)\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] ((-3*a + 2*b*x)*Sqrt[a*x^2 + b*x^3])/(4*b^2*Sqrt[x]) + (3*a^2*Log[Sqrt[x]])
/(2*b^(5/2)) - (3*a^2*Log[-(Sqrt[b]*x^(3/2)) + Sqrt[a*x^2 + b*x^3]])/(4*b^(
5/2))
```

fricas [A]   time = 0.41, size = 159, normalized size = 1.67

$$\left[\frac{3\,a^2\sqrt{b}\,x\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^3+ax^2}\,\sqrt{b}\,\sqrt{x}}{x}\right)+2\,\sqrt{bx^3+ax^2}\left(2\,b^2x-3\,ab\right)\sqrt{x}}{8\,b^3x},-\frac{3\,a^2\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-b}}{bx^{\frac{3}{2}}}\right)-\sqrt{bx^3+ax^2}\left(2\,b^2x-3\,ab\right)\sqrt{x}}{4\,b^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*x*log((2*b*x^2 + a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*sq
rt(x))/x) + 2*sqrt(b*x^3 + a*x^2)*(2*b^2*x - 3*a*b)*sqrt(x))/(b^3*x), -1/4*
(3*a^2*sqrt(-b)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b)/(b*x^(3/2))) - sqrt(b
*x^3 + a*x^2)*(2*b^2*x - 3*a*b)*sqrt(x))/(b^3*x)]
```

giac [A]   time = 0.18, size = 52, normalized size = 0.55

$$\frac{1}{4}\,\sqrt{bx+a}\,\sqrt{x}\left(\frac{2\,x}{b}-\frac{3\,a}{b^2}\right)-\frac{3\,a^2\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{4\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*x + a)*sqrt(x)*(2*x/b - 3*a/b^2) - 3/4*a^2*log(abs(-sqrt(b)*sqrt
(x) + sqrt(b*x + a)))/b^(5/2)
```

maple [A]   time = 0.05, size = 92, normalized size = 0.97

$$\frac{\left(4b^{\frac{7}{2}}x^3-2a\,b^{\frac{5}{2}}x^2-6a^2b^{\frac{3}{2}}x+3\sqrt{(bx+a)\,x}\;a^2b\ln\left(\frac{2bx+a+2\sqrt{b\,x^2+ax}\;\sqrt{b}}{2\sqrt{b}}\right)\right)\sqrt{x}}{8\sqrt{b\,x^3+a\,x^2}\;b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/8*x^(1/2)*(4*b^(7/2)*x^3-2*b^(5/2)*x^2*a-6*b^(3/2)*x*a^2+3*((b*x+a)*x)^(1
/2)*ln(1/2*(2*b*x+a+2*(b*x^2+a*x)^(1/2)*b^(1/2))/b^(1/2))*a^2*b)/(b*x^3+a*x
^2)^(1/2)/b^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{bx^3 + ax^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/sqrt(b*x^3 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\sqrt{b\,x^3 + a\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] int(x^(5/2)/(a*x^2 + b*x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{x^2 (a + bx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(5/2)/sqrt(x**2*(a + b*x)), x)
```

3.192 $\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=60

$$\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}$$

Rubi [A] time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2024, 2029, 206}

$$\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] Sqrt[a*x^2 + b*x^3]/(b*Sqrt[x]) - (a*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 +
 b*x^3]])/b^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx &= \frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx}{2b}\\
&= \frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b}\\
&= \frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 73, normalized size = 1.22

$$\frac{\sqrt{b}\,x^{3/2}(a+bx)-a^{3/2}x\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (Sqrt[b]*x^(3/2)*(a + b*x) - a^(3/2)*x*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*S
qrt[x])/Sqrt[a]])/(b^(3/2)*Sqrt[x^2*(a + b*x)])
```

IntegrateAlgebraic [A] time = 0.14, size = 75, normalized size = 1.25

$$\frac{a\log\left(\sqrt{ax^2+bx^3}-\sqrt{b}\,x^{3/2}\right)}{b^{3/2}}-\frac{2a\log\left(\sqrt{x}\right)}{b^{3/2}}+\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] Sqrt[a*x^2 + b*x^3]/(b*Sqrt[x]) - (2*a*Log[Sqrt[x]])/b^(3/2) + (a*Log[-(Sqr
t[b]*x^(3/2)) + Sqrt[a*x^2 + b*x^3]])/b^(3/2)
```

fricas [A] time = 0.43, size = 131, normalized size = 2.18

$$\left[\frac{a\sqrt{b}\,x\log\left(\frac{2\,bx^2+ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{b}\,\sqrt{x}}{x}\right)+2\,\sqrt{bx^3+ax^2}\,b\sqrt{x}}{2\,b^2x},\ \frac{a\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-b}}{bx^{\frac{3}{2}}}\right)+\sqrt{bx^3+ax^2}\,b\sqrt{x}}{b^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*sqrt(x
))/x) + 2*sqrt(b*x^3 + a*x^2)*b*sqrt(x))/(b^2*x), (a*sqrt(-b)*x*arctan(sqrt
(b*x^3 + a*x^2)*sqrt(-b)/(b*x^(3/2))) + sqrt(b*x^3 + a*x^2)*b*sqrt(x))/(b^2
*x)]
```

giac [A] time = 0.18, size = 38, normalized size = 0.63

$$\frac{a\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{3}{2}}}+\frac{\sqrt{bx+a}\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] a*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(3/2) + sqrt(b*x + a)*sqrt(x
)/b
```

maple [A] time = 0.05, size = 78, normalized size = 1.30

$$-\frac{\left(-2b^{\frac{5}{2}}x^2-2a\,b^{\frac{3}{2}}x+\sqrt{(bx+a)\,x}\;ab\ln\left(\frac{2bx+a+2\sqrt{b\,x^2+ax}\;\sqrt{b}}{2\sqrt{b}}\right)\right)\sqrt{x}}{2\sqrt{b\,x^3+a\,x^2}\;b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -1/2*x^(1/2)*(-2*b^(5/2)*x^2-2*a*b^(3/2)*x+a*((b*x+a)*x)^(1/2)*ln(1/2*(2*b*
x+a+2*(b*x^2+a*x)^(1/2)*b^(1/2))/b^(1/2))*b)/(b*x^3+a*x^2)^(1/2)/b^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{bx^3 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/sqrt(b*x^3 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{3/2}}{\sqrt{b\,x^3 + a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] int(x^(3/2)/(a*x^2 + b*x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{x^2\,(a + bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(3/2)/sqrt(x**2*(a + b*x)), x)
```

3.193 $\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}} \, dx$

Optimal. Leaf size=34

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2029, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}} \, dx &= 2\,\text{Subst}\left(\int \frac{1}{1-bx^2} \, dx, x, \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 1.62

$$\frac{2\sqrt{a}\,x\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*Sqrt[a]*x*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]
*Sqrt[x^2*(a + b*x)])
```

IntegrateAlgebraic [A] time = 0.09, size = 50, normalized size = 1.47

$$\frac{4\log\left(\sqrt{x}\right)}{\sqrt{b}} - \frac{2\log\left(\sqrt{ax^2+bx^3} - \sqrt{b}\,x^{3/2}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (4*Log[Sqrt[x]])/Sqrt[b] - (2*Log[-(Sqrt[b]*x^(3/2)) + Sqrt[a*x^2 + b*x^3]]
)/Sqrt[b]
```

fricas [A] time = 0.42, size = 77, normalized size = 2.26

$$\left[\frac{\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^3+ax^2}\,\sqrt{b}\,\sqrt{x}}{x}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-b}}{bx^{\frac{3}{2}}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((2*b*x^2 + a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*sqrt(x))/x)/sqrt(b), -2
*sqrt(-b)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b)/(b*x^(3/2)))/b]
```

giac [A] time = 0.17, size = 23, normalized size = 0.68

$$-\frac{2\,\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/sqrt(b)
```

maple [B] time = 0.05, size = 58, normalized size = 1.71

$$\frac{\sqrt{(bx+a)\,x}\,\sqrt{x}\,\ln\left(\frac{2bx+a+2\sqrt{b\,x^2+ax}\,\sqrt{b}}{2\sqrt{b}}\right)}{\sqrt{b\,x^3+a\,x^2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/(b*x^3+a*x^2)^(1/2)*x^(1/2)*((b*x+a)*x)^(1/2)*ln(1/2*(2*b*x+a+2*(b*x^2+a*
x)^(1/2)*b^(1/2))/b^(1/2))/b^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt(b*x^3 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{x}}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] int(x^(1/2)/(a*x^2 + b*x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{x^2\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x)/sqrt(x**2*(a + b*x)), x)
```

3.194 $\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=25

$$-\frac{2\sqrt{ax^2+bx^3}}{ax^{3/2}}$$

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2014}

$$-\frac{2\sqrt{ax^2+bx^3}}{ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^3])/(a*x^(3/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{ax^2+bx^3}}{ax^{3/2}}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.92

$$-\frac{2\sqrt{x^2(a+bx)}}{ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*Sqrt[x^2*(a + b*x)])/(a*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.09, size = 25, normalized size = 1.00

$$-\frac{2\sqrt{ax^2+bx^3}}{ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^3])/(a*x^(3/2))
```

fricas [A] time = 0.39, size = 21, normalized size = 0.84

$$-\frac{2\sqrt{bx^3+ax^2}}{ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*x^3 + a*x^2)/(a*x^(3/2))
```

giac [A] time = 0.24, size = 30, normalized size = 1.20

$$\frac{4\sqrt{b}}{\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 4*sqrt(b)/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)
```

maple [A] time = 0.04, size = 27, normalized size = 1.08

$$-\frac{2\,(bx+a)\sqrt{x}}{\sqrt{b\,x^3+a\,x^2}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -2*(b*x+a)*x^(1/2)/a/(b*x^3+a*x^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+ax^2}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{x}\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a*x^2 + b*x^3)^(1/2)),x)
```

```
[Out] int(1/(x^(1/2)*(a*x^2 + b*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*sqrt(x**2*(a + b*x))), x)
```

3.195 $\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=56

$$\frac{4b\sqrt{ax^2+bx^3}}{3a^2x^{3/2}}-\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}$$

Rubi [A] time = 0.08, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$\frac{4b\sqrt{ax^2+bx^3}}{3a^2x^{3/2}}-\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^3])/(3*a*x^(5/2)) + (4*b*Sqrt[a*x^2 + b*x^3])/(3*a^2*x
^(3/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx &= -\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}-\frac{(2b)\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx}{3a}\\ &= -\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}+\frac{4b\sqrt{ax^2+bx^3}}{3a^2x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.55

$$-\frac{2(a-2bx)\sqrt{x^2(a+bx)}}{3a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*(a - 2*b*x)*Sqrt[x^2*(a + b*x)])/(3*a^2*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.12, size = 35, normalized size = 0.62

$$\frac{2(2bx-a)\sqrt{ax^2+bx^3}}{3a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (2*(-a + 2*b*x)*Sqrt[a*x^2 + b*x^3])/(3*a^2*x^(5/2))
```

fricas [A] time = 0.40, size = 29, normalized size = 0.52

$$\frac{2\sqrt{bx^3+ax^2}\,(2\,bx-a)}{3\,a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x^3 + a*x^2)*(2*b*x - a)/(a^2*x^(5/2))
```

giac [A] time = 0.21, size = 55, normalized size = 0.98

$$\frac{8\left(3\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)b^{\frac{3}{2}}}{3\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)*b^(3/2)/((sqrt(b)*sqrt(x) -
 sqrt(b*x + a))^2 - a)^3
```

maple [A] time = 0.06, size = 33, normalized size = 0.59

$$-\frac{2\,(bx+a)\,(-2bx+a)}{3\sqrt{b\,x^3+a\,x^2}\;a^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -2/3*(b*x+a)*(-2*b*x+a)/x^(1/2)/a^2/(b*x^3+a*x^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+ax^2}\,x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^{3/2}\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a*x^2 + b*x^3)^(1/2)),x)
```

```
[Out] int(1/(x^(3/2)*(a*x^2 + b*x^3)^(1/2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(3/2)*sqrt(x**2*(a + b*x))), x)
```

3.196 $\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=86

$$-\frac{16b^2\sqrt{ax^2+bx^3}}{15a^3x^{3/2}}+\frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}}-\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}$$

Rubi [A] time = 0.12, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$-\frac{16b^2\sqrt{ax^2+bx^3}}{15a^3x^{3/2}}+\frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}}-\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^3])/(5*a*x^(7/2)) + (8*b*Sqrt[a*x^2 + b*x^3])/(15*a^2*
x^(5/2)) - (16*b^2*Sqrt[a*x^2 + b*x^3])/(15*a^3*x^(3/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx &= -\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}-\frac{(4b)\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx}{5a}\\
&= -\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}+\frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}}+\frac{\left(8b^2\right)\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx}{15a^2}\\
&= -\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}+\frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}}-\frac{16b^2\sqrt{ax^2+bx^3}}{15a^3x^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.51

$$-\frac{2\sqrt{x^2(a+bx)}\left(3a^2-4abx+8b^2x^2\right)}{15a^3x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*Sqrt[x^2*(a + b*x)]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^(7/2))
```

IntegrateAlgebraic [A] time = 0.14, size = 46, normalized size = 0.53

$$-\frac{2\left(3a^2-4abx+8b^2x^2\right)\sqrt{ax^2+bx^3}}{15a^3x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*Sqrt[a*x^2 + b*x^3]),x]
[Out] (-2*(3*a^2 - 4*a*b*x + 8*b^2*x^2)*Sqrt[a*x^2 + b*x^3])/(15*a^3*x^(7/2))
```

fricas [A] time = 0.40, size = 40, normalized size = 0.47

$$-\frac{2\left(8\,b^2x^2-4\,abx+3\,a^2\right)\sqrt{bx^3+ax^2}}{15\,a^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
[Out] -2/15*(8*b^2*x^2 - 4*a*b*x + 3*a^2)*sqrt(b*x^3 + a*x^2)/(a^3*x^(7/2))
```

giac [A] time = 0.20, size = 77, normalized size = 0.90

$$\frac{32\left(10\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^4-5\,a\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2+a^2\right)b^{\frac{5}{2}}}{15\left(\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] 32/15*(10*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 - 5*a*(sqrt(b)*sqrt(x) - sqrt
(b*x + a))^2 + a^2)*b^(5/2)/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^5
```

maple [A] time = 0.04, size = 46, normalized size = 0.53

$$-\frac{2\,(bx+a)\left(8b^2x^2-4abx+3a^2\right)}{15\sqrt{b\,x^3+a\,x^2}\;a^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x)
[Out] -2/15*(b*x+a)*(8*b^2*x^2-4*a*b*x+3*a^2)/x^(3/2)/a^3/(b*x^3+a*x^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+ax^2}\,x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{5/2}\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a*x^2 + b*x^3)^(1/2)),x)
```

```
[Out] int(1/(x^(5/2)*(a*x^2 + b*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}}\sqrt{x^2\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(5/2)*sqrt(x**2*(a + b*x))), x)
```

3.197 $\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=116

$$\frac{32b^3\sqrt{ax^2+bx^3}}{35a^4x^{3/2}}-\frac{16b^2\sqrt{ax^2+bx^3}}{35a^3x^{5/2}}+\frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}}-\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}$$

Rubi [A] time = 0.16, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$\frac{32b^3\sqrt{ax^2+bx^3}}{35a^4x^{3/2}}-\frac{16b^2\sqrt{ax^2+bx^3}}{35a^3x^{5/2}}+\frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}}-\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^3])/(7*a*x^(9/2)) + (12*b*Sqrt[a*x^2 + b*x^3])/(35*a^2
*x^(7/2)) - (16*b^2*Sqrt[a*x^2 + b*x^3])/(35*a^3*x^(5/2)) + (32*b^3*Sqrt[a*
x^2 + b*x^3])/(35*a^4*x^(3/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx &= -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}-\frac{(6b)\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx}{7a}\\
&= -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}+\frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}}+\frac{\left(24b^2\right)\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx}{35a^2}\\
&= -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}+\frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}}-\frac{16b^2\sqrt{ax^2+bx^3}}{35a^3x^{5/2}}-\frac{\left(16b^3\right)\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx}{35a^3}\\
&= -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}+\frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}}-\frac{16b^2\sqrt{ax^2+bx^3}}{35a^3x^{5/2}}+\frac{32b^3\sqrt{ax^2+bx^3}}{35a^4x^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.47

$$\frac{2\sqrt{x^2(a+bx)}\left(-5a^3+6a^2bx-8ab^2x^2+16b^3x^3\right)}{35a^4x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (2*Sqrt[x^2*(a + b*x)]*(-5*a^3 + 6*a^2*b*x - 8*a*b^2*x^2 + 16*b^3*x^3))/(35
*a^4*x^(9/2))
```

IntegrateAlgebraic [A] time = 0.14, size = 57, normalized size = 0.49

$$\frac{2\sqrt{ax^2+bx^3}\left(-5a^3+6a^2bx-8ab^2x^2+16b^3x^3\right)}{35a^4x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^3]*(-5*a^3 + 6*a^2*b*x - 8*a*b^2*x^2 + 16*b^3*x^3))/(35
*a^4*x^(9/2))
```

fricas [A] time = 0.40, size = 51, normalized size = 0.44

$$\frac{2\left(16\,b^3x^3-8\,ab^2x^2+6\,a^2bx-5\,a^3\right)\sqrt{bx^3+ax^2}}{35\,a^4x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x^3 + a*x^2)/(a^
4*x^(9/2))
```

giac [A] time = 0.21, size = 103, normalized size = 0.89

$$\frac{64\left(35\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^6-21\,a\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^4+7\,a^2\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a^3\right)b^{\frac{7}{2}}}{35\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 64/35*(35*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^6 - 21*a*(sqrt(b)*sqrt(x) - sqr
t(b*x + a))^4 + 7*a^2*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a^3)*b^(7/2)/((
sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^7
```

maple [A] time = 0.04, size = 57, normalized size = 0.49

$$-\frac{2\,(bx+a)\left(-16b^3x^3+8a\,b^2x^2-6a^2bx+5a^3\right)}{35\sqrt{b\,x^3+a\,x^2}\;a^4x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -2/35*(b*x+a)*(-16*b^3*x^3+8*a*b^2*x^2-6*a^2*b*x+5*a^3)/x^(5/2)/a^4/(b*x^3+
a*x^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+ax^2}\,x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{7/2}\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a*x^2 + b*x^3)^(1/2)),x)
[Out] int(1/(x^(7/2)*(a*x^2 + b*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{7}{2}}\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**3+a*x**2)**(1/2),x)
[Out] Integral(1/(x**(7/2)*sqrt(x**2*(a + b*x))), x)
```

3.198 $\int x^{-2-3n}\left(ax^2+bx^3\right)^n\,dx$

Optimal. Leaf size=32

$$-\frac{x^{-3(n+1)}\left(ax^2+bx^3\right)^{n+1}}{a(n+1)}$$

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2014}

$$-\frac{x^{-3(n+1)}\left(ax^2+bx^3\right)^{n+1}}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -((a*x^2 + b*x^3)^(1 + n)/(a*(1 + n)*x^(3*(1 + n))))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^{-2-3n}\left(ax^2+bx^3\right)^n\,dx=-\frac{x^{-3(1+n)}\left(ax^2+bx^3\right)^{1+n}}{a(1+n)}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.94

$$-\frac{x^{-3(n+1)}\left(x^2(a+bx)\right)^{n+1}}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -((x^2*(a + b*x))^(1 + n)/(a*(1 + n)*x^(3*(1 + n))))
```

IntegrateAlgebraic [F] time = 0.10, size = 0, normalized size = 0.00

$$\int x^{-2-3n}\left(ax^2+bx^3\right)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-2 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-2 - 3*n)*(a*x^2 + b*x^3)^n, x]
```

fricas [A] time = 0.41, size = 38, normalized size = 1.19

$$-\frac{\left(bx^2+ax\right)\left(bx^3+ax^2\right)^n x^{-3\,n-2}}{an+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
```

```
[Out] -(b*x^2 + a*x)*(b*x^3 + a*x^2)^n*x^(-3*n - 2)/(a*n + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + ax^2\right)^n x^{-3\,n-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 2), x)
```

maple [A] time = 0.05, size = 36, normalized size = 1.12

$$-\frac{(bx+a)\,x^{-3n-1}\left(b\,x^3+a\,x^2\right)^n}{(n+1)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2-3*n)*(b*x^3+a*x^2)^n,x)
```

```
[Out] -(b*x+a)*x^(-3*n-1)/a/(n+1)*(b*x^3+a*x^2)^n
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + ax^2\right)^n x^{-3\,n-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 2), x)
```

mupad [B] time = 5.28, size = 54, normalized size = 1.69

$$-\left(b\,x^3+a\,x^2\right)^n\left(\frac{x}{x^{3\,n+2}\,\left(n+1\right)}+\frac{b\,x^2}{a\,x^{3\,n+2}\,\left(n+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^n/x^(3*n + 2),x)
```

```
[Out] -(a*x^2 + b*x^3)^n*(x/(x^(3*n + 2)*(n + 1)) + (b*x^2)/(a*x^(3*n + 2)*(n + 1
)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-3n-2}\left(x^2\,(a+bx)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2-3*n)*(b*x**3+a*x**2)**n,x)
```

```
[Out] Integral(x**(-3*n - 2)*(x**2*(a + b*x))**n, x)
```

3.199 $\int x^{-3-3n}\left(ax^2+bx^3\right)^n\,dx$

Optimal. Leaf size=70

$$\frac{bx^{-3(n+1)}\left(ax^2+bx^3\right)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-3n-4}\left(ax^2+bx^3\right)^{n+1}}{a(n+2)}$$

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$\frac{bx^{-3(n+1)}\left(ax^2+bx^3\right)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-3n-4}\left(ax^2+bx^3\right)^{n+1}}{a(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -((x^(-4 - 3*n)*(a*x^2 + b*x^3)^(1 + n))/(a*(2 + n))) + (b*(a*x^2 + b*x^3)^
(1 + n))/(a^2*(1 + n)*(2 + n)*x^(3*(1 + n)))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-3-3n}\left(ax^2+bx^3\right)^n\,dx &= -\frac{x^{-4-3n}\left(ax^2+bx^3\right)^{1+n}}{a(2+n)}-\frac{b\int x^{-2-3n}\left(ax^2+bx^3\right)^n\,dx}{a(2+n)}\\ &= -\frac{x^{-4-3n}\left(ax^2+bx^3\right)^{1+n}}{a(2+n)}+\frac{bx^{-3(1+n)}\left(ax^2+bx^3\right)^{1+n}}{a^2(1+n)(2+n)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.63

$$-\frac{x^{-3n-4}(an+a-bx)\left(x^2(a+bx)\right)^{n+1}}{a^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -((x^(-4 - 3*n)*(a + a*n - b*x)*(x^2*(a + b*x))^(1 + n))/(a^2*(1 + n)*(2 +
n)))
```

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x^{-3-3n}\left(ax^2+bx^3\right)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-3 - 3*n)*(a*x^2 + b*x^3)^n,x]
[Out] Defer[IntegrateAlgebraic][x^(-3 - 3*n)*(a*x^2 + b*x^3)^n, x]
```

fricas [A] time = 0.42, size = 70, normalized size = 1.00

$$-\frac{\left(abnx^2-b^2x^3+\left(a^2n+a^2\right)x\right)\left(bx^3+ax^2\right)^n x^{-3\,n-3}}{a^2n^2+3\,a^2n+2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
[Out] -(a*b*n*x^2 - b^2*x^3 + (a^2*n + a^2)*x)*(b*x^3 + a*x^2)^n*x^(-3*n - 3)/(a^
2*n^2 + 3*a^2*n + 2*a^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^3+ax^2\right)^n x^{-3\,n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 3), x)
```

maple [A] time = 0.05, size = 50, normalized size = 0.71

$$-\frac{(an-bx+a)\,(bx+a)\,x^{-3n-2}\left(b\,x^3+a\,x^2\right)^n}{(n+2)\,(n+1)\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3-3*n)*(b*x^3+a*x^2)^n,x)
[Out] -(b*x^3+a*x^2)^n*x^(-2-3*n)*(a*n-b*x+a)*(b*x+a)/(n+2)/(n+1)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^3+ax^2\right)^n x^{-3\,n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 3), x)
```

mupad [B] time = 5.28, size = 98, normalized size = 1.40

$$-\left(b\,x^3+a\,x^2\right)^n\left(\frac{x\,\left(n+1\right)}{x^{3\,n+3}\,\left(n^2+3\,n+2\right)}-\frac{b^2\,x^3}{a^2\,x^{3\,n+3}\,\left(n^2+3\,n+2\right)}+\frac{b\,n\,x^2}{a\,x^{3\,n+3}\,\left(n^2+3\,n+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^n/x^(3*n + 3),x)
```

```
[Out] -(a*x^2 + b*x^3)^n*((x*(n + 1))/(x^(3*n + 3)*(3*n + n^2 + 2)) - (b^2*x^3)/(
a^2*x^(3*n + 3)*(3*n + n^2 + 2)) + (b*n*x^2)/(a*x^(3*n + 3)*(3*n + n^2 + 2)
))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3-3*n)*(b*x**3+a*x**2)**n,x)
```

```
[Out] Timed out
```

3.200 $\int x^{-4-3n} \left(ax^2+bx^3\right)^n \, dx$

Optimal. Leaf size=116

$$-\frac{2b^2x^{-3(n+1)}\left(ax^2+bx^3\right)^{n+1}}{a^3(n+1)(n+2)(n+3)}+\frac{2bx^{-3n-4}\left(ax^2+bx^3\right)^{n+1}}{a^2(n+2)(n+3)}-\frac{x^{-3n-5}\left(ax^2+bx^3\right)^{n+1}}{a(n+3)}$$

Rubi [A] time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$-\frac{2b^2x^{-3(n+1)}\left(ax^2+bx^3\right)^{n+1}}{a^3(n+1)(n+2)(n+3)}+\frac{2bx^{-3n-4}\left(ax^2+bx^3\right)^{n+1}}{a^2(n+2)(n+3)}-\frac{x^{-3n-5}\left(ax^2+bx^3\right)^{n+1}}{a(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-4 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -((x^(-5 - 3*n)*(a*x^2 + b*x^3)^(1 + n))/(a*(3 + n))) + (2*b*x^(-4 - 3*n)*(
a*x^2 + b*x^3)^(1 + n))/(a^2*(2 + n)*(3 + n)) - (2*b^2*(a*x^2 + b*x^3)^(1 +
 n))/(a^3*(1 + n)*(2 + n)*(3 + n)*x^(3*(1 + n)))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int x^{-4-3n}\left(ax^2+bx^3\right)^n\,dx &= -\frac{x^{-5-3n}\left(ax^2+bx^3\right)^{1+n}}{a(3+n)}-\frac{(2b)\int x^{-3-3n}\left(ax^2+bx^3\right)^n\,dx}{a(3+n)}\\
&= -\frac{x^{-5-3n}\left(ax^2+bx^3\right)^{1+n}}{a(3+n)}+\frac{2bx^{-4-3n}\left(ax^2+bx^3\right)^{1+n}}{a^2(2+n)(3+n)}+\frac{\left(2b^2\right)\int x^{-2-3n}\left(ax^2+bx^3\right)^{n}}{a^2(2+n)(3+n)}\\
&= -\frac{x^{-5-3n}\left(ax^2+bx^3\right)^{1+n}}{a(3+n)}+\frac{2bx^{-4-3n}\left(ax^2+bx^3\right)^{1+n}}{a^2(2+n)(3+n)}-\frac{2b^2x^{-3(1+n)}\left(ax^2+bx^3\right)^{1+n}}{a^3(1+n)(2+n)(3+n)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 0.62

$$-\frac{x^{-3(n+1)}(a+bx)\left(x^2(a+bx)\right)^n\left(a^2\left(n^2+3n+2\right)-2ab(n+1)x+2b^2x^2\right)}{a^3(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-4 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -(((a + b*x)*(x^2*(a + b*x))^n*(a^2*(2 + 3*n + n^2) - 2*a*b*(1 + n)*x + 2*b
^2*x^2))/(a^3*(1 + n)*(2 + n)*(3 + n)*x^(3*(1 + n))))
```

IntegrateAlgebraic [F] time = 0.09, size = 0, normalized size = 0.00

$$\int x^{-4-3n}\left(ax^2+bx^3\right)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-4 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-4 - 3*n)*(a*x^2 + b*x^3)^n, x]
```

fricas [A] time = 0.42, size = 111, normalized size = 0.96

$$\frac{\left(2\,ab^2nx^3-2\,b^3x^4-\left(a^2bn^2+a^2bn\right)x^2-\left(a^3n^2+3\,a^3n+2\,a^3\right)x\right)\left(bx^3+ax^2\right)^n x^{-3\,n-4}}{a^3n^3+6\,a^3n^2+11\,a^3n+6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
```

```
[Out] (2*a*b^2*n*x^3 - 2*b^3*x^4 - (a^2*b*n^2 + a^2*b*n)*x^2 - (a^3*n^2 + 3*a^3*n
 + 2*a^3)*x)*(b*x^3 + a*x^2)^n*x^(-3*n - 4)/(a^3*n^3 + 6*a^3*n^2 + 11*a^3*n
 + 6*a^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^3+ax^2\right)^n x^{-3\,n-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 4), x)
```

maple [A] time = 0.05, size = 84, normalized size = 0.72

$$-\frac{(bx+a)\left(a^2n^2-2abnx+2b^2x^2+3a^2n-2abx+2a^2\right)x^{-3n-3}\left(b\,x^3+a\,x^2\right)^n}{(n+3)\,(n+2)\,(n+1)\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-4-3*n)*(b*x^3+a*x^2)^n,x)
```

```
[Out] -x^(-3-3*n)*(b*x+a)*(a^2*n^2-2*a*b*n*x+2*b^2*x^2+3*a^2*n-2*a*b*x+2*a^2)*(b*
x^3+a*x^2)^n/(n+3)/(n+2)/(n+1)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^3+ax^2\right)^n x^{-3\,n-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 4), x)
```

mupad [B] time = 5.36, size = 157, normalized size = 1.35

$$-\left(b\,x^3+a\,x^2\right)^n\left(\frac{x\,\left(n^2+3\,n+2\right)}{x^{3\,n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{2\,b^3\,x^4}{a^3\,x^{3\,n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{2\,b^2\,n\,x^3}{a^2\,x^{3\,n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{b\,n\,x^2\,\left(n+1\right)}{a\,x^{3\,n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^n/x^(3*n + 4),x)
```

```
[Out] -(a*x^2 + b*x^3)^n*((x*(3*n + n^2 + 2))/(x^(3*n + 4)*(11*n + 6*n^2 + n^3 +
6)) + (2*b^3*x^4)/(a^3*x^(3*n + 4)*(11*n + 6*n^2 + n^3 + 6)) - (2*b^2*n*x^3
)/(a^2*x^(3*n + 4)*(11*n + 6*n^2 + n^3 + 6)) + (b*n*x^2*(n + 1))/(a*x^(3*n
+ 4)*(11*n + 6*n^2 + n^3 + 6)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-4-3*n)*(b*x**3+a*x**2)**n,x)
```

```
[Out] Timed out
```

3.201 $\int \frac{x^{11}}{\left(ax^2+bx^5\right)^3} dx$

Optimal. Leaf size=19

$$\frac{x^6}{6a\left(a+bx^3\right)^2}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 264}

$$\frac{x^6}{6a\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a*x^2 + b*x^5)^3,x]
```

```
[Out] x^6/(6*a*(a + b*x^3)^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\left(ax^2+bx^5\right)^3} dx &= \int \frac{x^5}{\left(a+bx^3\right)^3} dx\\ &= \frac{x^6}{6a\left(a+bx^3\right)^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.26

$$-\frac{a+2bx^3}{6b^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a*x^2 + b*x^5)^3,x]
```

```
[Out] -1/6*(a + 2*b*x^3)/(b^2*(a + b*x^3)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(ax^2+bx^5\right)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11/(a*x^2 + b*x^5)^3,x]
```

```
[Out] IntegrateAlgebraic[x^11/(a*x^2 + b*x^5)^3, x]
```

fricas [B] time = 0.38, size = 36, normalized size = 1.89

$$-\frac{2\,bx^3 + a}{6\,\left(b^4x^6 + 2\,ab^3x^3 + a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^5+a*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

giac [A] time = 0.18, size = 22, normalized size = 1.16

$$-\frac{2\,bx^3 + a}{6\,\left(bx^3 + a\right)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^5+a*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/6*(2*b*x^3 + a)/((b*x^3 + a)^2*b^2)
```

maple [A] time = 0.06, size = 31, normalized size = 1.63

$$\frac{a}{6\left(b\,x^3 + a\right)^2 b^2} - \frac{1}{3\left(b\,x^3 + a\right) b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^5+a*x^2)^3,x)
```

```
[Out] 1/6*a/b^2/(b*x^3+a)^2-1/3/b^2/(b*x^3+a)
```

maxima [B] time = 1.29, size = 36, normalized size = 1.89

$$-\frac{2\,bx^3 + a}{6\,\left(b^4x^6 + 2\,ab^3x^3 + a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^5+a*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(2*b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

mupad [B] time = 5.11, size = 37, normalized size = 1.95

$$-\frac{\frac{a}{6\,b^2} + \frac{x^3}{3\,b}}{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a*x^2 + b*x^5)^3,x)
```

```
[Out] -(a/(6*b^2) + x^3/(3*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)
```

sympy [B] time = 0.40, size = 36, normalized size = 1.89

$$\frac{-a-2bx^3}{6a^2b^2+12ab^3x^3+6b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**5+a*x**2)**3,x)
```

```
[Out] (-a - 2*b*x**3)/(6*a**2*b**2 + 12*a*b**3*x**3 + 6*b**4*x**6)
```

3.202 $\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=80

$$\frac{16a^2\sqrt{ax^2+bx^5}}{45b^3x}-\frac{8ax^2\sqrt{ax^2+bx^5}}{45b^2}+\frac{2x^5\sqrt{ax^2+bx^5}}{15b}$$

Rubi [A] time = 0.11, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 1588}

$$\frac{16a^2\sqrt{ax^2+bx^5}}{45b^3x}-\frac{8ax^2\sqrt{ax^2+bx^5}}{45b^2}+\frac{2x^5\sqrt{ax^2+bx^5}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (16*a^2*Sqrt[a*x^2 + b*x^5])/(45*b^3*x) - (8*a*x^2*Sqrt[a*x^2 + b*x^5])/(45
*b^2) + (2*x^5*Sqrt[a*x^2 + b*x^5])/(15*b)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx &= \frac{2x^5\sqrt{ax^2+bx^5}}{15b}-\frac{(4a)\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx}{5b}\\ &= -\frac{8ax^2\sqrt{ax^2+bx^5}}{45b^2}+\frac{2x^5\sqrt{ax^2+bx^5}}{15b}+\frac{\left(8a^2\right)\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx}{15b^2}\\ &= \frac{16a^2\sqrt{ax^2+bx^5}}{45b^3x}-\frac{8ax^2\sqrt{ax^2+bx^5}}{45b^2}+\frac{2x^5\sqrt{ax^2+bx^5}}{15b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.58

$$\frac{2\sqrt{x^2\left(a+bx^3\right)}\left(8a^2-4abx^3+3b^2x^6\right)}{45b^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/Sqrt[a*x^2 + b*x^5],x]
[Out] (2*Sqrt[x^2*(a + b*x^3)]*(8*a^2 - 4*a*b*x^3 + 3*b^2*x^6))/(45*b^3*x)
```

IntegrateAlgebraic [A] time = 0.04, size = 46, normalized size = 0.58

$$\frac{2\sqrt{ax^2+bx^5}\,\left(8a^2-4abx^3+3b^2x^6\right)}{45b^3x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^9/Sqrt[a*x^2 + b*x^5],x]
[Out] (2*Sqrt[a*x^2 + b*x^5]*(8*a^2 - 4*a*b*x^3 + 3*b^2*x^6))/(45*b^3*x)
```

fricas [A] time = 0.39, size = 42, normalized size = 0.52

$$\frac{2\left(3\,b^2x^6-4\,abx^3+8\,a^2\right)\sqrt{bx^5+ax^2}}{45\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/45*(3*b^2*x^6 - 4*a*b*x^3 + 8*a^2)*sqrt(b*x^5 + a*x^2)/(b^3*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^9/sqrt(b*x^5 + a*x^2), x)
```

maple [A] time = 0.05, size = 48, normalized size = 0.60

$$\frac{2\left(b\,x^3+a\right)\left(3b^2x^6-4ab\,x^3+8a^2\right)x}{45\sqrt{b\,x^5+a\,x^2}\;b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^5+a*x^2)^(1/2),x)
[Out] 2/45*(b*x^3+a)*(3*b^2*x^6-4*a*b*x^3+8*a^2)*x/b^3/(b*x^5+a*x^2)^(1/2)
```

maxima [A] time = 1.43, size = 46, normalized size = 0.58

$$\frac{2\left(3\,b^3x^9-ab^2x^6+4\,a^2bx^3+8\,a^3\right)}{45\,\sqrt{bx^3+a}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/45*(3*b^3*x^9 - a*b^2*x^6 + 4*a^2*b*x^3 + 8*a^3)/(sqrt(b*x^3 + a)*b^3)
```

mupad [B] time = 5.18, size = 42, normalized size = 0.52

$$\frac{2\,\sqrt{b\,x^5+a\,x^2}\,\left(8\,a^2-4\,a\,b\,x^3+3\,b^2\,x^6\right)}{45\,b^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] (2*(a*x^2 + b*x^5)^(1/2)*(8*a^2 + 3*b^2*x^6 - 4*a*b*x^3))/(45*b^3*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**9/sqrt(x**2*(a + b*x**3)), x)
```

3.203 $\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=52

$$\frac{2x^2\sqrt{ax^2+bx^5}}{9b}-\frac{4a\sqrt{ax^2+bx^5}}{9b^2x}$$

Rubi [A] time = 0.06, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 1588}

$$\frac{2x^2\sqrt{ax^2+bx^5}}{9b}-\frac{4a\sqrt{ax^2+bx^5}}{9b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-4*a*Sqrt[a*x^2 + b*x^5])/(9*b^2*x) + (2*x^2*Sqrt[a*x^2 + b*x^5])/(9*b)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx&=\frac{2x^2\sqrt{ax^2+bx^5}}{9b}-\frac{(2a)\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx}{3b}\\&=-\frac{4a\sqrt{ax^2+bx^5}}{9b^2x}+\frac{2x^2\sqrt{ax^2+bx^5}}{9b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.65

$$\frac{2\left(bx^3-2a\right)\sqrt{x^2\left(a+bx^3\right)}}{9b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*(-2*a + b*x^3)*Sqrt[x^2*(a + b*x^3)])/(9*b^2*x)
```

IntegrateAlgebraic [A] time = 0.04, size = 34, normalized size = 0.65

$$\frac{2\left(bx^3-2a\right)\sqrt{ax^2+bx^5}}{9b^2x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6/Sqrt[a*x^2 + b*x^5],x]
[Out] (2*(-2*a + b*x^3)*Sqrt[a*x^2 + b*x^5])/(9*b^2*x)
```

fricas [A] time = 0.40, size = 30, normalized size = 0.58

$$\frac{2\sqrt{bx^5+ax^2}\left(bx^3-2\,a\right)}{9\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/9*sqrt(b*x^5 + a*x^2)*(b*x^3 - 2*a)/(b^2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^6/sqrt(b*x^5 + a*x^2), x)
```

maple [A] time = 0.06, size = 37, normalized size = 0.71

$$-\frac{2\left(b\,x^3+a\right)\left(-b\,x^3+2a\right)x}{9\sqrt{b\,x^5+a\,x^2}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^5+a*x^2)^(1/2),x)
[Out] -2/9*(b*x^3+a)*(-b*x^3+2*a)*x/b^2/(b*x^5+a*x^2)^(1/2)
```

maxima [A] time = 1.44, size = 34, normalized size = 0.65

$$\frac{2\left(b^2x^6-abx^3-2\,a^2\right)}{9\,\sqrt{bx^3+a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/9*(b^2*x^6 - a*b*x^3 - 2*a^2)/(sqrt(b*x^3 + a)*b^2)
```

mupad [B] time = 5.33, size = 33, normalized size = 0.63

$$-\frac{\sqrt{b\,x^5+a\,x^2}\,\left(\frac{4\,a}{9\,b^2}-\frac{2\,x^3}{9\,b}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] -((a*x^2 + b*x^5)^(1/2)*((4*a)/(9*b^2) - (2*x^3)/(9*b)))/x
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**6/sqrt(x**2*(a + b*x**3)), x)
```

3.204 $\int \frac{x^3}{\sqrt{ax^2+bx^5}} dx$

Optimal. Leaf size=25

$$\frac{2\sqrt{ax^2+bx^5}}{3bx}$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1588}

$$\frac{2\sqrt{ax^2+bx^5}}{3bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x^2 + b*x^5],x]
[Out] (2*Sqrt[a*x^2 + b*x^5])/(3*b*x)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}} dx = \frac{2\sqrt{ax^2+bx^5}}{3bx}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{2\sqrt{x^2\left(a+bx^3\right)}}{3bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x^2 + b*x^5],x]
[Out] (2*Sqrt[x^2*(a + b*x^3)])/(3*b*x)
```

IntegrateAlgebraic [A] time = 0.04, size = 25, normalized size = 1.00

$$\frac{2\sqrt{ax^2+bx^5}}{3bx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[a*x^2 + b*x^5],x]
[Out] (2*Sqrt[a*x^2 + b*x^5])/(3*b*x)
```

fricas [A] time = 0.39, size = 21, normalized size = 0.84

$$\frac{2\sqrt{bx^5+ax^2}}{3\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(b*x^5 + a*x^2)/(b*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{bx^5 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^3/sqrt(b*x^5 + a*x^2), x)
```

maple [A] time = 0.05, size = 27, normalized size = 1.08

$$\frac{2\left(b\,x^3 + a\right)x}{3\sqrt{b\,x^5 + a\,x^2}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^5+a*x^2)^(1/2),x)
[Out] 2/3*(b*x^3+a)*x/b/(b*x^5+a*x^2)^(1/2)
```

maxima [A] time = 1.40, size = 14, normalized size = 0.56

$$\frac{2\sqrt{bx^3 + a}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/3*sqrt(b*x^3 + a)/b
```

mupad [B] time = 5.19, size = 21, normalized size = 0.84

$$\frac{2\sqrt{b\,x^5 + a\,x^2}}{3\,b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^5)^(1/2),x)
[Out] (2*(a*x^2 + b*x^5)^(1/2))/(3*b*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^2\left(a + bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(x**3/sqrt(x**2*(a + b*x**3)), x)
```

3.205 $\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=32

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{a}}$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2008, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^5]])/(3*Sqrt[a])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx &= -\left(\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^5}}\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.69

$$-\frac{2x\sqrt{a+bx^3}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-2*x*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]*Sqrt[x^2
*(a + b*x^3)])
```

IntegrateAlgebraic [A] time = 0.05, size = 32, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^5]])/(3*Sqrt[a])
```

fricas [A] time = 0.42, size = 75, normalized size = 2.34

$$\left[\frac{\log\left(\frac{bx^4+2\,ax-2\,\sqrt{bx^5+ax^2}\,\sqrt{a}}{x^4}\right)}{3\,\sqrt{a}}, \frac{2\,\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^5+ax^2}\,\sqrt{-a}}{ax}\right)}{3\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*log((b*x^4 + 2*a*x - 2*sqrt(b*x^5 + a*x^2)*sqrt(a))/x^4)/sqrt(a), 2/3*
sqrt(-a)*arctan(sqrt(b*x^5 + a*x^2)*sqrt(-a)/(a*x))/a]
```

giac [A] time = 0.19, size = 47, normalized size = 1.47

$$-\frac{2\,\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{3\,\sqrt{-a}} + \frac{2\,\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2/3*arctan(sqrt(b*x^3 + a)/
sqrt(-a))/(sqrt(-a)*sgn(x))
```

maple [A] time = 0.05, size = 43, normalized size = 1.34

$$-\frac{2\sqrt{b\,x^3+a}\;x\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{b\,x^5+a\,x^2}\;\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -2/3/(b*x^5+a*x^2)^(1/2)*x*(b*x^3+a)^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2))
/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*x^5 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] int(1/(a*x^2 + b*x^5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2 + bx^5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**2 + b*x**5), x)
```

3.206 $\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=59

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^5}}\right)}{3a^{3/2}}-\frac{\sqrt{ax^2+bx^5}}{3ax^4}$$

Rubi [A] time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2008, 206}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^5}}\right)}{3a^{3/2}}-\frac{\sqrt{ax^2+bx^5}}{3ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] -Sqrt[a*x^2 + b*x^5]/(3*a*x^4) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^5]
])/(3*a^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx &= -\frac{\sqrt{ax^2+bx^5}}{3ax^4}-\frac{b\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx}{2a}\\
&= -\frac{\sqrt{ax^2+bx^5}}{3ax^4}+\frac{b\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^5}}\right)}{3a}\\
&= -\frac{\sqrt{ax^2+bx^5}}{3ax^4}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^5}}\right)}{3a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 71, normalized size = 1.20

$$\frac{2b\sqrt{x^2\left(a+bx^3\right)}\left(\frac{\tanh^{-1}\left(\sqrt{\frac{bx^3}{a}+1}\right)}{2\sqrt{\frac{bx^3}{a}+1}}-\frac{a}{2bx^3}\right)}{3a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (2*b*Sqrt[x^2*(a + b*x^3)]*(-1/2*a/(b*x^3) + ArcTanh[Sqrt[1 + (b*x^3)/a]]/(
2*Sqrt[1 + (b*x^3)/a])))/(3*a^2*x)
```

IntegrateAlgebraic [A] time = 0.07, size = 59, normalized size = 1.00

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^5}}\right)}{3a^{3/2}}-\frac{\sqrt{ax^2+bx^5}}{3ax^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] -1/3*Sqrt[a*x^2 + b*x^5]/(a*x^4) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^
5]])/(3*a^(3/2))
```

fricas [A] time = 0.41, size = 127, normalized size = 2.15

$$\left[\frac{\sqrt{a}\,bx^4\log\left(\frac{bx^4+2\,ax+2\,\sqrt{bx^5+ax^2}\,\sqrt{a}}{x^4}\right)-2\,\sqrt{bx^5+ax^2}\,a}{6\,a^2x^4},-\frac{\sqrt{-a}\,bx^4\arctan\left(\frac{\sqrt{bx^5+ax^2}\,\sqrt{-a}}{ax}\right)+\sqrt{bx^5+ax^2}\,a}{3\,a^2x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(a)*b*x^4*log((b*x^4 + 2*a*x + 2*sqrt(b*x^5 + a*x^2)*sqrt(a))/x^4
) - 2*sqrt(b*x^5 + a*x^2)*a)/(a^2*x^4), -1/3*(sqrt(-a)*b*x^4*arctan(sqrt(b*
x^5 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^5 + a*x^2)*a)/(a^2*x^4)]
```

giac [A] time = 0.21, size = 57, normalized size = 0.97

$$-\frac{b\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}\,a\mathrm{sgn}(x)}-\frac{\sqrt{\frac{b}{x}+\frac{a}{x^4}}}{3\,ax\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*b*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a*sgn(x)) - 1/3*sqrt(b/x
+ a/x^4)/(a*x*sgn(x))
```

maple [A] time = 0.05, size = 66, normalized size = 1.12

$$-\frac{\sqrt{b\,x^3+a}\,\left(-ab\,x^3\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)+\sqrt{b\,x^3+a}\;a^{\frac{3}{2}}\right)}{3\sqrt{b\,x^5+a\,x^2}\;a^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -1/3/x^2*(b*x^3+a)^(1/2)*(-b*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a*x^3+(b*x^3+
a)^(1/2)*a^(3/2))/(b*x^5+a*x^2)^(1/2)/a^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^5 + ax^2}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\,\sqrt{b\,x^5 + a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x^2 + b*x^5)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a*x^2 + b*x^5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{x^2\left(a + bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x**2*(a + b*x**3))), x)
```

3.207 $\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{x}\,\sqrt{ax^2+bx^5}}{3b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3b^{3/2}}$$

Rubi [A] time = 0.09, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2024, 2029, 206}

$$\frac{\sqrt{x}\,\sqrt{ax^2+bx^5}}{3b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/Sqrt[a*x^2 + b*x^5],x]

[Out] (Sqrt[x]*Sqrt[a*x^2 + b*x^5])/(3*b) - (a*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a*x
^2 + b*x^5]])/(3*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx&=\frac{\sqrt{x}\,\sqrt{ax^2+bx^5}}{3b}-\frac{a\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx}{2b}\\&=\frac{\sqrt{x}\,\sqrt{ax^2+bx^5}}{3b}-\frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3b}\\&=\frac{\sqrt{x}\,\sqrt{ax^2+bx^5}}{3b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 81, normalized size = 1.25

$$\frac{\sqrt{b}\,x^{5/2}\left(a+bx^3\right)-ax\sqrt{a+bx^3}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a+bx^3}}\right)}{3b^{3/2}\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (Sqrt[b]*x^(5/2)*(a + b*x^3) - a*x*Sqrt[a + b*x^3]*ArcTanh[(Sqrt[b]*x^(3/2)
)/Sqrt[a + b*x^3]])/(3*b^(3/2)*Sqrt[x^2*(a + b*x^3)])
```

IntegrateAlgebraic [A] time = 0.29, size = 82, normalized size = 1.26

$$-\frac{a\log\left(\sqrt{ax^2+bx^5}+\sqrt{b}\,x^{5/2}\right)}{3b^{3/2}}+\frac{2a\log\left(\sqrt{x}\right)}{3b^{3/2}}+\frac{\sqrt{x}\,\sqrt{ax^2+bx^5}}{3b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(9/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (Sqrt[x]*Sqrt[a*x^2 + b*x^5])/(3*b) + (2*a*Log[Sqrt[x]])/(3*b^(3/2)) - (a*L
og[Sqrt[b]*x^(5/2) + Sqrt[a*x^2 + b*x^5]])/(3*b^(3/2))
```

fricas [A] time = 0.56, size = 148, normalized size = 2.28

$$\left[\frac{a\sqrt{b}\,\log\left(-8\,b^2x^6-8\,abx^3+4\,\sqrt{bx^5+ax^2}\left(2\,bx^3+a\right)\sqrt{b}\,\sqrt{x}-a^2\right)+4\,\sqrt{bx^5+ax^2}\,b\sqrt{x}}{12\,b^2},\frac{a\sqrt{-b}\,\arctan\left(\frac{2\,\sqrt{bx^5+ax^2}\,\sqrt{-b}\,\sqrt{x}}{2\,bx^3+a}\right)+2\,\sqrt{bx^5+ax^2}\,b\sqrt{x}}{6\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(a*sqrt(b)*log(-8*b^2*x^6 - 8*a*b*x^3 + 4*sqrt(b*x^5 + a*x^2)*(2*b*x^
3 + a)*sqrt(b)*sqrt(x) - a^2) + 4*sqrt(b*x^5 + a*x^2)*b*sqrt(x))/b^2, 1/6*(
a*sqrt(-b)*arctan(2*sqrt(b*x^5 + a*x^2)*sqrt(-b)*sqrt(x)/(2*b*x^3 + a)) + 2
*sqrt(b*x^5 + a*x^2)*b*sqrt(x))/b^2]
```

giac [A] time = 0.25, size = 44, normalized size = 0.68

$$\frac{\sqrt{bx^3+a}\,x^{\frac{3}{2}}}{3\,b}+\frac{a\log\left(\left|-\sqrt{b}\,x^{\frac{3}{2}}+\sqrt{bx^3+a}\right|\right)}{3\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(b*x^3 + a)*x^(3/2)/b + 1/3*a*log(abs(-sqrt(b)*x^(3/2) + sqrt(b*x^3
 + a)))/b^(3/2)
```

maple [C] time = 1.14, size = 3347, normalized size = 51.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] 1/3/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/b^3*(6*I*3^(1/2)*(-(I*3^(1/2)-3)/
(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(
1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*
3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(
1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a
*b^2-6*I*3^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(
1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*
3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/
2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1
/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a*b^2-12*I*3^(
1/2)*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x
)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/
(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(
1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*
3^(1/2))/(I*3^(1/2)-3))^(1/2))*x*a*b+12*I*3^(1/2)*(-a*b^2)^(1/3)*(-(I*3^(1/
2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*
b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*
b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))
*b*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3
^(1/2))/(I*3^(1/2)-3))^(1/2))*x*a*b+6*I*3^(1/2)*(-a*b^2)^(2/3)*(-(I*3^(1/2)
-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^
2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*
x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*
x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a
-6*I*3^(1/2)*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1
/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)
^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-
3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-
3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a-6*(-(
I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/
2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a
*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^
(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))
^(1/2))*x^2*a*b^2+6*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x
)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/
(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^
(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3
^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a*b^2+12*(-
a*b^2)^(1/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)
*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1
/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)
/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2)
)/(I*3^(1/2)-3))^(1/2))*x*a*b-12*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-
1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^
2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a
*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Elli
pticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),(I*3^
(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2
)-3))^(1/2))*x*a*b+I*3^(1/2)*((b*x^3+a)*x)^(1/2)*((-b*x+(-a*b^2)^(1/3))*(2*
b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/
```

```
3)-(-a*b^2)^(1/3))/b^2*x)^(1/2)*x*b^2-6*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)/(I*3
^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)
+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1
/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/
2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)
,((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a+6*(-a*b
^2)^(2/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((
2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)
-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(
-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*
(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a-3*x*((b*x^3+a)*x)^(1/2)
*b^2*((-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))
*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/b^2*x)^(1/2))/((b*x^3+a)*
x)^(1/2)/(I*3^(1/2)-3)/((-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/
3)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/b^2*x)^
(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{9}{2}}}{\sqrt{bx^5 + ax^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(9/2)/sqrt(b*x^5 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{9/2}}{\sqrt{b\,x^5 + a\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] int(x^(9/2)/(a*x^2 + b*x^5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{9}{2}}}{\sqrt{x^2 \left(a + bx^3\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(9/2)/sqrt(x**2*(a + b*x**3)), x)
```

3.208 $\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=36

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{b}}$$

Rubi [A] time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2029, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a*x^2 + b*x^5]])/(3*Sqrt[b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx &= \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 1.64

$$\frac{2x\sqrt{a+bx^3}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a+bx^3}}\right)}{3\sqrt{b}\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]])/(3*Sqrt[b]
*Sqrt[x^2*(a + b*x^3)])
```

IntegrateAlgebraic [A] time = 0.22, size = 53, normalized size = 1.47

$$\frac{2\log\left(\sqrt{ax^2+bx^5}+\sqrt{b}\,x^{5/2}\right)}{3\sqrt{b}} - \frac{4\log\left(\sqrt{x}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-4*Log[Sqrt[x]])/(3*Sqrt[b]) + (2*Log[Sqrt[b]*x^(5/2) + Sqrt[a*x^2 + b*x^5
]])/(3*Sqrt[b])
```

fricas [A] time = 0.56, size = 101, normalized size = 2.81

$$\left[\frac{\log\left(-8\,b^{2}x^{6}-8\,abx^{3}-4\,\sqrt{bx^{5}+ax^{2}}\left(2\,bx^{3}+a\right)\sqrt{b}\,\sqrt{x}-a^{2}\right)}{6\,\sqrt{b}},-\frac{\sqrt{-b}\,\arctan\left(\frac{2\,\sqrt{bx^{5}+ax^{2}}\,\sqrt{-b}\,\sqrt{x}}{2\,bx^{3}+a}\right)}{3\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log(-8*b^2*x^6 - 8*a*b*x^3 - 4*sqrt(b*x^5 + a*x^2)*(2*b*x^3 + a)*sqrt(
b)*sqrt(x) - a^2)/sqrt(b), -1/3*sqrt(-b)*arctan(2*sqrt(b*x^5 + a*x^2)*sqrt(
-b)*sqrt(x)/(2*b*x^3 + a))/b]
```

giac [A] time = 0.20, size = 41, normalized size = 1.14

$$-\frac{2\,\arctan\left(\frac{\sqrt{b+\frac{a}{x^{3}}}}{\sqrt{-b}}\right)}{3\,\sqrt{-b}}+\frac{2\,\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)}{3\,\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*arctan(sqrt(b + a/x^3)/sqrt(-b))/sqrt(-b) + 2/3*arctan(sqrt(b)/sqrt(-b
))/sqrt(-b)
```

maple [C] time = 1.24, size = 480, normalized size = 13.33

$$-\frac{4\left(b\,x^{3}+a\right)\left(i\sqrt{3}-1\right)\sqrt{-\frac{\left(i\sqrt{3}-3\right)bx}{\left(i\sqrt{3}-1\right)\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}}\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)^{2}\sqrt{\frac{2bx+i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}+\left(-a\,b^{2}\right)^{\frac{1}{3}}}{\left(1+i\sqrt{3}\right)\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}}\sqrt{\frac{-2bx+i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}-\left(-a\,b^{2}\right)^{\frac{1}{3}}}{\left(i\sqrt{3}-1\right)\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}}\left(\mathrm{EllipticF}\left(\sqrt{-\frac{\left(i\sqrt{3}-3\right)bx}{\left(i\sqrt{3}-1\right)\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}},\sqrt{\frac{\left(i\sqrt{3}+3\right)\left(i\sqrt{3}-1\right)}{\left(1+i\sqrt{3}\right)\left(i\sqrt{3}-3\right)}}\right)-\mathrm{EllipticPi}\left(\sqrt{-\frac{\left(i\sqrt{3}-3\right)bx}{\left(i\sqrt{3}-1\right)\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}},\frac{i\sqrt{3}-1}{i\sqrt{3}-3},\sqrt{\frac{\left(i\sqrt{3}+3\right)\left(i\sqrt{3}-1\right)}{\left(1+i\sqrt{3}\right)\left(i\sqrt{3}-3\right)}}\right)\right)x^{\frac{3}{2}}}{\sqrt{b\,x^{5}+a\,x^{2}}\,\sqrt{\left(b\,x^{3}+a\right)x}\,\left(i\sqrt{3}-3\right)\sqrt{\frac{\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\left(2bx+i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\left(-2bx+i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}-\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)x}{b^{2}}}\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -4*x^(3/2)*(b*x^3+a)*(I*3^(1/2)-1)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*
b^2)^(1/3))*b*x)^(1/2)*(-b*x+(-a*b^2)^(1/3))^2*((2*b*x+I*3^(1/2)*(-a*b^2)^(
1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*
3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*(EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^
(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))-Elli
pticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),(I*3^
(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2
)-3))^(1/2)))/(b*x^5+a*x^2)^(1/2)/b^2/((b*x^3+a)*x)^(1/2)/(I*3^(1/2)-3)/((-
b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x
+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/b^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{3}{2}}}{\sqrt{bx^{5}+ax^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/sqrt(b*x^5 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^{3/2}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] int(x^(3/2)/(a*x^2 + b*x^5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(3/2)/sqrt(x**2*(a + b*x**3)), x)
```

3.209 $\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=27

$$-\frac{2\sqrt{ax^2+bx^5}}{3ax^{5/2}}$$

Rubi [A] time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2014}

$$-\frac{2\sqrt{ax^2+bx^5}}{3ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^5])/(3*a*x^(5/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{ax^2+bx^5}}{3ax^{5/2}}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$-\frac{2\sqrt{x^2\left(a+bx^3\right)}}{3ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[x^2*(a + b*x^3)])/(3*a*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.24, size = 27, normalized size = 1.00

$$-\frac{2\sqrt{ax^2+bx^5}}{3ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^5])/(3*a*x^(5/2))
```

fricas [A] time = 0.41, size = 21, normalized size = 0.78

$$-\frac{2\sqrt{bx^5+ax^2}}{3\,ax^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] -2/3*sqrt(b*x^5 + a*x^2)/(a*x^(5/2))
```

giac [A] time = 0.20, size = 23, normalized size = 0.85

$$-\frac{2\sqrt{b+\frac{a}{x^3}}}{3\,a}+\frac{2\,\sqrt{b}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] -2/3*sqrt(b + a/x^3)/a + 2/3*sqrt(b)/a
```

maple [A] time = 0.04, size = 29, normalized size = 1.07

$$-\frac{2\left(b\,x^3+a\right)}{3\sqrt{b\,x^5+a\,x^2}\;a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x)
[Out] -2/3*(b*x^3+a)/x^(1/2)/a/(b*x^5+a*x^2)^(1/2)
```

maxima [A] time = 1.42, size = 26, normalized size = 0.96

$$-\frac{2\left(bx^4+ax\right)}{3\,\sqrt{bx^3+a}\,ax^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] -2/3*(b*x^4 + a*x)/(sqrt(b*x^3 + a)*a*x^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^{3/2}\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a*x^2 + b*x^5)^(1/2)),x)
[Out] int(1/(x^(3/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(1/(x**(3/2)*sqrt(x**2*(a + b*x**3))), x)
```

3.210 $\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=56

$$\frac{4b\sqrt{ax^2+bx^5}}{9a^2x^{5/2}}-\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}$$

Rubi [A] time = 0.08, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$\frac{4b\sqrt{ax^2+bx^5}}{9a^2x^{5/2}}-\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(9/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^5])/(9*a*x^(11/2)) + (4*b*Sqrt[a*x^2 + b*x^5])/(9*a^2*
x^(5/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx&=-\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}-\frac{(2b)\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx}{3a}\\&=-\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}+\frac{4b\sqrt{ax^2+bx^5}}{9a^2x^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.62

$$-\frac{2\left(a-2bx^3\right)\sqrt{x^2\left(a+bx^3\right)}}{9a^2x^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(9/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*(a - 2*b*x^3)*Sqrt[x^2*(a + b*x^3)])/(9*a^2*x^(11/2))
```

IntegrateAlgebraic [A] time = 0.29, size = 37, normalized size = 0.66

$$\frac{2\left(2bx^3-a\right)\sqrt{ax^2+bx^5}}{9a^2x^{11/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(9/2)*Sqrt[a*x^2 + b*x^5]),x]
[Out] (2*(-a + 2*b*x^3)*Sqrt[a*x^2 + b*x^5])/(9*a^2*x^(11/2))
```

fricas [A] time = 0.40, size = 31, normalized size = 0.55

$$\frac{2\sqrt{bx^5+ax^2}\left(2\,bx^3-a\right)}{9\,a^2x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/9*sqrt(b*x^5 + a*x^2)*(2*b*x^3 - a)/(a^2*x^(11/2))
```

giac [A] time = 0.23, size = 38, normalized size = 0.68

$$-\frac{2\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}}{9\,a^2}+\frac{2\sqrt{b+\frac{a}{x^3}}\,b}{3\,a^2}-\frac{4\,b^{\frac{3}{2}}}{9\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] -2/9*(b + a/x^3)^(3/2)/a^2 + 2/3*sqrt(b + a/x^3)*b/a^2 - 4/9*b^(3/2)/a^2
```

maple [A] time = 0.04, size = 37, normalized size = 0.66

$$-\frac{2\left(b\,x^3+a\right)\left(-2b\,x^3+a\right)}{9\sqrt{b\,x^5+a\,x^2}\;a^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x)
[Out] -2/9*(b*x^3+a)*(-2*b*x^3+a)/x^(7/2)/a^2/(b*x^5+a*x^2)^(1/2)
```

maxima [A] time = 1.42, size = 38, normalized size = 0.68

$$\frac{2\left(2\,b^2x^7+abx^4-a^2x\right)}{9\,\sqrt{bx^3+a}\,a^2x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/9*(2*b^2*x^7 + a*b*x^4 - a^2*x)/(sqrt(b*x^3 + a)*a^2*x^(11/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^{9/2}\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(9/2)*(a*x^2 + b*x^5)^(1/2)),x)
[Out] int(1/(x^(9/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{9}{2}}\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(9/2)/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(1/(x**(9/2)*sqrt(x**2*(a + b*x**3))), x)
```

3.211 $\int \frac{x}{ax^3+bx^4} dx$

Optimal. Leaf size=28

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 44}

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^3 + b*x^4),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{ax^3+bx^4}\,dx &= \int \frac{1}{x^2(a+bx)}\,dx\\ &= \int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx\\ &= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^3 + b*x^4),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{ax^3+bx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a*x^3 + b*x^4),x]
[Out] IntegrateAlgebraic[x/(a*x^3 + b*x^4), x]
```

fricas [A] time = 0.38, size = 26, normalized size = 0.93

$$\frac{bx\log(bx+a) - bx\log(x) - a}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x^3),x, algorithm="fricas")
[Out] (b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

giac [A] time = 0.15, size = 30, normalized size = 1.07

$$\frac{b\log(|bx+a|)}{a^2} - \frac{b\log(|x|)}{a^2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x^3),x, algorithm="giac")
[Out] b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

maple [A] time = 0.05, size = 29, normalized size = 1.04

$$-\frac{b\ln(x)}{a^2} + \frac{b\ln(bx+a)}{a^2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a*x^3),x)
[Out] -1/a^2*b*ln(x)+1/a^2*b*ln(b*x+a)-1/a/x
```

maxima [A] time = 1.36, size = 28, normalized size = 1.00

$$\frac{b\log(bx+a)}{a^2} - \frac{b\log(x)}{a^2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x^3),x, algorithm="maxima")
[Out] b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

mupad [B] time = 5.21, size = 25, normalized size = 0.89

$$\frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2} - \frac{1}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^3 + b*x^4),x)
[Out] (2*b*atanh((2*b*x)/a + 1))/a^2 - 1/(a*x)
```

sympy [A] time = 0.21, size = 19, normalized size = 0.68

$$-\frac{1}{ax} + \frac{b\left(-\log(x) + \log\left(\frac{a}{b}+x\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a*x**3),x)
[Out] -1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

3.212 $\int \frac{1}{ax^3+bx^4}\,dx$

Optimal. Leaf size=42

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 44}

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^4)^(-1),x]
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax^3+bx^4}\,dx &= \int \frac{1}{x^3(a+bx)}\,dx\\ &= \int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx\\ &= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 42, normalized size = 1.00

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^4)^(-1),x]
[Out] -1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ax^3+bx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x^3 + b*x^4)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a*x^3 + b*x^4)^(-1), x]
```

fricas [A] time = 0.39, size = 41, normalized size = 0.98

$$-\frac{2\,b^{2}x^{2}\log\left(bx+a\right)-2\,b^{2}x^{2}\log\left(x\right)-2\,abx+a^{2}}{2\,a^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x^3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

giac [A] time = 0.14, size = 45, normalized size = 1.07

$$-\frac{b^{2}\log\left(|bx+a|\right)}{a^{3}}+\frac{b^{2}\log\left(|x|\right)}{a^{3}}+\frac{2\,abx-a^{2}}{2\,a^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x^3),x, algorithm="giac")
```

```
[Out] -b^2*log(abs(b*x + a))/a^3 + b^2*log(abs(x))/a^3 + 1/2*(2*a*b*x - a^2)/(a^3
*x^2)
```

maple [A] time = 0.05, size = 41, normalized size = 0.98

$$\frac{b^{2}\ln\left(x\right)}{a^{3}}-\frac{b^{2}\ln\left(bx+a\right)}{a^{3}}+\frac{b}{a^{2}x}-\frac{1}{2a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a*x^3),x)
```

```
[Out] 1/a^3*b^2*ln(x)-1/a^3*b^2*ln(b*x+a)+1/a^2*b/x-1/2/a/x^2
```

maxima [A] time = 1.36, size = 40, normalized size = 0.95

$$-\frac{b^{2}\log\left(bx+a\right)}{a^{3}}+\frac{b^{2}\log\left(x\right)}{a^{3}}+\frac{2\,bx-a}{2\,a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x^3),x, algorithm="maxima")
```

```
[Out] -b^2*log(b*x + a)/a^3 + b^2*log(x)/a^3 + 1/2*(2*b*x - a)/(a^2*x^2)
```

mupad [B] time = 5.72, size = 38, normalized size = 0.90

$$-\frac{\frac{a^{2}}{2}-a\,b\,x}{a^{3}\,x^{2}}-\frac{2\,b^{2}\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^3 + b*x^4),x)
```

```
[Out] - (a^2/2 - a*b*x)/(a^3*x^2) - (2*b^2*atanh((2*b*x)/a + 1))/a^3
```

sympy [A] time = 0.22, size = 31, normalized size = 0.74

$$\frac{-a+2bx}{2a^{2}x^{2}}+\frac{b^{2}\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a*x**3),x)
```

```
[Out] (-a + 2*b*x)/(2*a**2*x**2) + b**2*(log(x) - log(a/b + x))/a**3
```

3.213 $\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx$

Optimal. Leaf size=112

$$-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{8b^{7/2}}+\frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x}-\frac{5a\sqrt{ax^3+bx^4}}{12b^2}+\frac{x\sqrt{ax^3+bx^4}}{3b}$$

Rubi [A] time = 0.18, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2024, 2029, 206}

$$\frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{8b^{7/2}}-\frac{5a\sqrt{ax^3+bx^4}}{12b^2}+\frac{x\sqrt{ax^3+bx^4}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (-5*a*Sqrt[a*x^3 + b*x^4])/(12*b^2) + (5*a^2*Sqrt[a*x^3 + b*x^4])/(8*b^3*x)
 + (x*Sqrt[a*x^3 + b*x^4])/(3*b) - (5*a^3*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3
+ b*x^4]])/(8*b^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx = \frac{x\sqrt{ax^3+bx^4}}{3b} - \frac{(5a)\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx}{6b}$$

$$= -\frac{5a\sqrt{ax^3+bx^4}}{12b^2} + \frac{x\sqrt{ax^3+bx^4}}{3b} + \frac{\left(5a^2\right)\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx}{8b^2}$$

$$= -\frac{5a\sqrt{ax^3+bx^4}}{12b^2} + \frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x} + \frac{x\sqrt{ax^3+bx^4}}{3b} - \frac{\left(5a^3\right)\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx}{16b^3}$$

$$= -\frac{5a\sqrt{ax^3+bx^4}}{12b^2} + \frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x} + \frac{x\sqrt{ax^3+bx^4}}{3b} - \frac{\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{ax^3+bx}}\right.}{8b^3}$$

$$= -\frac{5a\sqrt{ax^3+bx^4}}{12b^2} + \frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x} + \frac{x\sqrt{ax^3+bx^4}}{3b} - \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{8b^{7/2}}$$

Mathematica [A] time = 0.20, size = 94, normalized size = 0.84

$$\frac{\sqrt{x^3(a+bx)}\left(\sqrt{b}\sqrt{x}\left(15a^2-10abx+8b^2x^2\right)-\frac{15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{24b^{7/2}x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (Sqrt[x^3*(a + b*x)]*(Sqrt[b]*Sqrt[x]*(15*a^2 - 10*a*b*x + 8*b^2*x^2) - (15
*a^(5/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a]))/(24*b^(7/2
)*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.33, size = 109, normalized size = 0.97

$$-\frac{5a^3\log(x)}{16b^{7/2}} + \frac{5a^3\log\left(-2b^{7/2}\sqrt{ax^3+bx^4}+ab^3x+2b^4x^2\right)}{16b^{7/2}} + \frac{\left(15a^2-10abx+8b^2x^2\right)\sqrt{ax^3+bx^4}}{24b^3x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] ((15*a^2 - 10*a*b*x + 8*b^2*x^2)*Sqrt[a*x^3 + b*x^4])/(24*b^3*x) - (5*a^3*L
og[x])/(16*b^(7/2)) + (5*a^3*Log[a*b^3*x + 2*b^4*x^2 - 2*b^(7/2)*Sqrt[a*x^3
 + b*x^4]])/(16*b^(7/2))
```

fricas [A] time = 0.42, size = 171, normalized size = 1.53

$$\left[\frac{15\,a^3\sqrt{b}\,x\log\left(\frac{2\,bx^2+ax-2\,\sqrt{bx^4+ax^3}\,\sqrt{b}}{x}\right)+2\left(8\,b^3x^2-10\,ab^2x+15\,a^2b\right)\sqrt{bx^4+ax^3}}{48\,b^4x}, \frac{15\,a^3\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx^4+ax^3}\,\sqrt{-b}}{bx^2}\right)+\left(8\,b^3x^2-10\,ab^2x+15\,a^2b\right)\sqrt{bx^4+ax^3}}{24\,b^4x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^4 + a*x^3)*sqrt(b))
/x) + 2*(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^4 + a*x^3))/(b^4*x), 1
/24*(15*a^3*sqrt(-b)*x*arctan(sqrt(b*x^4 + a*x^3)*sqrt(-b)/(b*x^2)) + (8*b^
3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^4 + a*x^3))/(b^4*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{bx^4 + ax^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/sqrt(b*x^4 + a*x^3), x)
```

maple [A] time = 0.05, size = 120, normalized size = 1.07

$$\frac{\sqrt{(bx+a)\,x}\,\left(16\sqrt{b\,x^2+ax}\;b^{\frac{7}{2}}x^2-15a^3b\ln\left(\frac{2bx+a+2\sqrt{b\,x^2+ax}\;\sqrt{b}}{2\sqrt{b}}\right)-20\sqrt{b\,x^2+ax}\;a\,b^{\frac{5}{2}}x+30\sqrt{b\,x^2+ax}\;a^2b^{\frac{3}{2}}\right)x}{48\sqrt{b\,x^4+a\,x^3}\;b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^4+a*x^3)^(1/2),x)
[Out] 1/48*x*((b*x+a)*x)^(1/2)*(16*x^2*(b*x^2+a*x)^(1/2)*b^(7/2)-20*(b*x^2+a*x)^(
1/2)*b^(5/2)*x*a+30*(b*x^2+a*x)^(1/2)*b^(3/2)*a^2-15*ln(1/2*(2*b*x+a+2*(b*x
^2+a*x)^(1/2)*b^(1/2))/b^(1/2))*a^3*b)/(b*x^4+a*x^3)^(1/2)/b^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{bx^4 + ax^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/sqrt(b*x^4 + a*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{b\,x^4 + a\,x^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^3 + b*x^4)^(1/2),x)
[Out] int(x^4/(a*x^3 + b*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x^3\,(a + bx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**4+a*x**3)**(1/2),x)
[Out] Integral(x**4/sqrt(x**3*(a + b*x)), x)
```

3.214 $\int \frac{x^3}{\sqrt{ax^3+bx^4}} \, dx$

Optimal. Leaf size=86

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{4b^{5/2}}-\frac{3a\sqrt{ax^3+bx^4}}{4b^2x}+\frac{\sqrt{ax^3+bx^4}}{2b}$$

Rubi [A] time = 0.13, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2024, 2029, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{4b^{5/2}}-\frac{3a\sqrt{ax^3+bx^4}}{4b^2x}+\frac{\sqrt{ax^3+bx^4}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] Sqrt[a*x^3 + b*x^4]/(2*b) - (3*a*Sqrt[a*x^3 + b*x^4])/(4*b^2*x) + (3*a^2*Ar
cTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/(4*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{ax^3+bx^4}} \, dx &= \frac{\sqrt{ax^3+bx^4}}{2b}-\frac{(3a)\int \frac{x^2}{\sqrt{ax^3+bx^4}} \, dx}{4b}\\ &= \frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\sqrt{ax^3+bx^4}}{4b^2x}+\frac{\left(3a^2\right)\int \frac{x}{\sqrt{ax^3+bx^4}} \, dx}{8b^2}\\ &= \frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\sqrt{ax^3+bx^4}}{4b^2x}+\frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2} \, dx,x,\frac{x^2}{\sqrt{ax^3+bx^4}}\right)}{4b^2}\\ &= \frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\sqrt{ax^3+bx^4}}{4b^2x}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{4b^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 92, normalized size = 1.07

$$\frac{3a^{5/2}x^{3/2}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+\sqrt{b}x^2\left(-3a^2-abx+2b^2x^2\right)}{4b^{5/2}\sqrt{x^3(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (Sqrt[b]*x^2*(-3*a^2 - a*b*x + 2*b^2*x^2) + 3*a^(5/2)*x^(3/2)*Sqrt[1 + (b*x
)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(5/2)*Sqrt[x^3*(a + b*x)])
```

IntegrateAlgebraic [A] time = 0.30, size = 98, normalized size = 1.14

$$\frac{3a^2\log(x)}{8b^{5/2}}-\frac{3a^2\log\left(-2b^{5/2}\sqrt{ax^3+bx^4}+ab^2x+2b^3x^2\right)}{8b^{5/2}}+\frac{(2bx-3a)\sqrt{ax^3+bx^4}}{4b^2x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] ((-3*a + 2*b*x)*Sqrt[a*x^3 + b*x^4])/(4*b^2*x) + (3*a^2*Log[x])/(8*b^(5/2))
 - (3*a^2*Log[a*b^2*x + 2*b^3*x^2 - 2*b^(5/2)*Sqrt[a*x^3 + b*x^4]])/(8*b^(5
/2))
```

fricas [A] time = 0.40, size = 150, normalized size = 1.74

$$\left[\frac{3a^2\sqrt{b}x\log\left(\frac{2bx^2+ax+2\sqrt{bx^4+ax^3}\sqrt{b}}{x}\right)+2\sqrt{bx^4+ax^3}\left(2b^2x-3ab\right)}{8b^3x}, -\frac{3a^2\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^4+ax^3}\sqrt{-b}}{bx^2}\right)-\sqrt{bx^4+ax^3}\left(2b^2x-3ab\right)}{4b^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*x*log((2*b*x^2 + a*x + 2*sqrt(b*x^4 + a*x^3)*sqrt(b))/x
) + 2*sqrt(b*x^4 + a*x^3)*(2*b^2*x - 3*a*b))/(b^3*x), -1/4*(3*a^2*sqrt(-b)*
x*arctan(sqrt(b*x^4 + a*x^3)*sqrt(-b)/(b*x^2)) - sqrt(b*x^4 + a*x^3)*(2*b^2
*x - 3*a*b))/(b^3*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{bx^4+ax^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(b*x^4 + a*x^3), x)
```

maple [A] time = 0.05, size = 98, normalized size = 1.14

$$\frac{\sqrt{(bx+a)x}\left(3a^2b\ln\left(\frac{2bx+a+2\sqrt{bx^2+ax}\sqrt{b}}{2\sqrt{b}}\right)+4\sqrt{bx^2+ax}\,b^{\frac{5}{2}}x-6\sqrt{bx^2+ax}\,a\,b^{\frac{3}{2}}\right)x}{8\sqrt{bx^4+ax^3}\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a*x^3)^(1/2),x)
```

```
[Out] 1/8*x*((b*x+a)*x)^(1/2)*(4*x*(b*x^2+a*x)^(1/2)*b^(5/2)-6*(b*x^2+a*x)^(1/2)*
b^(3/2)*a+3*ln(1/2*(2*b*x+a+2*(b*x^2+a*x)^(1/2)*b^(1/2))/b^(1/2))*a^2*b)/(b
*x^4+a*x^3)^(1/2)/b^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{bx^4 + ax^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(b*x^4 + a*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{b\,x^4 + a\,x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^3 + b*x^4)^(1/2),x)
```

```
[Out] int(x^3/(a*x^3 + b*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^3\,(a + bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**3*(a + b*x)), x)
```

3.215 $\int \frac{x^2}{\sqrt{ax^3+bx^4}} \, dx$

Optimal. Leaf size=56

$$\frac{\sqrt{ax^3+bx^4}}{bx} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{b^{3/2}}$$

Rubi [A] time = 0.08, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2024, 2029, 206}

$$\frac{\sqrt{ax^3+bx^4}}{bx} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] Sqrt[a*x^3 + b*x^4]/(b*x) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/
b^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{ax^3+bx^4}} \, dx &= \frac{\sqrt{ax^3+bx^4}}{bx} - \frac{a\int \frac{x}{\sqrt{ax^3+bx^4}} \, dx}{2b} \\
&= \frac{\sqrt{ax^3+bx^4}}{bx} - \frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2} \, dx, x, \frac{x^2}{\sqrt{ax^3+bx^4}}\right)}{b} \\
&= \frac{\sqrt{ax^3+bx^4}}{bx} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 75, normalized size = 1.34

$$\frac{\sqrt{b}\,x^2(a+bx)-a^{3/2}x^{3/2}\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}\sqrt{x^3(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (Sqrt[b]*x^2*(a + b*x) - a^(3/2)*x^(3/2)*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]
*Sqrt[x])/Sqrt[a]])/(b^(3/2)*Sqrt[x^3*(a + b*x)])
```

IntegrateAlgebraic [A] time = 0.25, size = 81, normalized size = 1.45

$$-\frac{a\log(x)}{2b^{3/2}}+\frac{a\log\left(-2b^{3/2}\sqrt{ax^3+bx^4}+abx+2b^2x^2\right)}{2b^{3/2}}+\frac{\sqrt{ax^3+bx^4}}{bx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] Sqrt[a*x^3 + b*x^4]/(b*x) - (a*Log[x])/(2*b^(3/2)) + (a*Log[a*b*x + 2*b^2*x
^2 - 2*b^(3/2)*Sqrt[a*x^3 + b*x^4]])/(2*b^(3/2))
```

fricas [A] time = 0.41, size = 122, normalized size = 2.18

$$\left[\frac{a\sqrt{b}\,x\log\left(\frac{2\,bx^2+ax-2\,\sqrt{bx^4+ax^3}\,\sqrt{b}}{x}\right)+2\,\sqrt{bx^4+ax^3}\,b}{2\,b^2x},\ \frac{a\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx^4+ax^3}\,\sqrt{-b}}{bx^2}\right)+\sqrt{bx^4+ax^3}\,b}{b^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^4 + a*x^3)*sqrt(b))/x) +
2*sqrt(b*x^4 + a*x^3)*b)/(b^2*x), (a*sqrt(-b)*x*arctan(sqrt(b*x^4 + a*x^3)*
sqrt(-b)/(b*x^2)) + sqrt(b*x^4 + a*x^3)*b)/(b^2*x)]
```

giac [A] time = 0.23, size = 48, normalized size = 0.86

$$\frac{\frac{a\sqrt{b+\frac{a}{x}}\,x}{b}+\frac{a^2\arctan\left(\frac{\sqrt{b+\frac{a}{x}}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] (a*sqrt(b + a/x)*x/b + a^2*arctan(sqrt(b + a/x)/sqrt(-b))/(sqrt(-b)*b))/a
```

maple [A] time = 0.05, size = 78, normalized size = 1.39

$$\frac{\sqrt{(bx+a)\,x}\,\left(-ab\ln\left(\frac{2bx+a+2\sqrt{b\,x^2+ax}\,\sqrt{b}}{2\sqrt{b}}\right)+2\sqrt{b\,x^2+ax}\,b^{\frac{3}{2}}\right)x}{2\sqrt{b\,x^4+a\,x^3}\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^4+a*x^3)^(1/2),x)
```

```
[Out] 1/2*x*((b*x+a)*x)^(1/2)*(2*(b*x^2+a*x)^(1/2)*b^(3/2)-a*ln(1/2*(2*b*x+a+2*(b
*x^2+a*x)^(1/2)*b^(1/2))/b^(1/2))*b)/(b*x^4+a*x^3)^(1/2)/b^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{bx^4 + ax^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(b*x^4 + a*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{b\,x^4 + a\,x^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^3 + b*x^4)^(1/2),x)
```

```
[Out] int(x^2/(a*x^3 + b*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^3\,(a + bx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x**3*(a + b*x)), x)
```

3.216 $\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx$

Optimal. Leaf size=32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2029, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{ax^3+bx^4}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 1.84

$$\frac{2\sqrt{a}\,x^{3/2}\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt{x^3(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (2*Sqrt[a]*x^(3/2)*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[x^3*(a + b*x)])
```

IntegrateAlgebraic [A] time = 0.18, size = 49, normalized size = 1.53

$$\frac{\log(x)}{\sqrt{b}} - \frac{\log\left(-2\sqrt{b}\,\sqrt{ax^3+bx^4}+ax+2bx^2\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] Log[x]/Sqrt[b] - Log[a*x + 2*b*x^2 - 2*Sqrt[b]*Sqrt[a*x^3 + b*x^4]]/Sqrt[b]
```

fricas [A] time = 0.41, size = 74, normalized size = 2.31

$$\left[\frac{\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^4+ax^3}\,\sqrt{b}}{x}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^4+ax^3}\,\sqrt{-b}}{bx^2}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((2*b*x^2 + a*x + 2*sqrt(b*x^4 + a*x^3)*sqrt(b))/x)/sqrt(b), -2*sqrt(-b
)*arctan(sqrt(b*x^4 + a*x^3)*sqrt(-b)/(b*x^2))/b]
```

giac [A] time = 0.28, size = 23, normalized size = 0.72

$$-\frac{2\,\arctan\left(\frac{\sqrt{b+\frac{a}{x}}}{\sqrt{-b}}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(b + a/x)/sqrt(-b))/sqrt(-b)
```

maple [B] time = 0.05, size = 56, normalized size = 1.75

$$\frac{\sqrt{(bx+a)\,x}\;x\ln\left(\frac{2bx+a+2\,\sqrt{b\,x^2+ax}\,\sqrt{b}}{2\,\sqrt{b}}\right)}{\sqrt{b\,x^4+a\,x^3}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a*x^3)^(1/2),x)
```

```
[Out] 1/(b*x^4+a*x^3)^(1/2)*x*((b*x+a)*x)^(1/2)*ln(1/2*(2*b*x+a+2*(b*x^2+a*x)^(1/
2)*b^(1/2))/b^(1/2))/b^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{bx^4+ax^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*x^4 + a*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\sqrt{b\,x^4+a\,x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^3 + b*x^4)^(1/2),x)
```

```
[Out] int(x/(a*x^3 + b*x^4)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^3 (a + bx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**3*(a + b*x)), x)
```

3.217 $\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx$

Optimal. Leaf size=23

$$-\frac{2\sqrt{ax^3+bx^4}}{ax^2}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2000}

$$-\frac{2\sqrt{ax^3+bx^4}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(a*x^2)
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx = -\frac{2\sqrt{ax^3+bx^4}}{ax^2}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.91

$$-\frac{2\sqrt{x^3(a+bx)}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (-2*Sqrt[x^3*(a + b*x)])/(a*x^2)
```

IntegrateAlgebraic [A] time = 0.14, size = 23, normalized size = 1.00

$$-\frac{2\sqrt{ax^3+bx^4}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(a*x^2)
```

fricas [A] time = 0.39, size = 21, normalized size = 0.91

$$-\frac{2\sqrt{bx^4+ax^3}}{ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(b*x^4 + a*x^3)/(a*x^2)
```

giac [A] time = 0.21, size = 27, normalized size = 1.17

$$\frac{2}{\left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
[Out] 2/((sqrt(b)*x - sqrt(b*x^2 + a*x))*sgn(x))
```

maple [A] time = 0.05, size = 25, normalized size = 1.09

$$-\frac{2\,(bx+a)\,x}{\sqrt{b\,x^4+a\,x^3}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a*x^3)^(1/2),x)
[Out] -2*(b*x+a)*x/a/(b*x^4+a*x^3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4+ax^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*x^4 + a*x^3), x)
```

mupad [B] time = 5.14, size = 21, normalized size = 0.91

$$-\frac{2\sqrt{b\,x^4+a\,x^3}}{a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^3 + b*x^4)^(1/2),x)
[Out] -(2*(a*x^3 + b*x^4)^(1/2))/(a*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a*x**3)**(1/2),x)
[Out] Integral(1/sqrt(a*x**3 + b*x**4), x)
```

3.218 $\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx$

Optimal. Leaf size=52

$$\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2}-\frac{2\sqrt{ax^3+bx^4}}{3ax^3}$$

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2000}

$$\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2}-\frac{2\sqrt{ax^3+bx^4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(3*a*x^3) + (4*b*Sqrt[a*x^3 + b*x^4])/(3*a^2*x^2)
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx &= -\frac{2\sqrt{ax^3+bx^4}}{3ax^3}-\frac{(2b)\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx}{3a}\\ &= -\frac{2\sqrt{ax^3+bx^4}}{3ax^3}+\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.56

$$-\frac{2(a-2bx)\sqrt{x^3(a+bx)}}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*(a - 2*b*x)*Sqrt[x^3*(a + b*x)])/(3*a^2*x^3)
```

IntegrateAlgebraic [A] time = 0.17, size = 33, normalized size = 0.63

$$\frac{2(2bx-a)\sqrt{ax^3+bx^4}}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a*x^3 + b*x^4]),x]
[Out] (2*(-a + 2*b*x)*Sqrt[a*x^3 + b*x^4])/(3*a^2*x^3)
```

fricas [A] time = 0.39, size = 29, normalized size = 0.56

$$\frac{2\sqrt{bx^4+ax^3}\,(2\,bx-a)}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(b*x^4 + a*x^3)*(2*b*x - a)/(a^2*x^3)
```

giac [A] time = 0.23, size = 27, normalized size = 0.52

$$-\frac{2\left(\left(b+\frac{a}{x}\right)^{\frac{3}{2}}-3\,\sqrt{b+\frac{a}{x}}\,b\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
[Out] -2/3*((b + a/x)^(3/2) - 3*sqrt(b + a/x)*b)/a^2
```

maple [A] time = 0.05, size = 30, normalized size = 0.58

$$-\frac{2\,(bx+a)\,(-2bx+a)}{3\sqrt{b\,x^4+a\,x^3}\;a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^4+a*x^3)^(1/2),x)
[Out] -2/3*(b*x+a)*(-2*b*x+a)/a^2/(b*x^4+a*x^3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4+ax^3}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^4 + a*x^3)*x), x)
```

mupad [B] time = 5.06, size = 42, normalized size = 0.81

$$-\frac{2\,a\,\sqrt{b\,x^4+a\,x^3}-4\,b\,x\,\sqrt{b\,x^4+a\,x^3}}{3\,a^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^3 + b*x^4)^(1/2)),x)
[Out] -(2*a*(a*x^3 + b*x^4)^(1/2) - 4*b*x*(a*x^3 + b*x^4)^(1/2))/(3*a^2*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x^3\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**3*(a + b*x))), x)
```

3.219 $\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx$

Optimal. Leaf size=80

$$-\frac{16b^2\sqrt{ax^3+bx^4}}{15a^3x^2}+\frac{8b\sqrt{ax^3+bx^4}}{15a^2x^3}-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}$$

Rubi [A] time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2000}

$$-\frac{16b^2\sqrt{ax^3+bx^4}}{15a^3x^2}+\frac{8b\sqrt{ax^3+bx^4}}{15a^2x^3}-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(5*a*x^4) + (8*b*Sqrt[a*x^3 + b*x^4])/(15*a^2*x^3)
 - (16*b^2*Sqrt[a*x^3 + b*x^4])/(15*a^3*x^2)
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx &= -\frac{2\sqrt{ax^3+bx^4}}{5ax^4}-\frac{(4b)\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx}{5a}\\
&= -\frac{2\sqrt{ax^3+bx^4}}{5ax^4}+\frac{8b\sqrt{ax^3+bx^4}}{15a^2x^3}+\frac{\left(8b^2\right)\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx}{15a^2}\\
&= -\frac{2\sqrt{ax^3+bx^4}}{5ax^4}+\frac{8b\sqrt{ax^3+bx^4}}{15a^2x^3}-\frac{16b^2\sqrt{ax^3+bx^4}}{15a^3x^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.52

$$-\frac{2\sqrt{x^3(a+bx)}\left(3a^2-4abx+8b^2x^2\right)}{15a^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[x^3*(a + b*x)]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^4)
```

IntegrateAlgebraic [A] time = 0.18, size = 44, normalized size = 0.55

$$-\frac{2\left(3a^2-4abx+8b^2x^2\right)\sqrt{ax^3+bx^4}}{15a^3x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[a*x^3 + b*x^4]),x]
[Out] (-2*(3*a^2 - 4*a*b*x + 8*b^2*x^2)*Sqrt[a*x^3 + b*x^4])/(15*a^3*x^4)
```

fricas [A] time = 0.39, size = 40, normalized size = 0.50

$$-\frac{2\sqrt{bx^4+ax^3}\left(8\,b^2x^2-4\,abx+3\,a^2\right)}{15\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
[Out] -2/15*sqrt(b*x^4 + a*x^3)*(8*b^2*x^2 - 4*a*b*x + 3*a^2)/(a^3*x^4)
```

giac [A] time = 0.24, size = 43, normalized size = 0.54

$$-\frac{2\left(3\left(b+\frac{a}{x}\right)^{\frac{5}{2}}-10\left(b+\frac{a}{x}\right)^{\frac{3}{2}}b+15\sqrt{b+\frac{a}{x}}\,b^2\right)}{15\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
[Out] -2/15*(3*(b + a/x)^(5/2) - 10*(b + a/x)^(3/2)*b + 15*sqrt(b + a/x)*b^2)/a^3
```

maple [A] time = 0.04, size = 46, normalized size = 0.58

$$-\frac{2\left(bx+a\right)\left(8b^2x^2-4abx+3a^2\right)}{15\sqrt{b\,x^4+a\,x^3}\;a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^4+a*x^3)^(1/2),x)
[Out] -2/15*(b*x+a)*(8*b^2*x^2-4*a*b*x+3*a^2)/x/a^3/(b*x^4+a*x^3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4+ax^3}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^4 + a*x^3)*x^2), x)
```

mupad [B] time = 5.14, size = 40, normalized size = 0.50

$$-\frac{2\sqrt{b\,x^4+a\,x^3}\,\left(3\,a^2-4\,a\,b\,x+8\,b^2\,x^2\right)}{15\,a^3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x^3 + b*x^4)^(1/2)),x)
```

```
[Out] -(2*(a*x^3 + b*x^4)^(1/2)*(3*a^2 + 8*b^2*x^2 - 4*a*b*x))/(15*a^3*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{x^3\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**3*(a + b*x))), x)
```

3.220 $\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx$

Optimal. Leaf size=108

$$\frac{32b^3\sqrt{ax^3+bx^4}}{35a^4x^2}-\frac{16b^2\sqrt{ax^3+bx^4}}{35a^3x^3}+\frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4}-\frac{2\sqrt{ax^3+bx^4}}{7ax^5}$$

Rubi [A] time = 0.13, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2000}

$$\frac{32b^3\sqrt{ax^3+bx^4}}{35a^4x^2}-\frac{16b^2\sqrt{ax^3+bx^4}}{35a^3x^3}+\frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4}-\frac{2\sqrt{ax^3+bx^4}}{7ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(7*a*x^5) + (12*b*Sqrt[a*x^3 + b*x^4])/(35*a^2*x^4
) - (16*b^2*Sqrt[a*x^3 + b*x^4])/(35*a^3*x^3) + (32*b^3*Sqrt[a*x^3 + b*x^4]
)/(35*a^4*x^2)
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx &= -\frac{2\sqrt{ax^3+bx^4}}{7ax^5}-\frac{(6b)\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx}{7a}\\ &= -\frac{2\sqrt{ax^3+bx^4}}{7ax^5}+\frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4}+\frac{\left(24b^2\right)\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx}{35a^2}\\ &= -\frac{2\sqrt{ax^3+bx^4}}{7ax^5}+\frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4}-\frac{16b^2\sqrt{ax^3+bx^4}}{35a^3x^3}-\frac{\left(16b^3\right)\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx}{35a^3}\\ &= -\frac{2\sqrt{ax^3+bx^4}}{7ax^5}+\frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4}-\frac{16b^2\sqrt{ax^3+bx^4}}{35a^3x^3}+\frac{32b^3\sqrt{ax^3+bx^4}}{35a^4x^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.49

$$\frac{2\sqrt{x^3(a+bx)}\left(-5a^3+6a^2bx-8ab^2x^2+16b^3x^3\right)}{35a^4x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (2*Sqrt[x^3*(a + b*x)]*(-5*a^3 + 6*a^2*b*x - 8*a*b^2*x^2 + 16*b^3*x^3))/(35
*a^4*x^5)
```

IntegrateAlgebraic [A] time = 0.18, size = 55, normalized size = 0.51

$$\frac{2 \left(-5a^3+6a^2bx-8ab^2x^2+16b^3x^3\right) \sqrt{ax^3+bx^4}}{35a^4x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (2*(-5*a^3 + 6*a^2*b*x - 8*a*b^2*x^2 + 16*b^3*x^3)*Sqrt[a*x^3 + b*x^4])/(35
*a^4*x^5)
```

fricas [A] time = 0.40, size = 51, normalized size = 0.47

$$\frac{2 \left(16 \, b^3 x^3 - 8 \, a b^2 x^2 + 6 \, a^2 b x - 5 \, a^3\right) \sqrt{b x^4 + a x^3}}{35 \, a^4 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x^4 + a*x^3)/(a^
4*x^5)
```

giac [A] time = 0.27, size = 57, normalized size = 0.53

$$-\frac{2 \left(5 \left(b + \frac{a}{x}\right)^{\frac{7}{2}} - 21 \left(b + \frac{a}{x}\right)^{\frac{5}{2}} b + 35 \left(b + \frac{a}{x}\right)^{\frac{3}{2}} b^2 - 35 \sqrt{b + \frac{a}{x}} \, b^3\right)}{35 \, a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] -2/35*(5*(b + a/x)^(7/2) - 21*(b + a/x)^(5/2)*b + 35*(b + a/x)^(3/2)*b^2 -
35*sqrt(b + a/x)*b^3)/a^4
```

maple [A] time = 0.05, size = 57, normalized size = 0.53

$$-\frac{2 \left(b x + a\right) \left(-16 b^3 x^3 + 8 a \, b^2 x^2 - 6 a^2 b x + 5 a^3\right)}{35 \sqrt{b \, x^4 + a \, x^3} \; a^4 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^4+a*x^3)^(1/2),x)
```

```
[Out] -2/35*(b*x+a)*(-16*b^3*x^3+8*a*b^2*x^2-6*a^2*b*x+5*a^3)/x^2/a^4/(b*x^4+a*x^
3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b x^4 + a x^3} \, x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a*x^3)*x^3), x)
```

mupad [B] time = 5.14, size = 92, normalized size = 0.85

$$\frac{12\,b\,\sqrt{b\,x^4+a\,x^3}}{35\,a^2\,x^4}-\frac{2\,\sqrt{b\,x^4+a\,x^3}}{7\,a\,x^5}-\frac{16\,b^2\,\sqrt{b\,x^4+a\,x^3}}{35\,a^3\,x^3}+\frac{32\,b^3\,\sqrt{b\,x^4+a\,x^3}}{35\,a^4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x^3 + b*x^4)^(1/2)),x)
```

```
[Out] (12*b*(a*x^3 + b*x^4)^(1/2))/(35*a^2*x^4) - (2*(a*x^3 + b*x^4)^(1/2))/(7*a*
x^5) - (16*b^2*(a*x^3 + b*x^4)^(1/2))/(35*a^3*x^3) + (32*b^3*(a*x^3 + b*x^4
)^(1/2))/(35*a^4*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{x^3\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x**3*(a + b*x))), x)
```

3.221 $\int \frac{1}{x^4\sqrt{ax^3+bx^4}}\,dx$

Optimal. Leaf size=136

$$-\frac{256b^4\sqrt{ax^3+bx^4}}{315a^5x^2}+\frac{128b^3\sqrt{ax^3+bx^4}}{315a^4x^3}-\frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4}+\frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5}-\frac{2\sqrt{ax^3+bx^4}}{9ax^6}$$

Rubi [A] time = 0.17, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2000}

$$-\frac{256b^4\sqrt{ax^3+bx^4}}{315a^5x^2}+\frac{128b^3\sqrt{ax^3+bx^4}}{315a^4x^3}-\frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4}+\frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5}-\frac{2\sqrt{ax^3+bx^4}}{9ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(9*a*x^6) + (16*b*Sqrt[a*x^3 + b*x^4])/(63*a^2*x^5
) - (32*b^2*Sqrt[a*x^3 + b*x^4])/(105*a^3*x^4) + (128*b^3*Sqrt[a*x^3 + b*x^
4])/(315*a^4*x^3) - (256*b^4*Sqrt[a*x^3 + b*x^4])/(315*a^5*x^2)
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\sqrt{ax^3+bx^4}}\,dx &= -\frac{2\sqrt{ax^3+bx^4}}{9ax^6}-\frac{(8b)\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx}{9a}\\
&= -\frac{2\sqrt{ax^3+bx^4}}{9ax^6}+\frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5}+\frac{\left(16b^2\right)\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx}{21a^2}\\
&= -\frac{2\sqrt{ax^3+bx^4}}{9ax^6}+\frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5}-\frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4}-\frac{\left(64b^3\right)\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx}{105a^3}\\
&= -\frac{2\sqrt{ax^3+bx^4}}{9ax^6}+\frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5}-\frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4}+\frac{128b^3\sqrt{ax^3+bx^4}}{315a^4x^3}+\frac{\left(128b^4\right)}{}\\
&= -\frac{2\sqrt{ax^3+bx^4}}{9ax^6}+\frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5}-\frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4}+\frac{128b^3\sqrt{ax^3+bx^4}}{315a^4x^3}-\frac{256b^4\sqrt{a}}{315}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 64, normalized size = 0.47

$$-\frac{2\sqrt{x^3(a+bx)}\left(35a^4-40a^3bx+48a^2b^2x^2-64ab^3x^3+128b^4x^4\right)}{315a^5x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[x^3*(a + b*x)]*(35*a^4 - 40*a^3*b*x + 48*a^2*b^2*x^2 - 64*a*b^3*x^
3 + 128*b^4*x^4))/(315*a^5*x^6)
```

IntegrateAlgebraic [A] time = 0.21, size = 66, normalized size = 0.49

$$-\frac{2\sqrt{ax^3+bx^4}\,\left(35a^4-40a^3bx+48a^2b^2x^2-64ab^3x^3+128b^4x^4\right)}{315a^5x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^4]*(35*a^4 - 40*a^3*b*x + 48*a^2*b^2*x^2 - 64*a*b^3*x^
3 + 128*b^4*x^4))/(315*a^5*x^6)
```

fricas [A] time = 0.39, size = 62, normalized size = 0.46

$$-\frac{2\left(128\,b^4x^4-64\,ab^3x^3+48\,a^2b^2x^2-40\,a^3bx+35\,a^4\right)\sqrt{bx^4+ax^3}}{315\,a^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(128*b^4*x^4 - 64*a*b^3*x^3 + 48*a^2*b^2*x^2 - 40*a^3*b*x + 35*a^4)*
sqrt(b*x^4 + a*x^3)/(a^5*x^6)
```

giac [A] time = 0.23, size = 71, normalized size = 0.52

$$-\frac{2\left(35\left(b+\frac{a}{x}\right)^{\frac{9}{2}}-180\left(b+\frac{a}{x}\right)^{\frac{7}{2}}b+378\left(b+\frac{a}{x}\right)^{\frac{5}{2}}b^2-420\left(b+\frac{a}{x}\right)^{\frac{3}{2}}b^3+315\sqrt{b+\frac{a}{x}}\,b^4\right)}{315\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] -2/315*(35*(b + a/x)^(9/2) - 180*(b + a/x)^(7/2)*b + 378*(b + a/x)^(5/2)*b^
2 - 420*(b + a/x)^(3/2)*b^3 + 315*sqrt(b + a/x)*b^4)/a^5
```

maple [A] time = 0.05, size = 68, normalized size = 0.50

$$-\frac{2\left(bx+a\right)\left(128b^4x^4-64a\,b^3x^3+48b^2x^2a^2-40bx\,a^3+35a^4\right)}{315\sqrt{b\,x^4+a\,x^3}\;a^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^4+a*x^3)^(1/2),x)
```

```
[Out] -2/315*(b*x+a)*(128*b^4*x^4-64*a*b^3*x^3+48*a^2*b^2*x^2-40*a^3*b*x+35*a^4)/
x^3/a^5/(b*x^4+a*x^3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{bx^4+ax^3}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a*x^3)*x^4), x)
```

mupad [B] time = 5.14, size = 116, normalized size = 0.85

$$\frac{16\,b\,\sqrt{b\,x^4+a\,x^3}}{63\,a^2\,x^5}-\frac{2\,\sqrt{b\,x^4+a\,x^3}}{9\,a\,x^6}-\frac{32\,b^2\,\sqrt{b\,x^4+a\,x^3}}{105\,a^3\,x^4}+\frac{128\,b^3\,\sqrt{b\,x^4+a\,x^3}}{315\,a^4\,x^3}-\frac{256\,b^4\,\sqrt{b\,x^4+a\,x^3}}{315\,a^5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a*x^3 + b*x^4)^(1/2)),x)
```

```
[Out] (16*b*(a*x^3 + b*x^4)^(1/2))/(63*a^2*x^5) - (2*(a*x^3 + b*x^4)^(1/2))/(9*a*
x^6) - (32*b^2*(a*x^3 + b*x^4)^(1/2))/(105*a^3*x^4) + (128*b^3*(a*x^3 + b*x
^4)^(1/2))/(315*a^4*x^3) - (256*b^4*(a*x^3 + b*x^4)^(1/2))/(315*a^5*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{x^3\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(x**3*(a + b*x))), x)
```

3.222 $\int \frac{1}{x^3+bx^5} \, dx$

Optimal. Leaf size=26

$$\frac{1}{2}b\log\left(bx^2+1\right)-b\log(x)-\frac{1}{2x^2}$$

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1593, 266, 44}

$$\frac{1}{2}b\log\left(bx^2+1\right)-b\log(x)-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3 + b*x^5)^(-1),x]
```

```
[Out] -1/(2*x^2) - b*Log[x] + (b*Log[1 + b*x^2])/2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3+bx^5} \, dx &= \int \frac{1}{x^3\left(1+bx^2\right)} \, dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(1+bx)} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{1+bx}\right) \, dx, x, x^2\right)\\
&= -\frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1+bx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{1}{2}b\log\left(bx^2+1\right)-b\log(x)-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3 + b*x^5)^(-1),x]
```

```
[Out] -1/2*1/x^2 - b*Log[x] + (b*Log[1 + b*x^2])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 + bx^5}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3 + b*x^5)^(-1),x]
[Out] IntegrateAlgebraic[(x^3 + b*x^5)^(-1), x]
```

fricas [A] time = 0.40, size = 28, normalized size = 1.08

$$\frac{bx^2 \log\left(bx^2 + 1\right) - 2\,bx^2 \log(x) - 1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+x^3),x, algorithm="fricas")
[Out] 1/2*(b*x^2*log(b*x^2 + 1) - 2*b*x^2*log(x) - 1)/x^2
```

giac [A] time = 0.15, size = 32, normalized size = 1.23

$$-\frac{1}{2}\,b \log\left(x^2\right) + \frac{1}{2}\,b \log\left(\left|bx^2 + 1\right|\right) + \frac{bx^2 - 1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+x^3),x, algorithm="giac")
[Out] -1/2*b*log(x^2) + 1/2*b*log(abs(b*x^2 + 1)) + 1/2*(b*x^2 - 1)/x^2
```

maple [A] time = 0.04, size = 23, normalized size = 0.88

$$-b \ln(x) + \frac{b \ln\left(b\,x^2 + 1\right)}{2} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^5+x^3),x)
[Out] -1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2+1)
```

maxima [A] time = 1.34, size = 22, normalized size = 0.85

$$\frac{1}{2}\,b \log\left(bx^2 + 1\right) - b \log(x) - \frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+x^3),x, algorithm="maxima")
[Out] 1/2*b*log(b*x^2 + 1) - b*log(x) - 1/2/x^2
```

mupad [B] time = 0.05, size = 22, normalized size = 0.85

$$\frac{b\,\ln\left(b\,x^2 + 1\right)}{2} - b\,\ln(x) - \frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^5 + x^3),x)
```

```
[Out] (b*log(b*x^2 + 1))/2 - b*log(x) - 1/(2*x^2)
```

sympy [A] time = 0.22, size = 22, normalized size = 0.85

$$-b\log(x) + \frac{b\log\left(x^2 + \frac{1}{b}\right)}{2} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**5+x**3),x)
```

```
[Out] -b*log(x) + b*log(x**2 + 1/b)/2 - 1/(2*x**2)
```

3.223 $\int \frac{1}{-x^3+bx^5} \, dx$

Optimal. Leaf size=27

$$\frac{1}{2}b\log\left(1-bx^2\right)-b\log(x)+\frac{1}{2x^2}$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 266, 44}

$$\frac{1}{2}b\log\left(1-bx^2\right)-b\log(x)+\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^3 + b*x^5)^(-1),x]
```

```
[Out] 1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-x^3+bx^5} \, dx &= \int \frac{1}{x^3\left(-1+bx^2\right)} \, dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(-1+bx)} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{-1+bx}\right) dx, x, x^2\right)\\
&= \frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1-bx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{1}{2}b\log\left(1-bx^2\right)-b\log(x)+\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^3 + b*x^5)^(-1),x]
```

```
[Out] 1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{-x^3 + bx^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-x^3 + b*x^5)^(-1),x]
[Out] IntegrateAlgebraic[(-x^3 + b*x^5)^(-1), x]
```

fricas [A] time = 0.39, size = 28, normalized size = 1.04

$$\frac{bx^2 \log\left(bx^2 - 1\right) - 2\,bx^2 \log(x) + 1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5-x^3),x, algorithm="fricas")
[Out] 1/2*(b*x^2*log(b*x^2 - 1) - 2*b*x^2*log(x) + 1)/x^2
```

giac [A] time = 0.18, size = 32, normalized size = 1.19

$$-\frac{1}{2}\,b\log\left(x^2\right) + \frac{1}{2}\,b\log\left(\left|bx^2 - 1\right|\right) + \frac{bx^2 + 1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5-x^3),x, algorithm="giac")
[Out] -1/2*b*log(x^2) + 1/2*b*log(abs(b*x^2 - 1)) + 1/2*(b*x^2 + 1)/x^2
```

maple [A] time = 0.05, size = 23, normalized size = 0.85

$$-b\ln(x) + \frac{b\ln\left(b\,x^2 - 1\right)}{2} + \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^5-x^3),x)
[Out] 1/2*b*ln(b*x^2-1)+1/2/x^2-b*ln(x)
```

maxima [A] time = 1.32, size = 22, normalized size = 0.81

$$\frac{1}{2}\,b\log\left(bx^2 - 1\right) - b\log(x) + \frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5-x^3),x, algorithm="maxima")
[Out] 1/2*b*log(b*x^2 - 1) - b*log(x) + 1/2/x^2
```

mupad [B] time = 5.21, size = 22, normalized size = 0.81

$$\frac{b\,\ln\left(b\,x^2 - 1\right)}{2} - b\,\ln(x) + \frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^5 - x^3),x)
```

```
[Out] (b*log(b*x^2 - 1))/2 - b*log(x) + 1/(2*x^2)
```

sympy [**A**] time = 0.23, size = 22, normalized size = 0.81

$$-b\log(x) + \frac{b\log\left(x^2 - \frac{1}{b}\right)}{2} + \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**5-x**3),x)
```

```
[Out] -b*log(x) + b*log(x**2 - 1/b)/2 + 1/(2*x**2)
```

3.224 $\int \frac{1}{ax+bx} \, dx$

Optimal. Leaf size=8

$$\frac{\log(x)}{a+b}$$

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6, 12, 29}

$$\frac{\log(x)}{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x)^(-1),x]
```

```
[Out] Log[x]/(a + b)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{ax+bx} \, dx &= \int \frac{1}{(a+b)x} \, dx \\
&= \frac{\int \frac{1}{x} \, dx}{a+b} \\
&= \frac{\log(x)}{a+b}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.75

$$\frac{\log(ax+bx)}{a+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x)^(-1),x]
```

```
[Out] Log[a*x + b*x]/(a + b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ax+bx} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x + b*x)^(-1),x]
[Out] IntegrateAlgebraic[(a*x + b*x)^(-1), x]
```

fricas [**A**] time = 0.39, size = 8, normalized size = 1.00

$$\frac{\log(x)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x),x, algorithm="fricas")
[Out] log(x)/(a + b)
```

giac [**A**] time = 0.15, size = 15, normalized size = 1.88

$$\frac{\log(|ax+bx|)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x),x, algorithm="giac")
[Out] log(abs(a*x + b*x))/(a + b)
```

maple [**A**] time = 0.04, size = 9, normalized size = 1.12

$$\frac{\ln(x)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x),x)
[Out] ln(x)/(a+b)
```

maxima [**A**] time = 1.33, size = 14, normalized size = 1.75

$$\frac{\log(ax+bx)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x),x, algorithm="maxima")
[Out] log(a*x + b*x)/(a + b)
```

mupad [**B**] time = 5.27, size = 8, normalized size = 1.00

$$\frac{\ln(x)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x),x)
[Out] log(x)/(a + b)
```

sympy [**A**] time = 0.07, size = 5, normalized size = 0.62

$$\frac{\log(x)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x),x)
[Out] log(x)/(a + b)
```

3.225 $\int \frac{1}{(ax+bx)^2}\,dx$

Optimal. Leaf size=10

$$-\frac{1}{x(a+b)^2}$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6, 12, 30}

$$-\frac{1}{x(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x)^(-2),x]
```

```
[Out] -(1/((a + b)^2*x))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ax+bx)^2}\,dx &= \int \frac{1}{(a+b)^2x^2}\,dx\\ &= \frac{\int \frac{1}{x^2}\,dx}{(a+b)^2}\\ &= -\frac{1}{(a+b)^2x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$-\frac{1}{x(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x)^(-2),x]
```

```
[Out] -(1/((a + b)^2*x))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x + b*x)^(-2),x]
[Out] IntegrateAlgebraic[(a*x + b*x)^(-2), x]
```

fricas [A] time = 0.37, size = 18, normalized size = 1.80

$$-\frac{1}{\left(a^{2}+2\,ab+b^{2}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^2,x, algorithm="fricas")
[Out] -1/((a^2 + 2*a*b + b^2)*x)
```

giac [A] time = 0.15, size = 16, normalized size = 1.60

$$-\frac{1}{(ax+bx)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^2,x, algorithm="giac")
[Out] -1/((a*x + b*x)*(a + b))
```

maple [A] time = 0.03, size = 11, normalized size = 1.10

$$-\frac{1}{(a+b)^{2}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x)^2,x)
[Out] -1/(a+b)^2/x
```

maxima [A] time = 1.34, size = 16, normalized size = 1.60

$$-\frac{1}{(ax+bx)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^2,x, algorithm="maxima")
[Out] -1/((a*x + b*x)*(a + b))
```

mupad [B] time = 0.03, size = 10, normalized size = 1.00

$$-\frac{1}{x\,(a+b)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x)^2,x)
[Out] -1/(x*(a + b)^2)
```

sympy [A] time = 0.08, size = 15, normalized size = 1.50

$$-\frac{1}{x\left(a^{2}+2ab+b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)**2,x)
```

```
[Out] -1/(x*(a**2 + 2*a*b + b**2))
```

3.226 $\int \frac{1}{(ax+bx)^3} dx$

Optimal. Leaf size=12

$$-\frac{1}{2x^2(a+b)^3}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6, 12, 30}

$$-\frac{1}{2x^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x)^(-3),x]
```

```
[Out] -1/(2*(a + b)^3*x^2)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ax+bx)^3} dx &= \int \frac{1}{(a+b)^3x^3} dx\\ &= \frac{\int \frac{1}{x^3} dx}{(a+b)^3}\\ &= -\frac{1}{2(a+b)^3x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{2x^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x)^(-3),x]
```

```
[Out] -1/2*1/((a + b)^3*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax+bx)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x + b*x)^(-3),x]
```

```
[Out] IntegrateAlgebraic[(a*x + b*x)^(-3), x]
```

fricas [B] time = 0.37, size = 26, normalized size = 2.17

$$-\frac{1}{2\left(a^3+3\,a^2b+3\,ab^2+b^3\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x^2)
```

giac [A] time = 0.15, size = 16, normalized size = 1.33

$$-\frac{1}{2\,(ax+bx)^2(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^3,x, algorithm="giac")
```

```
[Out] -1/2/((a*x + b*x)^2*(a + b))
```

maple [A] time = 0.05, size = 11, normalized size = 0.92

$$-\frac{1}{2\,(a+b)^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x)^3,x)
```

```
[Out] -1/2/(a+b)^3/x^2
```

maxima [A] time = 1.31, size = 16, normalized size = 1.33

$$-\frac{1}{2\,(ax+bx)^2(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2/((a*x + b*x)^2*(a + b))
```

mupad [B] time = 0.04, size = 26, normalized size = 2.17

$$-\frac{1}{2\,x^2\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x)^3,x)
```

```
[Out] -1/(2*x^2*(3*a*b^2 + 3*a^2*b + a^3 + b^3))
```

sympy [B] time = 0.09, size = 27, normalized size = 2.25

$$-\frac{1}{2x^2\left(a^3+3a^2b+3ab^2+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)**3,x)
```

```
[Out] -1/(2*x**2*(a**3 + 3*a**2*b + 3*a*b**2 + b**3))
```

3.227 $\int \frac{1}{ax^2+bx^2}\,dx$

Optimal. Leaf size=10

$$-\frac{1}{x(a+b)}$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6, 12, 30}

$$-\frac{1}{x(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^2)^(-1),x]
[Out] -(1/((a + b)*x))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax^2+bx^2}\,dx &= \int \frac{1}{(a+b)x^2}\,dx\\ &= \frac{\int \frac{1}{x^2}\,dx}{a+b}\\ &= -\frac{1}{(a+b)x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$-\frac{1}{x(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^2)^(-1),x]
[Out] -(1/((a + b)*x))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ax^2+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x^2)^(-1),x]
[Out] IntegrateAlgebraic[(a*x^2 + b*x^2)^(-1), x]
```

fricas [A] time = 0.37, size = 10, normalized size = 1.00

$$-\frac{1}{(a+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x^2),x, algorithm="fricas")
[Out] -1/((a + b)*x)
```

giac [A] time = 0.15, size = 10, normalized size = 1.00

$$-\frac{1}{(a+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x^2),x, algorithm="giac")
[Out] -1/((a + b)*x)
```

maple [A] time = 0.04, size = 11, normalized size = 1.10

$$-\frac{1}{(a+b)\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2+b*x^2),x)
[Out] -1/(a+b)/x
```

maxima [A] time = 1.36, size = 10, normalized size = 1.00

$$-\frac{1}{(a+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x^2),x, algorithm="maxima")
[Out] -1/((a + b)*x)
```

mupad [B] time = 0.03, size = 10, normalized size = 1.00

$$-\frac{1}{x\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^2),x)
[Out] -1/(x*(a + b))
```

sympy [A] time = 0.08, size = 7, normalized size = 0.70

$$-\frac{1}{x\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**2+b*x**2),x)
```

```
[Out] -1/(x*(a + b))
```

3.228 $\int \frac{1}{ax^n+bx^n} dx$

Optimal. Leaf size=20

$$\frac{x^{1-n}}{(1-n)(a+b)}$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6, 12, 30}

$$\frac{x^{1-n}}{(1-n)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^n + b*x^n)^(-1),x]
```

```
[Out] x^(1 - n)/((a + b)*(1 - n))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax^n+bx^n}\,dx &= \int \frac{x^{-n}}{a+b}\,dx\\ &= \frac{\int x^{-n}\,dx}{a+b}\\ &= \frac{x^{1-n}}{(a+b)(1-n)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$\frac{x^{1-n}}{(1-n)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^n + b*x^n)^(-1),x]
```

```
[Out] x^(1 - n)/((a + b)*(1 - n))
```

IntegrateAlgebraic [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{ax^n+bx^n}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x^n + b*x^n)^(-1),x]
[Out] Defer[IntegrateAlgebraic][(a*x^n + b*x^n)^(-1), x]
```

fricas [A] time = 0.40, size = 22, normalized size = 1.10

$$-\frac{x}{((a+b)n-a-b)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n),x, algorithm="fricas")
[Out] -x/(((a + b)*n - a - b)*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ax^n+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n),x, algorithm="giac")
[Out] integrate(1/(a*x^n + b*x^n), x)
```

maple [A] time = 0.04, size = 19, normalized size = 0.95

$$-\frac{x\,x^{-n}}{(n-1)\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^n+b*x^n),x)
[Out] -x/(n-1)/(x^n)/(a+b)
```

maxima [A] time = 1.35, size = 21, normalized size = 1.05

$$-\frac{x}{(a(n-1)+b(n-1))x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n),x, algorithm="maxima")
[Out] -x/((a*(n - 1) + b*(n - 1))*x^n)
```

mupad [B] time = 5.18, size = 19, normalized size = 0.95

$$-\frac{x^{1-n}}{(a+b)\,(n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^n + b*x^n),x)
[Out] -x^(1 - n)/((a + b)*(n - 1))
```

sympy [A] time = 0.64, size = 32, normalized size = 1.60

$$\begin{cases} -\frac{x}{anx^n-ax^n+bnx^n-bx^n} & \text{for}\ n\neq 1 \\ \frac{\log(x)}{a+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**n+b*x**n),x)
```

```
[Out] Piecewise((-x/(a*n*x**n - a*x**n + b*n*x**n - b*x**n), Ne(n, 1)), (log(x)/(
a + b), True))
```

3.229 $\int \frac{1}{(ax^n+bx^n)^2}\,dx$

Optimal. Leaf size=20

$$\frac{x^{1-2n}}{(1-2n)(a+b)^2}$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6, 12, 30}

$$\frac{x^{1-2n}}{(1-2n)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^n + b*x^n)^(-2),x]
[Out] x^(1 - 2*n)/((a + b)^2*(1 - 2*n))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ax^n+bx^n)^2}\,dx &= \int \frac{x^{-2n}}{(a+b)^2}\,dx\\ &= \frac{\int x^{-2n}\,dx}{(a+b)^2}\\ &= \frac{x^{1-2n}}{(a+b)^2(1-2n)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$\frac{x^{1-2n}}{(1-2n)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^n + b*x^n)^(-2),x]
[Out] x^(1 - 2*n)/((a + b)^2*(1 - 2*n))
```

IntegrateAlgebraic [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax^n+bx^n)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x^n + b*x^n)^(-2),x]
[Out] Defer[IntegrateAlgebraic][(a*x^n + b*x^n)^(-2), x]
```

fricas [A] time = 0.41, size = 36, normalized size = 1.80

$$\frac{x}{\left(a^2+2\,ab+b^2-2\left(a^2+2\,ab+b^2\right)n\right)x^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n)^2,x, algorithm="fricas")
[Out] x/((a^2 + 2*a*b + b^2 - 2*(a^2 + 2*a*b + b^2)*n)*x^(2*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax^n+bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n)^2,x, algorithm="giac")
[Out] integrate((a*x^n + b*x^n)^(-2), x)
```

maple [A] time = 0.04, size = 21, normalized size = 1.05

$$-\frac{x\,x^{-2n}}{(2n-1)\,(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^n+b*x^n)^2,x)
[Out] -x/(2*n-1)/(x^n)^2/(a+b)^2
```

maxima [A] time = 1.36, size = 40, normalized size = 2.00

$$-\frac{x}{\left(a^2(2\,n-1)+2\,ab(2\,n-1)+b^2(2\,n-1)\right)x^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n)^2,x, algorithm="maxima")
[Out] -x/((a^2*(2*n - 1) + 2*a*b*(2*n - 1) + b^2*(2*n - 1))*x^(2*n))
```

mupad [B] time = 5.14, size = 21, normalized size = 1.05

$$-\frac{x^{1-2\,n}}{(a+b)^2\,(2\,n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^n + b*x^n)^2,x)
[Out] -x^(1 - 2*n)/((a + b)^2*(2*n - 1))
```

sympy [A] time = 1.03, size = 82, normalized size = 4.10

$$\begin{cases} -\frac{x}{2a^2nx^{2n}-a^2x^{2n}+4abnx^{2n}-2abx^{2n}+2b^2nx^{2n}-b^2x^{2n}} & \text{for}\ n \neq \frac{1}{2} \\ \frac{\log(x)}{a^2+2ab+b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**n+b*x**n)**2,x)
```

```
[Out] Piecewise((-x/(2*a**2*n*x**(2*n) - a**2*x**(2*n) + 4*a*b*n*x**(2*n) - 2*a*b
*x**(2*n) + 2*b**2*n*x**(2*n) - b**2*x**(2*n)), Ne(n, 1/2)), (log(x)/(a**2
+ 2*a*b + b**2), True))
```

3.230 $\int \frac{1}{(ax^n+bx^n)^3} dx$

Optimal. Leaf size=20

$$\frac{x^{1-3n}}{(1-3n)(a+b)^3}$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6, 12, 30}

$$\frac{x^{1-3n}}{(1-3n)(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^n + b*x^n)^(-3),x]
```

```
[Out] x^(1 - 3*n)/((a + b)^3*(1 - 3*n))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ax^n+bx^n)^3}\,dx &= \int \frac{x^{-3n}}{(a+b)^3}\,dx\\ &= \frac{\int x^{-3n}\,dx}{(a+b)^3}\\ &= \frac{x^{1-3n}}{(a+b)^3(1-3n)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$\frac{x^{1-3n}}{(1-3n)(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^n + b*x^n)^(-3),x]
```

```
[Out] x^(1 - 3*n)/((a + b)^3*(1 - 3*n))
```

IntegrateAlgebraic [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax^n+bx^n)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x^n + b*x^n)^(-3),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a*x^n + b*x^n)^(-3), x]
```

fricas [B] time = 0.42, size = 52, normalized size = 2.60

$$\frac{x}{\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}-3\,\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)n\right)x^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] x/((a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*n)*x^
(3*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax^{n}+bx^{n})^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((a*x^n + b*x^n)^(-3), x)
```

maple [A] time = 0.04, size = 21, normalized size = 1.05

$$-\frac{x\,x^{-3n}}{(3n-1)\,(a+b)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^n+b*x^n)^3,x)
```

```
[Out] -x/(3*n-1)/(x^n)^3/(a+b)^3
```

maxima [B] time = 1.40, size = 53, normalized size = 2.65

$$-\frac{x}{\left(a^{3}(3\,n-1)+3\,a^{2}b(3\,n-1)+3\,ab^{2}(3\,n-1)+b^{3}(3\,n-1)\right)x^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] -x/((a^3*(3*n - 1) + 3*a^2*b*(3*n - 1) + 3*a*b^2*(3*n - 1) + b^3*(3*n - 1))
*x^(3*n))
```

mupad [B] time = 5.12, size = 21, normalized size = 1.05

$$-\frac{x^{1-3\,n}}{(a+b)^{3}\,(3\,n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^n + b*x^n)^3,x)
```

```
[Out] -x^(1 - 3*n)/((a + b)^3*(3*n - 1))
```

sympy [A] time = 1.35, size = 119, normalized size = 5.95

$$\begin{cases} -\frac{x}{3a^3nx^{3n}-a^3x^{3n}+9a^2bnx^{3n}-3a^2bx^{3n}+9ab^2nx^{3n}-3ab^2x^{3n}+3b^3nx^{3n}-b^3x^{3n}} & \text{for } n \neq \frac{1}{3} \\ \frac{\log(x)}{a^3+3a^2b+3ab^2+b^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**n+b*x**n)**3,x)
```

```
[Out] Piecewise((-x/(3*a**3*n*x**(3*n) - a**3*x**(3*n) + 9*a**2*b*n*x**(3*n) - 3*
a**2*b*x**(3*n) + 9*a*b**2*n*x**(3*n) - 3*a*b**2*x**(3*n) + 3*b**3*n*x**(3*
n) - b**3*x**(3*n)), Ne(n, 1/3)), (log(x)/(a**3 + 3*a**2*b + 3*a*b**2 + b**
3), True))
```

3.231 $\int \left(ax+bx^{14}\right)^{12} dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 261}

$$\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^14)^12,x]
```

```
[Out] (a + b*x^13)^13/(169*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(ax+bx^{14}\right)^{12} dx &= \int x^{12}\left(a+bx^{13}\right)^{12} dx\\ &= \frac{\left(a+bx^{13}\right)^{13}}{169b}\end{aligned}$$

Mathematica [B] time = 0.00, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{13}}{13}+\frac{6}{13}a^{11}bx^{26}+\frac{22}{13}a^{10}b^2x^{39}+\frac{55}{13}a^9b^3x^{52}+\frac{99}{13}a^8b^4x^{65}+\frac{132}{13}a^7b^5x^{78}+\frac{132}{13}a^6b^6x^{91}+\frac{99}{13}a^5b^7x^{104}+\frac{55}{13}a^4b^8x^{117}+\frac{22}{13}a^3b^9x^{130}+\frac{6}{13}a^2b^{10}x^{143}+\frac{1}{13}ab^{11}x^{156}+\frac{b^{12}x^{169}}{169}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^14)^12,x]
```

```
[Out] (a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x
^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)
/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13
 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax+bx^{14}\right)^{12} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x + b*x^14)^12,x]
```

```
[Out] IntegrateAlgebraic[(a*x + b*x^14)^12, x]
```

fricas [B] time = 0.35, size = 134, normalized size = 8.38

$$\frac{1}{169}x^{169}b^{12} + \frac{1}{13}x^{156}b^{11}a + \frac{6}{13}x^{143}b^{10}a^2 + \frac{22}{13}x^{130}b^9a^3 + \frac{55}{13}x^{117}b^8a^4 + \frac{99}{13}x^{104}b^7a^5 + \frac{132}{13}x^{91}b^6a^6 + \frac{132}{13}x^{78}b^5a^7 + \frac{99}{13}x^{65}b^4a^8 + \frac{55}{13}x^{52}b^3a^9 + \frac{22}{13}x^{39}b^2a^{10} + \frac{6}{13}x^{26}ba^{11} + \frac{1}{13}x^{13}a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/169*x^169*b^12 + 1/13*x^156*b^11*a + 6/13*x^143*b^10*a^2 + 22/13*x^130*b^
9*a^3 + 55/13*x^117*b^8*a^4 + 99/13*x^104*b^7*a^5 + 132/13*x^91*b^6*a^6 + 1
32/13*x^78*b^5*a^7 + 99/13*x^65*b^4*a^8 + 55/13*x^52*b^3*a^9 + 22/13*x^39*b
^2*a^10 + 6/13*x^26*b*a^11 + 1/13*x^13*a^12
```

giac [B] time = 0.14, size = 134, normalized size = 8.38

$$\frac{1}{169}b^{12}x^{169} + \frac{1}{13}ab^{11}x^{156} + \frac{6}{13}a^2b^{10}x^{143} + \frac{22}{13}a^3b^9x^{130} + \frac{55}{13}a^4b^8x^{117} + \frac{99}{13}a^5b^7x^{104} + \frac{132}{13}a^6b^6x^{91} + \frac{132}{13}a^7b^5x^{78} + \frac{99}{13}a^8b^4x^{65} + \frac{55}{13}a^9b^3x^{52} + \frac{22}{13}a^{10}b^2x^{39} + \frac{6}{13}a^{11}bx^{26} + \frac{1}{13}a^{12}x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

maple [B] time = 0.05, size = 135, normalized size = 8.44

$$\frac{1}{169}b^{12}x^{169} + \frac{1}{13}a\,b^{11}x^{156} + \frac{6}{13}a^2b^{10}x^{143} + \frac{22}{13}a^3b^9x^{130} + \frac{55}{13}a^4b^8x^{117} + \frac{99}{13}a^5b^7x^{104} + \frac{132}{13}a^6b^6x^{91} + \frac{132}{13}a^7b^5x^{78} + \frac{99}{13}a^8b^4x^{65} + \frac{55}{13}a^9b^3x^{52} + \frac{22}{13}a^{10}b^2x^{39} + \frac{6}{13}a^{11}b\,x^{26} + \frac{1}{13}a^{12}x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^14+a*x)^12,x)
```

```
[Out] 1/169*b^12*x^169+1/13*a*b^11*x^156+6/13*a^2*b^10*x^143+22/13*a^3*b^9*x^130+
55/13*a^4*b^8*x^117+99/13*a^5*b^7*x^104+132/13*a^6*b^6*x^91+132/13*a^7*b^5*
x^78+99/13*a^8*b^4*x^65+55/13*a^9*b^3*x^52+22/13*a^10*b^2*x^39+6/13*a^11*b*
x^26+1/13*a^12*x^13
```

maxima [B] time = 1.31, size = 134, normalized size = 8.38

$$\frac{1}{169}b^{12}x^{169} + \frac{1}{13}ab^{11}x^{156} + \frac{6}{13}a^2b^{10}x^{143} + \frac{22}{13}a^3b^9x^{130} + \frac{55}{13}a^4b^8x^{117} + \frac{99}{13}a^5b^7x^{104} + \frac{132}{13}a^6b^6x^{91} + \frac{132}{13}a^7b^5x^{78} + \frac{99}{13}a^8b^4x^{65} + \frac{55}{13}a^9b^3x^{52} + \frac{22}{13}a^{10}b^2x^{39} + \frac{6}{13}a^{11}bx^{26} + \frac{1}{13}a^{12}x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

mupad [B] time = 5.16, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{13}}{13} + \frac{6\,a^{11}\,b\,x^{26}}{13} + \frac{22\,a^{10}\,b^2\,x^{39}}{13} + \frac{55\,a^9\,b^3\,x^{52}}{13} + \frac{99\,a^8\,b^4\,x^{65}}{13} + \frac{132\,a^7\,b^5\,x^{78}}{13} + \frac{132\,a^6\,b^6\,x^{91}}{13} + \frac{99\,a^5\,b^7\,x^{104}}{13} + \frac{55\,a^4\,b^8\,x^{117}}{13} + \frac{22\,a^3\,b^9\,x^{130}}{13} + \frac{6\,a^2\,b^{10}\,x^{143}}{13} + \frac{a\,b^{11}\,x^{156}}{13} + \frac{b^{12}\,x^{169}}{169}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^14)^12,x)
```

[Out] (a^12*x^13)/13 + (b^12*x^169)/169 + (6*a^11*b*x^26)/13 + (a*b^11*x^156)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13 + (6*a^2*b^10*x^143)/13

sympy [B] time = 0.12, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{13}}{13}+\frac{6a^{11}bx^{26}}{13}+\frac{22a^{10}b^2x^{39}}{13}+\frac{55a^9b^3x^{52}}{13}+\frac{99a^8b^4x^{65}}{13}+\frac{132a^7b^5x^{78}}{13}+\frac{132a^6b^6x^{91}}{13}+\frac{99a^5b^7x^{104}}{13}+\frac{55a^4b^8x^{117}}{13}+\frac{22a^3b^9x^{130}}{13}+\frac{6a^2b^{10}x^{143}}{13}+\frac{ab^{11}x^{156}}{13}+\frac{b^{12}x^{169}}{169}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**14+a*x)**12,x)

[Out] a**12*x**13/13 + 6*a**11*b*x**26/13 + 22*a**10*b**2*x**39/13 + 55*a**9*b**3*x**52/13 + 99*a**8*b**4*x**65/13 + 132*a**7*b**5*x**78/13 + 132*a**6*b**6*x**91/13 + 99*a**5*b**7*x**104/13 + 55*a**4*b**8*x**117/13 + 22*a**3*b**9*x**130/13 + 6*a**2*b**10*x**143/13 + a*b**11*x**156/13 + b**12*x**169/169

3.232 $\int x^{12} \left(ax + bx^{26}\right)^{12} dx$

Optimal. Leaf size=16

$$\frac{\left(a + bx^{25}\right)^{13}}{325b}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 261}

$$\frac{\left(a + bx^{25}\right)^{13}}{325b}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a*x + b*x^26)^12,x]
```

```
[Out] (a + b*x^25)^13/(325*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{12} \left(ax + bx^{26}\right)^{12} dx &= \int x^{24} \left(a + bx^{25}\right)^{12} dx \\ &= \frac{\left(a + bx^{25}\right)^{13}}{325b}\end{aligned}$$

Mathematica [B] time = 0.02, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25} + \frac{6}{25}a^{11}bx^{50} + \frac{22}{25}a^{10}b^2x^{75} + \frac{11}{5}a^9b^3x^{100} + \frac{99}{25}a^8b^4x^{125} + \frac{132}{25}a^7b^5x^{150} + \frac{132}{25}a^6b^6x^{175} + \frac{99}{25}a^5b^7x^{200} + \frac{11}{5}a^4b^8x^{225} + \frac{22}{25}a^3b^9x^{250} + \frac{6}{25}a^2b^{10}x^{275} + \frac{1}{25}ab^{11}x^{300} + \frac{b^{12}x^{325}}{325}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a*x + b*x^26)^12,x]
```

```
[Out] (a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x
^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^1
75)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/
25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{12} \left(ax + bx^{26}\right)^{12} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^12*(a*x + b*x^26)^12,x]
```

```
[Out] IntegrateAlgebraic[x^12*(a*x + b*x^26)^12, x]
```

fricas [B] time = 0.34, size = 134, normalized size = 8.38

$$\frac{1}{325}x^{325}b^{12} + \frac{1}{25}x^{300}b^{11}a + \frac{6}{25}x^{275}b^{10}a^2 + \frac{22}{25}x^{250}b^9a^3 + \frac{11}{5}x^{225}b^8a^4 + \frac{99}{25}x^{200}b^7a^5 + \frac{132}{25}x^{175}b^6a^6 + \frac{132}{25}x^{150}b^5a^7 + \frac{99}{25}x^{125}b^4a^8 + \frac{11}{5}x^{100}b^3a^9 + \frac{22}{25}x^{75}b^2a^{10} + \frac{6}{25}x^{50}ba^{11} + \frac{1}{25}x^{25}a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/325*x^325*b^12 + 1/25*x^300*b^11*a + 6/25*x^275*b^10*a^2 + 22/25*x^250*b^
9*a^3 + 11/5*x^225*b^8*a^4 + 99/25*x^200*b^7*a^5 + 132/25*x^175*b^6*a^6 + 1
32/25*x^150*b^5*a^7 + 99/25*x^125*b^4*a^8 + 11/5*x^100*b^3*a^9 + 22/25*x^75
*b^2*a^10 + 6/25*x^50*b*a^11 + 1/25*x^25*a^12
```

giac [B] time = 0.15, size = 134, normalized size = 8.38

$$\frac{1}{325}b^{12}x^{325} + \frac{1}{25}ab^{11}x^{300} + \frac{6}{25}a^2b^{10}x^{275} + \frac{22}{25}a^3b^9x^{250} + \frac{11}{5}a^4b^8x^{225} + \frac{99}{25}a^5b^7x^{200} + \frac{132}{25}a^6b^6x^{175} + \frac{132}{25}a^7b^5x^{150} + \frac{99}{25}a^8b^4x^{125} + \frac{11}{5}a^9b^3x^{100} + \frac{22}{25}a^{10}b^2x^{75} + \frac{6}{25}a^{11}bx^{50} + \frac{1}{25}a^{12}x^{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

maple [B] time = 0.04, size = 135, normalized size = 8.44

$$\frac{1}{325}b^{12}x^{325} + \frac{1}{25}a\,b^{11}x^{300} + \frac{6}{25}a^2b^{10}x^{275} + \frac{22}{25}a^3b^9x^{250} + \frac{11}{5}a^4b^8x^{225} + \frac{99}{25}a^5b^7x^{200} + \frac{132}{25}a^6b^6x^{175} + \frac{132}{25}a^7b^5x^{150} + \frac{99}{25}a^8b^4x^{125} + \frac{11}{5}a^9b^3x^{100} + \frac{22}{25}a^{10}b^2x^{75} + \frac{6}{25}a^{11}b\,x^{50} + \frac{1}{25}a^{12}x^{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b*x^26+a*x)^12,x)
```

```
[Out] 1/325*b^12*x^325+1/25*a*b^11*x^300+6/25*a^2*b^10*x^275+22/25*a^3*b^9*x^250+
11/5*a^4*b^8*x^225+99/25*a^5*b^7*x^200+132/25*a^6*b^6*x^175+132/25*a^7*b^5*
x^150+99/25*a^8*b^4*x^125+11/5*a^9*b^3*x^100+22/25*a^10*b^2*x^75+6/25*a^11*
b*x^50+1/25*a^12*x^25
```

maxima [B] time = 1.37, size = 134, normalized size = 8.38

$$\frac{1}{325}b^{12}x^{325} + \frac{1}{25}ab^{11}x^{300} + \frac{6}{25}a^2b^{10}x^{275} + \frac{22}{25}a^3b^9x^{250} + \frac{11}{5}a^4b^8x^{225} + \frac{99}{25}a^5b^7x^{200} + \frac{132}{25}a^6b^6x^{175} + \frac{132}{25}a^7b^5x^{150} + \frac{99}{25}a^8b^4x^{125} + \frac{11}{5}a^9b^3x^{100} + \frac{22}{25}a^{10}b^2x^{75} + \frac{6}{25}a^{11}bx^{50} + \frac{1}{25}a^{12}x^{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

mupad [B] time = 5.16, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{25}}{25} + \frac{6\,a^{11}\,b\,x^{50}}{25} + \frac{22\,a^{10}\,b^2\,x^{75}}{25} + \frac{11\,a^9\,b^3\,x^{100}}{5} + \frac{99\,a^8\,b^4\,x^{125}}{25} + \frac{132\,a^7\,b^5\,x^{150}}{25} + \frac{132\,a^6\,b^6\,x^{175}}{25} + \frac{99\,a^5\,b^7\,x^{200}}{25} + \frac{11\,a^4\,b^8\,x^{225}}{5} + \frac{22\,a^3\,b^9\,x^{250}}{25} + \frac{6\,a^2\,b^{10}\,x^{275}}{25} + \frac{a\,b^{11}\,x^{300}}{25} + \frac{b^{12}\,x^{325}}{325}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a*x + b*x^26)^12,x)
```

```
[Out] (a^12*x^25)/25 + (b^12*x^325)/325 + (6*a^11*b*x^50)/25 + (a*b^11*x^300)/25
+ (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x^100)/5 + (99*a^8*b^4*x^125)/25 + (1
32*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^175)/25 + (99*a^5*b^7*x^200)/25 + (11
*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/25 + (6*a^2*b^10*x^275)/25
```

sympy [B] time = 0.13, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25}+\frac{6a^{11}bx^{50}}{25}+\frac{22a^{10}b^{2}x^{75}}{25}+\frac{11a^{9}b^{3}x^{100}}{5}+\frac{99a^{8}b^{4}x^{125}}{25}+\frac{132a^{7}b^{5}x^{150}}{25}+\frac{132a^{6}b^{6}x^{175}}{25}+\frac{99a^{5}b^{7}x^{200}}{25}+\frac{11a^{4}b^{8}x^{225}}{5}+\frac{22a^{3}b^{9}x^{250}}{25}+\frac{6a^{2}b^{10}x^{275}}{25}+\frac{ab^{11}x^{300}}{25}+\frac{b^{12}x^{325}}{325}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b*x**26+a*x)**12,x)
```

```
[Out] a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**3
*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b**
6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b**9
*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/325
```

3.233 $\int x^{24} \left(ax+bx^{38}\right)^{12} \, dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 261}

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

Antiderivative was successfully verified.

```
[In] Int[x^24*(a*x + b*x^38)^12,x]
```

```
[Out] (a + b*x^37)^13/(481*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{24} \left(ax+bx^{38}\right)^{12} \, dx &= \int x^{36} \left(a+bx^{37}\right)^{12} \, dx\\ &= \frac{\left(a+bx^{37}\right)^{13}}{481b}\end{aligned}$$

Mathematica [B] time = 0.01, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{333}+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^2b^{10}x^{407}+\frac{1}{37}ab^{11}x^{444}+\frac{b^{12}x^{481}}{481}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^24*(a*x + b*x^38)^12,x]
```

```
[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{24} \left(ax+bx^{38}\right)^{12} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^24*(a*x + b*x^38)^12,x]
```

```
[Out] IntegrateAlgebraic[x^24*(a*x + b*x^38)^12, x]
```

fricas [B] time = 0.34, size = 134, normalized size = 8.38

$$\frac{1}{481}x^{481}b^{12}+\frac{1}{37}x^{444}b^{11}a+\frac{6}{37}x^{407}b^{10}a^{2}+\frac{22}{37}x^{370}b^{9}a^{3}+\frac{55}{37}x^{333}b^{8}a^{4}+\frac{99}{37}x^{296}b^{7}a^{5}+\frac{132}{37}x^{259}b^{6}a^{6}+\frac{132}{37}x^{222}b^{5}a^{7}+\frac{99}{37}x^{185}b^{4}a^{8}+\frac{55}{37}x^{148}b^{3}a^{9}+\frac{22}{37}x^{111}b^{2}a^{10}+\frac{6}{37}x^{74}ba^{11}+\frac{1}{37}x^{37}a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/481*x^481*b^12 + 1/37*x^444*b^11*a + 6/37*x^407*b^10*a^2 + 22/37*x^370*b^
9*a^3 + 55/37*x^333*b^8*a^4 + 99/37*x^296*b^7*a^5 + 132/37*x^259*b^6*a^6 +
132/37*x^222*b^5*a^7 + 99/37*x^185*b^4*a^8 + 55/37*x^148*b^3*a^9 + 22/37*x^
111*b^2*a^10 + 6/37*x^74*b*a^11 + 1/37*x^37*a^12
```

giac [B] time = 0.15, size = 134, normalized size = 8.38

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}ab^{11}x^{444}+\frac{6}{37}a^{2}b^{10}x^{407}+\frac{22}{37}a^{3}b^{9}x^{370}+\frac{55}{37}a^{4}b^{8}x^{333}+\frac{99}{37}a^{5}b^{7}x^{296}+\frac{132}{37}a^{6}b^{6}x^{259}+\frac{132}{37}a^{7}b^{5}x^{222}+\frac{99}{37}a^{8}b^{4}x^{185}+\frac{55}{37}a^{9}b^{3}x^{148}+\frac{22}{37}a^{10}b^{2}x^{111}+\frac{6}{37}a^{11}bx^{74}+\frac{1}{37}a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

maple [B] time = 0.04, size = 135, normalized size = 8.44

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}a\,b^{11}x^{444}+\frac{6}{37}a^{2}b^{10}x^{407}+\frac{22}{37}a^{3}b^{9}x^{370}+\frac{55}{37}a^{4}b^{8}x^{333}+\frac{99}{37}a^{5}b^{7}x^{296}+\frac{132}{37}a^{6}b^{6}x^{259}+\frac{132}{37}a^{7}b^{5}x^{222}+\frac{99}{37}a^{8}b^{4}x^{185}+\frac{55}{37}a^{9}b^{3}x^{148}+\frac{22}{37}a^{10}b^{2}x^{111}+\frac{6}{37}a^{11}b\,x^{74}+\frac{1}{37}a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24*(b*x^38+a*x)^12,x)
```

```
[Out] 1/481*b^12*x^481+1/37*a*b^11*x^444+6/37*a^2*b^10*x^407+22/37*a^3*b^9*x^370+
55/37*a^4*b^8*x^333+99/37*a^5*b^7*x^296+132/37*a^6*b^6*x^259+132/37*a^7*b^5
*x^222+99/37*a^8*b^4*x^185+55/37*a^9*b^3*x^148+22/37*a^10*b^2*x^111+6/37*a^
11*b*x^74+1/37*a^12*x^37
```

maxima [B] time = 1.31, size = 134, normalized size = 8.38

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}ab^{11}x^{444}+\frac{6}{37}a^{2}b^{10}x^{407}+\frac{22}{37}a^{3}b^{9}x^{370}+\frac{55}{37}a^{4}b^{8}x^{333}+\frac{99}{37}a^{5}b^{7}x^{296}+\frac{132}{37}a^{6}b^{6}x^{259}+\frac{132}{37}a^{7}b^{5}x^{222}+\frac{99}{37}a^{8}b^{4}x^{185}+\frac{55}{37}a^{9}b^{3}x^{148}+\frac{22}{37}a^{10}b^{2}x^{111}+\frac{6}{37}a^{11}bx^{74}+\frac{1}{37}a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

mupad [B] time = 5.15, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{37}}{37}+\frac{6\,a^{11}\,b\,x^{74}}{37}+\frac{22\,a^{10}\,b^2\,x^{111}}{37}+\frac{55\,a^9\,b^3\,x^{148}}{37}+\frac{99\,a^8\,b^4\,x^{185}}{37}+\frac{132\,a^7\,b^5\,x^{222}}{37}+\frac{132\,a^6\,b^6\,x^{259}}{37}+\frac{99\,a^5\,b^7\,x^{296}}{37}+\frac{55\,a^4\,b^8\,x^{333}}{37}+\frac{22\,a^3\,b^9\,x^{370}}{37}+\frac{6\,a^2\,b^{10}\,x^{407}}{37}+\frac{a\,b^{11}\,x^{444}}{37}+\frac{b^{12}\,x^{481}}{481}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24*(a*x + b*x^38)^12,x)
```

```
[Out] (a^12*x^37)/37 + (b^12*x^481)/481 + (6*a^11*b*x^74)/37 + (a*b^11*x^444)/37
+ (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37 +
(132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37 + (
55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37
```

sympy [**B**] time = 0.14, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37} + \frac{6a^{11}bx^{74}}{37} + \frac{22a^{10}b^2x^{111}}{37} + \frac{55a^9b^3x^{148}}{37} + \frac{99a^8b^4x^{185}}{37} + \frac{132a^7b^5x^{222}}{37} + \frac{132a^6b^6x^{259}}{37} + \frac{99a^5b^7x^{296}}{37} + \frac{55a^4b^8x^{333}}{37} + \frac{22a^3b^9x^{370}}{37} + \frac{6a^2b^{10}x^{407}}{37} + \frac{ab^{11}x^{444}}{37} + \frac{b^{12}x^{481}}{481}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**24*(b*x**38+a*x)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

3.234 $\int x^{12(-1+m)} \left(ax + bx^{2+12m}\right)^{12} \, dx$

Optimal. Leaf size=27

$$\frac{\left(a + bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1584, 261}

$$\frac{\left(a + bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(12*(-1 + m))*(a*x + b*x^(2 + 12*m))^12,x]
```

```
[Out] (a + b*x^(1 + 12*m))^13/(13*b*(1 + 12*m))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{12(-1+m)} \left(ax + bx^{2+12m}\right)^{12} \, dx &= \int x^{12+12(-1+m)} \left(a + bx^{1+12m}\right)^{12} \, dx \\ &= \frac{\left(a + bx^{1+12m}\right)^{13}}{13b(1+12m)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 24, normalized size = 0.89

$$\frac{\left(a + bx^{12m+1}\right)^{13}}{156bm + 13b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(12*(-1 + m))*(a*x + b*x^(2 + 12*m))^12,x]
```

```
[Out] (a + b*x^(1 + 12*m))^13/(13*b + 156*b*m)
```

IntegrateAlgebraic [B] time = 0.07, size = 196, normalized size = 7.26

$$\frac{x^{12m+1}\left(13a^{12} + 78a^{11}bx^{12m+1} + 286a^{10}b^2x^{24m+2} + 715a^9b^3x^{36m+3} + 1287a^8b^4x^{48m+4} + 1716a^7b^5x^{60m+5} + 1716a^6b^6x^{72m+6} + 1287a^5b^7x^{84m+7} + 715a^4b^8x^{96m+8} + 286a^3b^9x^{108m+9} + 78a^2b^{10}x^{120m+10} + 13ab^{11}x^{132m+11} + b^{12}x^{144m+12}\right)}{13(12m+1)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(12*(-1 + m))*(a*x + b*x^(2 + 12*m))^12,x]
```

```
[Out] (x^(1 + 12*m)*(13*a^12 + 78*a^11*b*x^(1 + 12*m) + 286*a^10*b^2*x^(2 + 24*m)
 + 715*a^9*b^3*x^(3 + 36*m) + 1287*a^8*b^4*x^(4 + 48*m) + 1716*a^7*b^5*x^(5
 + 60*m) + 1716*a^6*b^6*x^(6 + 72*m) + 1287*a^5*b^7*x^(7 + 84*m) + 715*a^4*
b^8*x^(8 + 96*m) + 286*a^3*b^9*x^(9 + 108*m) + 78*a^2*b^10*x^(10 + 120*m) +
 13*a*b^11*x^(11 + 132*m) + b^12*x^(12 + 144*m)))/(13*(1 + 12*m))
```

fricas [**B**] time = 0.44, size = 231, normalized size = 8.56

$$\frac{13\,a^{12}x^{12}x^{12\,m+2}+78\,a^{11}bx^{11}x^{24\,m+4}+286\,a^{10}b^{2}x^{10}x^{36\,m+6}+715\,a^{9}b^{3}x^{9}x^{48\,m+8}+1287\,a^{8}b^{4}x^{8}x^{60\,m+10}+1716\,a^{7}b^{5}x^{7}x^{72\,m+12}+1716\,a^{6}b^{6}x^{6}x^{84\,m+14}+1287\,a^{5}b^{7}x^{5}x^{96\,m+16}+715\,a^{4}b^{8}x^{4}x^{108\,m+18}+286\,a^{3}b^{9}x^{3}x^{120\,m+20}+78\,a^{2}b^{10}x^{2}x^{132\,m+22}+13\,ab^{11}xx^{144\,m+24}+b^{12}x^{156\,m+26}}{13\,(12\,m+1)x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x, algorithm="fricas")
```

```
[Out] 1/13*(13*a^12*x^12*x^(12*m + 2) + 78*a^11*b*x^11*x^(24*m + 4) + 286*a^10*b^
2*x^10*x^(36*m + 6) + 715*a^9*b^3*x^9*x^(48*m + 8) + 1287*a^8*b^4*x^8*x^(60
*m + 10) + 1716*a^7*b^5*x^7*x^(72*m + 12) + 1716*a^6*b^6*x^6*x^(84*m + 14)
+ 1287*a^5*b^7*x^5*x^(96*m + 16) + 715*a^4*b^8*x^4*x^(108*m + 18) + 286*a^3
*b^9*x^3*x^(120*m + 20) + 78*a^2*b^10*x^2*x^(132*m + 22) + 13*a*b^11*x*x^(1
44*m + 24) + b^12*x^(156*m + 26))/((12*m + 1)*x^13)
```

giac [**B**] time = 0.51, size = 285, normalized size = 10.56

$$\frac{13\,a^{12}x^{12}e^{\left(12\,m\log(x)+2\,\log(x)\right)}+78\,a^{11}bx^{11}e^{\left(24\,m\log(x)+4\,\log(x)\right)}+286\,a^{10}b^{2}x^{10}e^{\left(36\,m\log(x)+6\,\log(x)\right)}+715\,a^{9}b^{3}x^{9}e^{\left(48\,m\log(x)+8\,\log(x)\right)}+1287\,a^{8}b^{4}x^{8}e^{\left(60\,m\log(x)+10\,\log(x)\right)}+1716\,a^{7}b^{5}x^{7}e^{\left(72\,m\log(x)+12\,\log(x)\right)}+1716\,a^{6}b^{6}x^{6}e^{\left(84\,m\log(x)+14\,\log(x)\right)}+1287\,a^{5}b^{7}x^{5}e^{\left(96\,m\log(x)+16\,\log(x)\right)}+715\,a^{4}b^{8}x^{4}e^{\left(108\,m\log(x)+18\,\log(x)\right)}+286\,a^{3}b^{9}x^{3}e^{\left(120\,m\log(x)+20\,\log(x)\right)}+78\,a^{2}b^{10}x^{2}e^{\left(132\,m\log(x)+22\,\log(x)\right)}+13\,ab^{11}xe^{\left(144\,m\log(x)+24\,\log(x)\right)}+b^{12}e^{\left(156\,m\log(x)+26\,\log(x)\right)}}{13\left(12\,mx^{13}+x^{13}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x, algorithm="giac")
```

```
[Out] 1/13*(13*a^12*x^12*e^(12*m*log(x) + 2*log(x)) + 78*a^11*b*x^11*e^(24*m*log(
x) + 4*log(x)) + 286*a^10*b^2*x^10*e^(36*m*log(x) + 6*log(x)) + 715*a^9*b^3
*x^9*e^(48*m*log(x) + 8*log(x)) + 1287*a^8*b^4*x^8*e^(60*m*log(x) + 10*log(
x)) + 1716*a^7*b^5*x^7*e^(72*m*log(x) + 12*log(x)) + 1716*a^6*b^6*x^6*e^(84
*m*log(x) + 14*log(x)) + 1287*a^5*b^7*x^5*e^(96*m*log(x) + 16*log(x)) + 715
*a^4*b^8*x^4*e^(108*m*log(x) + 18*log(x)) + 286*a^3*b^9*x^3*e^(120*m*log(x)
 + 20*log(x)) + 78*a^2*b^10*x^2*e^(132*m*log(x) + 22*log(x)) + 13*a*b^11*x*
e^(144*m*log(x) + 24*log(x)) + b^12*e^(156*m*log(x) + 26*log(x)))/(12*m*x^1
3 + x^13)
```

maple [**B**] time = 0.11, size = 339, normalized size = 12.56

$$\frac{a^{12}x^{12m+2}}{(12m+1)\,x}+\frac{6a^{11}b\,x^{24m+4}}{(12m+1)\,x^{2}}+\frac{22a^{10}b^{2}x^{36m+6}}{(12m+1)\,x^{3}}+\frac{55a^{9}b^{3}x^{48m+8}}{(12m+1)\,x^{4}}+\frac{99a^{8}b^{4}x^{60m+10}}{(12m+1)\,x^{5}}+\frac{132a^{7}b^{5}x^{72m+12}}{(12m+1)\,x^{6}}+\frac{132a^{6}b^{6}x^{84m+14}}{(12m+1)\,x^{7}}+\frac{99a^{5}b^{7}x^{96m+16}}{(12m+1)\,x^{8}}+\frac{55a^{4}b^{8}x^{108m+18}}{(12m+1)\,x^{9}}+\frac{22a^{3}b^{9}x^{120m+20}}{(12m+1)\,x^{10}}+\frac{6a^{2}b^{10}x^{132m+22}}{(12m+1)\,x^{11}}+\frac{a\,b^{11}x^{144m+24}}{(12m+1)\,x^{12}}+\frac{b^{12}x^{156m+26}}{13\,(12m+1)\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(12*m-12)*(a*x+b*x^(2+12*m))^12,x)
```

```
[Out] 1/13/(12*m+1)*b^12/x^13*(x^(2+12*m))^13+1/(12*m+1)*a*b^11/x^12*(x^(2+12*m))
^12+6/(12*m+1)*a^2*b^10/x^11*(x^(2+12*m))^11+22/(12*m+1)*a^3*b^9/x^10*(x^(2
+12*m))^10+55/(12*m+1)*a^4*b^8/x^9*(x^(2+12*m))^9+99/(12*m+1)*a^5*b^7/x^8*(
x^(2+12*m))^8+132/(12*m+1)*a^6*b^6/x^7*(x^(2+12*m))^7+132/(12*m+1)*a^7*b^5/
x^6*(x^(2+12*m))^6+99/(12*m+1)*a^8*b^4/x^5*(x^(2+12*m))^5+55/(12*m+1)*a^9*b
^3/x^4*(x^(2+12*m))^4+22/(12*m+1)*a^10*b^2/x^3*(x^(2+12*m))^3+6/(12*m+1)*a^
11*b/x^2*(x^(2+12*m))^2+1/(12*m+1)*a^12/x*x^(2+12*m)
```

maxima [**B**] time = 1.43, size = 275, normalized size = 10.19

$$\frac{b^{12}x^{156\,m+13}}{13\,(12\,m+1)}+\frac{ab^{11}x^{144\,m+12}}{12\,m+1}+\frac{6\,a^{2}b^{10}x^{132\,m+11}}{12\,m+1}+\frac{22\,a^{3}b^{9}x^{120\,m+10}}{12\,m+1}+\frac{55\,a^{4}b^{8}x^{108\,m+9}}{12\,m+1}+\frac{99\,a^{5}b^{7}x^{96\,m+8}}{12\,m+1}+\frac{132\,a^{6}b^{6}x^{84\,m+7}}{12\,m+1}+\frac{132\,a^{7}b^{5}x^{72\,m+6}}{12\,m+1}+\frac{99\,a^{8}b^{4}x^{60\,m+5}}{12\,m+1}+\frac{55\,a^{9}b^{3}x^{48\,m+4}}{12\,m+1}+\frac{22\,a^{10}b^{2}x^{36\,m+3}}{12\,m+1}+\frac{6\,a^{11}bx^{24\,m+2}}{12\,m+1}+\frac{a^{12}x^{12\,m+1}}{12\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x, algorithm="maxima")
```

```
[Out] 1/13*b^12*x^(156*m + 13)/(12*m + 1) + a*b^11*x^(144*m + 12)/(12*m + 1) + 6*
a^2*b^10*x^(132*m + 11)/(12*m + 1) + 22*a^3*b^9*x^(120*m + 10)/(12*m + 1) +
 55*a^4*b^8*x^(108*m + 9)/(12*m + 1) + 99*a^5*b^7*x^(96*m + 8)/(12*m + 1) +
 132*a^6*b^6*x^(84*m + 7)/(12*m + 1) + 132*a^7*b^5*x^(72*m + 6)/(12*m + 1)
+ 99*a^8*b^4*x^(60*m + 5)/(12*m + 1) + 55*a^9*b^3*x^(48*m + 4)/(12*m + 1) +
 22*a^10*b^2*x^(36*m + 3)/(12*m + 1) + 6*a^11*b*x^(24*m + 2)/(12*m + 1) + a
^12*x^(12*m + 1)/(12*m + 1)
```

mupad [**B**] time = 5.87, size = 287, normalized size = 10.63

$$\frac{b^{12}\,x^{156\,m}\,x^{13}}{156\,m+13}+\frac{13\,a^{12}\,x\,x^{12\,m}}{156\,m+13}+\frac{78\,a^{11}\,b\,x^{24\,m}\,x^2}{156\,m+13}+\frac{13\,a\,b^{11}\,x^{144\,m}\,x^{12}}{156\,m+13}+\frac{286\,a^{10}\,b^2\,x^{36\,m}\,x^3}{156\,m+13}+\frac{715\,a^9\,b^3\,x^{48\,m}\,x^4}{156\,m+13}+\frac{1287\,a^8\,b^4\,x^{60\,m}\,x^5}{156\,m+13}+\frac{1716\,a^7\,b^5\,x^{72\,m}\,x^6}{156\,m+13}+\frac{1716\,a^6\,b^6\,x^{84\,m}\,x^7}{156\,m+13}+\frac{1287\,a^5\,b^7\,x^{96\,m}\,x^8}{156\,m+13}+\frac{715\,a^4\,b^8\,x^{108\,m}\,x^9}{156\,m+13}+\frac{286\,a^3\,b^9\,x^{120\,m}\,x^{10}}{156\,m+13}+\frac{78\,a^2\,b^{10}\,x^{132\,m}\,x^{11}}{156\,m+13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(12*m - 12)*(a*x + b*x^(12*m + 2))^12,x)
```

```
[Out] (b^12*x^(156*m)*x^13)/(156*m + 13) + (13*a^12*x*x^(12*m))/(156*m + 13) + (7
8*a^11*b*x^(24*m)*x^2)/(156*m + 13) + (13*a*b^11*x^(144*m)*x^12)/(156*m + 1
3) + (286*a^10*b^2*x^(36*m)*x^3)/(156*m + 13) + (715*a^9*b^3*x^(48*m)*x^4)/
(156*m + 13) + (1287*a^8*b^4*x^(60*m)*x^5)/(156*m + 13) + (1716*a^7*b^5*x^(
72*m)*x^6)/(156*m + 13) + (1716*a^6*b^6*x^(84*m)*x^7)/(156*m + 13) + (1287*
a^5*b^7*x^(96*m)*x^8)/(156*m + 13) + (715*a^4*b^8*x^(108*m)*x^9)/(156*m + 1
3) + (286*a^3*b^9*x^(120*m)*x^10)/(156*m + 13) + (78*a^2*b^10*x^(132*m)*x^1
1)/(156*m + 13)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-12+12*m)*(a*x+b*x**(2+12*m))**12,x)
```

```
[Out] Timed out
```

3.235 $\int \left(ax+bx^{14}\right)^{12}\,dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 261}

$$\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^14)^12,x]
```

```
[Out] (a + b*x^13)^13/(169*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(ax+bx^{14}\right)^{12}\,dx&=\int x^{12}\left(a+bx^{13}\right)^{12}\,dx\\&=\frac{\left(a+bx^{13}\right)^{13}}{169b}\end{aligned}$$

Mathematica [B] time = 0.00, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{13}}{13}+\frac{6}{13}a^{11}bx^{26}+\frac{22}{13}a^{10}b^2x^{39}+\frac{55}{13}a^9b^3x^{52}+\frac{99}{13}a^8b^4x^{65}+\frac{132}{13}a^7b^5x^{78}+\frac{132}{13}a^6b^6x^{91}+\frac{99}{13}a^5b^7x^{104}+\frac{55}{13}a^4b^8x^{117}+\frac{22}{13}a^3b^9x^{130}+\frac{6}{13}a^2b^{10}x^{143}+\frac{1}{13}ab^{11}x^{156}+\frac{b^{12}x^{169}}{169}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^14)^12,x]
```

```
[Out] (a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x
^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)
/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13
 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax+bx^{14}\right)^{12}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x + b*x^14)^12,x]
```

```
[Out] IntegrateAlgebraic[(a*x + b*x^14)^12, x]
```

fricas [B] time = 0.34, size = 134, normalized size = 8.38

$$\frac{1}{169}x^{169}b^{12} + \frac{1}{13}x^{156}b^{11}a + \frac{6}{13}x^{143}b^{10}a^2 + \frac{22}{13}x^{130}b^9a^3 + \frac{55}{13}x^{117}b^8a^4 + \frac{99}{13}x^{104}b^7a^5 + \frac{132}{13}x^{91}b^6a^6 + \frac{132}{13}x^{78}b^5a^7 + \frac{99}{13}x^{65}b^4a^8 + \frac{55}{13}x^{52}b^3a^9 + \frac{22}{13}x^{39}b^2a^{10} + \frac{6}{13}x^{26}ba^{11} + \frac{1}{13}x^{13}a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/169*x^169*b^12 + 1/13*x^156*b^11*a + 6/13*x^143*b^10*a^2 + 22/13*x^130*b^
9*a^3 + 55/13*x^117*b^8*a^4 + 99/13*x^104*b^7*a^5 + 132/13*x^91*b^6*a^6 + 1
32/13*x^78*b^5*a^7 + 99/13*x^65*b^4*a^8 + 55/13*x^52*b^3*a^9 + 22/13*x^39*b
^2*a^10 + 6/13*x^26*b*a^11 + 1/13*x^13*a^12
```

giac [B] time = 0.17, size = 134, normalized size = 8.38

$$\frac{1}{169}b^{12}x^{169} + \frac{1}{13}ab^{11}x^{156} + \frac{6}{13}a^2b^{10}x^{143} + \frac{22}{13}a^3b^9x^{130} + \frac{55}{13}a^4b^8x^{117} + \frac{99}{13}a^5b^7x^{104} + \frac{132}{13}a^6b^6x^{91} + \frac{132}{13}a^7b^5x^{78} + \frac{99}{13}a^8b^4x^{65} + \frac{55}{13}a^9b^3x^{52} + \frac{22}{13}a^{10}b^2x^{39} + \frac{6}{13}a^{11}bx^{26} + \frac{1}{13}a^{12}x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

maple [B] time = 0.05, size = 135, normalized size = 8.44

$$\frac{1}{169}b^{12}x^{169} + \frac{1}{13}a\,b^{11}x^{156} + \frac{6}{13}a^2b^{10}x^{143} + \frac{22}{13}a^3b^9x^{130} + \frac{55}{13}a^4b^8x^{117} + \frac{99}{13}a^5b^7x^{104} + \frac{132}{13}a^6b^6x^{91} + \frac{132}{13}a^7b^5x^{78} + \frac{99}{13}a^8b^4x^{65} + \frac{55}{13}a^9b^3x^{52} + \frac{22}{13}a^{10}b^2x^{39} + \frac{6}{13}a^{11}b\,x^{26} + \frac{1}{13}a^{12}x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^14+a*x)^12,x)
```

```
[Out] 1/169*b^12*x^169+1/13*a*b^11*x^156+6/13*a^2*b^10*x^143+22/13*a^3*b^9*x^130+
55/13*a^4*b^8*x^117+99/13*a^5*b^7*x^104+132/13*a^6*b^6*x^91+132/13*a^7*b^5*
x^78+99/13*a^8*b^4*x^65+55/13*a^9*b^3*x^52+22/13*a^10*b^2*x^39+6/13*a^11*b*
x^26+1/13*a^12*x^13
```

maxima [B] time = 1.33, size = 134, normalized size = 8.38

$$\frac{1}{169}b^{12}x^{169} + \frac{1}{13}ab^{11}x^{156} + \frac{6}{13}a^2b^{10}x^{143} + \frac{22}{13}a^3b^9x^{130} + \frac{55}{13}a^4b^8x^{117} + \frac{99}{13}a^5b^7x^{104} + \frac{132}{13}a^6b^6x^{91} + \frac{132}{13}a^7b^5x^{78} + \frac{99}{13}a^8b^4x^{65} + \frac{55}{13}a^9b^3x^{52} + \frac{22}{13}a^{10}b^2x^{39} + \frac{6}{13}a^{11}bx^{26} + \frac{1}{13}a^{12}x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

mupad [B] time = 0.00, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{13}}{13} + \frac{6\,a^{11}\,b\,x^{26}}{13} + \frac{22\,a^{10}\,b^2\,x^{39}}{13} + \frac{55\,a^9\,b^3\,x^{52}}{13} + \frac{99\,a^8\,b^4\,x^{65}}{13} + \frac{132\,a^7\,b^5\,x^{78}}{13} + \frac{132\,a^6\,b^6\,x^{91}}{13} + \frac{99\,a^5\,b^7\,x^{104}}{13} + \frac{55\,a^4\,b^8\,x^{117}}{13} + \frac{22\,a^3\,b^9\,x^{130}}{13} + \frac{6\,a^2\,b^{10}\,x^{143}}{13} + \frac{a\,b^{11}\,x^{156}}{13} + \frac{b^{12}\,x^{169}}{169}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^14)^12,x)
```

```
[Out] (a^12*x^13)/13 + (b^12*x^169)/169 + (6*a^11*b*x^26)/13 + (a*b^11*x^156)/13
+ (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x^52)/13 + (99*a^8*b^4*x^65)/13 + (13
2*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)/13 + (99*a^5*b^7*x^104)/13 + (55*a^
4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13 + (6*a^2*b^10*x^143)/13
```

sympy [**B**] time = 0.13, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{13}}{13}+\frac{6a^{11}bx^{26}}{13}+\frac{22a^{10}b^{2}x^{39}}{13}+\frac{55a^{9}b^{3}x^{52}}{13}+\frac{99a^{8}b^{4}x^{65}}{13}+\frac{132a^{7}b^{5}x^{78}}{13}+\frac{132a^{6}b^{6}x^{91}}{13}+\frac{99a^{5}b^{7}x^{104}}{13}+\frac{55a^{4}b^{8}x^{117}}{13}+\frac{22a^{3}b^{9}x^{130}}{13}+\frac{6a^{2}b^{10}x^{143}}{13}+\frac{ab^{11}x^{156}}{13}+\frac{b^{12}x^{169}}{169}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**14+a*x)**12,x)
```

```
[Out] a**12*x**13/13 + 6*a**11*b*x**26/13 + 22*a**10*b**2*x**39/13 + 55*a**9*b**3
*x**52/13 + 99*a**8*b**4*x**65/13 + 132*a**7*b**5*x**78/13 + 132*a**6*b**6*
x**91/13 + 99*a**5*b**7*x**104/13 + 55*a**4*b**8*x**117/13 + 22*a**3*b**9*x
**130/13 + 6*a**2*b**10*x**143/13 + a*b**11*x**156/13 + b**12*x**169/169
```

3.236 $\int (ax^2+bx^{27})^{12}\,dx$

Optimal. Leaf size=16

$$\frac{(a+bx^{25})^{13}}{325b}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 261}

$$\frac{(a+bx^{25})^{13}}{325b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^27)^12,x]
```

```
[Out] (a + b*x^25)^13/(325*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int (ax^2+bx^{27})^{12}\,dx &= \int x^{24}(a+bx^{25})^{12}\,dx\\ &= \frac{(a+bx^{25})^{13}}{325b}\end{aligned}$$

Mathematica [B] time = 0.01, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25}+\frac{6}{25}a^{11}bx^{50}+\frac{22}{25}a^{10}b^2x^{75}+\frac{11}{5}a^9b^3x^{100}+\frac{99}{25}a^8b^4x^{125}+\frac{132}{25}a^7b^5x^{150}+\frac{132}{25}a^6b^6x^{175}+\frac{99}{25}a^5b^7x^{200}+\frac{11}{5}a^4b^8x^{225}+\frac{22}{25}a^3b^9x^{250}+\frac{6}{25}a^2b^{10}x^{275}+\frac{1}{25}ab^{11}x^{300}+\frac{b^{12}x^{325}}{325}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^27)^12,x]
```

```
[Out] (a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x
^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^1
75)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/
25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ax^2+bx^{27})^{12}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x^27)^12,x]
```

```
[Out] IntegrateAlgebraic[(a*x^2 + b*x^27)^12, x]
```

fricas [B] time = 0.34, size = 134, normalized size = 8.38

$$\frac{1}{325}x^{325}b^{12}+\frac{1}{25}x^{300}b^{11}a+\frac{6}{25}x^{275}b^{10}a^{2}+\frac{22}{25}x^{250}b^{9}a^{3}+\frac{11}{5}x^{225}b^{8}a^{4}+\frac{99}{25}x^{200}b^{7}a^{5}+\frac{132}{25}x^{175}b^{6}a^{6}+\frac{132}{25}x^{150}b^{5}a^{7}+\frac{99}{25}x^{125}b^{4}a^{8}+\frac{11}{5}x^{100}b^{3}a^{9}+\frac{22}{25}x^{75}b^{2}a^{10}+\frac{6}{25}x^{50}ba^{11}+\frac{1}{25}x^{25}a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^27+a*x^2)^12,x, algorithm="fricas")
```

```
[Out] 1/325*x^325*b^12 + 1/25*x^300*b^11*a + 6/25*x^275*b^10*a^2 + 22/25*x^250*b^
9*a^3 + 11/5*x^225*b^8*a^4 + 99/25*x^200*b^7*a^5 + 132/25*x^175*b^6*a^6 + 1
32/25*x^150*b^5*a^7 + 99/25*x^125*b^4*a^8 + 11/5*x^100*b^3*a^9 + 22/25*x^75
*b^2*a^10 + 6/25*x^50*b*a^11 + 1/25*x^25*a^12
```

giac [B] time = 0.15, size = 134, normalized size = 8.38

$$\frac{1}{325}b^{12}x^{325}+\frac{1}{25}ab^{11}x^{300}+\frac{6}{25}a^{2}b^{10}x^{275}+\frac{22}{25}a^{3}b^{9}x^{250}+\frac{11}{5}a^{4}b^{8}x^{225}+\frac{99}{25}a^{5}b^{7}x^{200}+\frac{132}{25}a^{6}b^{6}x^{175}+\frac{132}{25}a^{7}b^{5}x^{150}+\frac{99}{25}a^{8}b^{4}x^{125}+\frac{11}{5}a^{9}b^{3}x^{100}+\frac{22}{25}a^{10}b^{2}x^{75}+\frac{6}{25}a^{11}bx^{50}+\frac{1}{25}a^{12}x^{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^27+a*x^2)^12,x, algorithm="giac")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

maple [B] time = 0.04, size = 135, normalized size = 8.44

$$\frac{1}{325}b^{12}x^{325}+\frac{1}{25}a\,b^{11}x^{300}+\frac{6}{25}a^{2}b^{10}x^{275}+\frac{22}{25}a^{3}b^{9}x^{250}+\frac{11}{5}a^{4}b^{8}x^{225}+\frac{99}{25}a^{5}b^{7}x^{200}+\frac{132}{25}a^{6}b^{6}x^{175}+\frac{132}{25}a^{7}b^{5}x^{150}+\frac{99}{25}a^{8}b^{4}x^{125}+\frac{11}{5}a^{9}b^{3}x^{100}+\frac{22}{25}a^{10}b^{2}x^{75}+\frac{6}{25}a^{11}b\,x^{50}+\frac{1}{25}a^{12}x^{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^27+a*x^2)^12,x)
```

```
[Out] 1/325*b^12*x^325+1/25*a*b^11*x^300+6/25*a^2*b^10*x^275+22/25*a^3*b^9*x^250+
11/5*a^4*b^8*x^225+99/25*a^5*b^7*x^200+132/25*a^6*b^6*x^175+132/25*a^7*b^5*
x^150+99/25*a^8*b^4*x^125+11/5*a^9*b^3*x^100+22/25*a^10*b^2*x^75+6/25*a^11*
b*x^50+1/25*a^12*x^25
```

maxima [B] time = 1.34, size = 134, normalized size = 8.38

$$\frac{1}{325}b^{12}x^{325}+\frac{1}{25}ab^{11}x^{300}+\frac{6}{25}a^{2}b^{10}x^{275}+\frac{22}{25}a^{3}b^{9}x^{250}+\frac{11}{5}a^{4}b^{8}x^{225}+\frac{99}{25}a^{5}b^{7}x^{200}+\frac{132}{25}a^{6}b^{6}x^{175}+\frac{132}{25}a^{7}b^{5}x^{150}+\frac{99}{25}a^{8}b^{4}x^{125}+\frac{11}{5}a^{9}b^{3}x^{100}+\frac{22}{25}a^{10}b^{2}x^{75}+\frac{6}{25}a^{11}bx^{50}+\frac{1}{25}a^{12}x^{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^27+a*x^2)^12,x, algorithm="maxima")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

mupad [B] time = 5.19, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{25}}{25}+\frac{6\,a^{11}\,b\,x^{50}}{25}+\frac{22\,a^{10}\,b^2\,x^{75}}{25}+\frac{11\,a^9\,b^3\,x^{100}}{5}+\frac{99\,a^8\,b^4\,x^{125}}{25}+\frac{132\,a^7\,b^5\,x^{150}}{25}+\frac{132\,a^6\,b^6\,x^{175}}{25}+\frac{99\,a^5\,b^7\,x^{200}}{25}+\frac{11\,a^4\,b^8\,x^{225}}{5}+\frac{22\,a^3\,b^9\,x^{250}}{25}+\frac{6\,a^2\,b^{10}\,x^{275}}{25}+\frac{a\,b^{11}\,x^{300}}{25}+\frac{b^{12}\,x^{325}}{325}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^27)^12,x)
```

```
[Out] (a^12*x^25)/25 + (b^12*x^325)/325 + (6*a^11*b*x^50)/25 + (a*b^11*x^300)/25
+ (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x^100)/5 + (99*a^8*b^4*x^125)/25 + (1
32*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^175)/25 + (99*a^5*b^7*x^200)/25 + (11
*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/25 + (6*a^2*b^10*x^275)/25
```

sympy [B] time = 0.13, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25} + \frac{6a^{11}bx^{50}}{25} + \frac{22a^{10}b^{2}x^{75}}{25} + \frac{11a^{9}b^{3}x^{100}}{5} + \frac{99a^{8}b^{4}x^{125}}{25} + \frac{132a^{7}b^{5}x^{150}}{25} + \frac{132a^{6}b^{6}x^{175}}{25} + \frac{99a^{5}b^{7}x^{200}}{25} + \frac{11a^{4}b^{8}x^{225}}{5} + \frac{22a^{3}b^{9}x^{250}}{25} + \frac{6a^{2}b^{10}x^{275}}{25} + \frac{ab^{11}x^{300}}{25} + \frac{b^{12}x^{325}}{325}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**27+a*x**2)**12,x)
```

```
[Out] a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**3
*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b**
6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b**9
*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/325
```

3.237 $\int \left(ax^3+bx^{40}\right)^{12}\,dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 261}

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^40)^12,x]
```

```
[Out] (a + b*x^37)^13/(481*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(ax^3+bx^{40}\right)^{12}\,dx &= \int x^{36}\left(a+bx^{37}\right)^{12}\,dx\\ &= \frac{\left(a+bx^{37}\right)^{13}}{481b}\end{aligned}$$

Mathematica [B] time = 0.01, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{333}+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^2b^{10}x^{407}+\frac{1}{37}ab^{11}x^{444}+\frac{b^{12}x^{481}}{481}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^40)^12,x]
```

```
[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^3+bx^{40}\right)^{12}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x^3 + b*x^40)^12,x]
```

```
[Out] IntegrateAlgebraic[(a*x^3 + b*x^40)^12, x]
```

fricas [B] time = 0.34, size = 134, normalized size = 8.38

$$\frac{1}{481}x^{481}b^{12}+\frac{1}{37}x^{444}b^{11}a+\frac{6}{37}x^{407}b^{10}a^2+\frac{22}{37}x^{370}b^9a^3+\frac{55}{37}x^{333}b^8a^4+\frac{99}{37}x^{296}b^7a^5+\frac{132}{37}x^{259}b^6a^6+\frac{132}{37}x^{222}b^5a^7+\frac{99}{37}x^{185}b^4a^8+\frac{55}{37}x^{148}b^3a^9+\frac{22}{37}x^{111}b^2a^{10}+\frac{6}{37}x^{74}ba^{11}+\frac{1}{37}x^{37}a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^40+a*x^3)^12,x, algorithm="fricas")
```

```
[Out] 1/481*x^481*b^12 + 1/37*x^444*b^11*a + 6/37*x^407*b^10*a^2 + 22/37*x^370*b^
9*a^3 + 55/37*x^333*b^8*a^4 + 99/37*x^296*b^7*a^5 + 132/37*x^259*b^6*a^6 +
132/37*x^222*b^5*a^7 + 99/37*x^185*b^4*a^8 + 55/37*x^148*b^3*a^9 + 22/37*x^
111*b^2*a^10 + 6/37*x^74*b*a^11 + 1/37*x^37*a^12
```

giac [B] time = 0.16, size = 134, normalized size = 8.38

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}ab^{11}x^{444}+\frac{6}{37}a^2b^{10}x^{407}+\frac{22}{37}a^3b^9x^{370}+\frac{55}{37}a^4b^8x^{333}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^6b^6x^{259}+\frac{132}{37}a^7b^5x^{222}+\frac{99}{37}a^8b^4x^{185}+\frac{55}{37}a^9b^3x^{148}+\frac{22}{37}a^{10}b^2x^{111}+\frac{6}{37}a^{11}bx^{74}+\frac{1}{37}a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^40+a*x^3)^12,x, algorithm="giac")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

maple [B] time = 0.04, size = 135, normalized size = 8.44

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}a\,b^{11}x^{444}+\frac{6}{37}a^2b^{10}x^{407}+\frac{22}{37}a^3b^9x^{370}+\frac{55}{37}a^4b^8x^{333}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^6b^6x^{259}+\frac{132}{37}a^7b^5x^{222}+\frac{99}{37}a^8b^4x^{185}+\frac{55}{37}a^9b^3x^{148}+\frac{22}{37}a^{10}b^2x^{111}+\frac{6}{37}a^{11}b\,x^{74}+\frac{1}{37}a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^40+a*x^3)^12,x)
```

```
[Out] 1/481*b^12*x^481+1/37*a*b^11*x^444+6/37*a^2*b^10*x^407+22/37*a^3*b^9*x^370+
55/37*a^4*b^8*x^333+99/37*a^5*b^7*x^296+132/37*a^6*b^6*x^259+132/37*a^7*b^5
*x^222+99/37*a^8*b^4*x^185+55/37*a^9*b^3*x^148+22/37*a^10*b^2*x^111+6/37*a^
11*b*x^74+1/37*a^12*x^37
```

maxima [B] time = 1.35, size = 134, normalized size = 8.38

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}ab^{11}x^{444}+\frac{6}{37}a^2b^{10}x^{407}+\frac{22}{37}a^3b^9x^{370}+\frac{55}{37}a^4b^8x^{333}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^6b^6x^{259}+\frac{132}{37}a^7b^5x^{222}+\frac{99}{37}a^8b^4x^{185}+\frac{55}{37}a^9b^3x^{148}+\frac{22}{37}a^{10}b^2x^{111}+\frac{6}{37}a^{11}bx^{74}+\frac{1}{37}a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^40+a*x^3)^12,x, algorithm="maxima")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

mupad [B] time = 5.16, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{37}}{37}+\frac{6\,a^{11}\,b\,x^{74}}{37}+\frac{22\,a^{10}\,b^2\,x^{111}}{37}+\frac{55\,a^9\,b^3\,x^{148}}{37}+\frac{99\,a^8\,b^4\,x^{185}}{37}+\frac{132\,a^7\,b^5\,x^{222}}{37}+\frac{132\,a^6\,b^6\,x^{259}}{37}+\frac{99\,a^5\,b^7\,x^{296}}{37}+\frac{55\,a^4\,b^8\,x^{333}}{37}+\frac{22\,a^3\,b^9\,x^{370}}{37}+\frac{6\,a^2\,b^{10}\,x^{407}}{37}+\frac{a\,b^{11}\,x^{444}}{37}+\frac{b^{12}\,x^{481}}{481}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3 + b*x^40)^12,x)
```

```
[Out] (a^12*x^37)/37 + (b^12*x^481)/481 + (6*a^11*b*x^74)/37 + (a*b^11*x^444)/37
+ (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37 +
(132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37 + (
55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37
```

sympy [B] time = 0.13, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37}+\frac{6a^{11}bx^{74}}{37}+\frac{22a^{10}b^2x^{111}}{37}+\frac{55a^9b^3x^{148}}{37}+\frac{99a^8b^4x^{185}}{37}+\frac{132a^7b^5x^{222}}{37}+\frac{132a^6b^6x^{259}}{37}+\frac{99a^5b^7x^{296}}{37}+\frac{55a^4b^8x^{333}}{37}+\frac{22a^3b^9x^{370}}{37}+\frac{6a^2b^{10}x^{407}}{37}+\frac{ab^{11}x^{444}}{37}+\frac{b^{12}x^{481}}{481}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**40+a*x**3)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

3.238 $\int \left(ax^m + bx^{1+13m}\right)^{12} dx$

Optimal. Leaf size=27

$$\frac{\left(a + bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1593, 261}

$$\frac{\left(a + bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m + b*x^(1 + 13*m))^12,x]
```

```
[Out] (a + b*x^(1 + 12*m))^13/(13*b*(1 + 12*m))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \left(ax^m + bx^{1+13m}\right)^{12} dx &= \int x^{12m} \left(a + bx^{1+12m}\right)^{12} dx \\ &= \frac{\left(a + bx^{1+12m}\right)^{13}}{13b(1+12m)} \end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 0.89

$$\frac{\left(a + bx^{12m+1}\right)^{13}}{156bm + 13b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m + b*x^(1 + 13*m))^12,x]
```

```
[Out] (a + b*x^(1 + 12*m))^13/(13*b + 156*b*m)
```

IntegrateAlgebraic [B] time = 0.06, size = 196, normalized size = 7.26

$$\frac{x^{12m+1} \left(13a^{12} + 78a^{11}bx^{12m+1} + 286a^{10}b^2x^{24m+2} + 715a^9b^3x^{36m+3} + 1287a^8b^4x^{48m+4} + 1716a^7b^5x^{60m+5} + 1716a^6b^6x^{72m+6} + 1287a^5b^7x^{84m+7} + 715a^4b^8x^{96m+8} + 286a^3b^9x^{108m+9} + 78a^2b^{10}x^{120m+10} + 13ab^{11}x^{132m+11} + b^{12}x^{144m+12}\right)}{13(12m+1)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x^m + b*x^(1 + 13*m))^12,x]
```

```
[Out] (x^(1 + 12*m)*(13*a^12 + 78*a^11*b*x^(1 + 12*m) + 286*a^10*b^2*x^(2 + 24*m)
 + 715*a^9*b^3*x^(3 + 36*m) + 1287*a^8*b^4*x^(4 + 48*m) + 1716*a^7*b^5*x^(5
 + 60*m) + 1716*a^6*b^6*x^(6 + 72*m) + 1287*a^5*b^7*x^(7 + 84*m) + 715*a^4*
b^8*x^(8 + 96*m) + 286*a^3*b^9*x^(9 + 108*m) + 78*a^2*b^10*x^(10 + 120*m) +
 13*a*b^11*x^(11 + 132*m) + b^12*x^(12 + 144*m)))/(13*(1 + 12*m))
```

fricas [B] time = 0.42, size = 205, normalized size = 7.59

$$\frac{b^{12}x^{13}x^{156\,m}+13\,ab^{11}x^{12}x^{144\,m}+78\,a^{2}b^{10}x^{11}x^{132\,m}+286\,a^{3}b^{9}x^{10}x^{120\,m}+715\,a^{4}b^{8}x^{9}x^{108\,m}+1287\,a^{5}b^{7}x^{8}x^{96\,m}+1716\,a^{6}b^{6}x^{7}x^{84\,m}+1716\,a^{7}b^{5}x^{6}x^{72\,m}+1287\,a^{8}b^{4}x^{5}x^{60\,m}+715\,a^{9}b^{3}x^{4}x^{48\,m}+286\,a^{10}b^{2}x^{3}x^{36\,m}+78\,a^{11}bx^{2}x^{24\,m}+13\,a^{12}xx^{12\,m}}{13\,(12\,m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(1+13*m))^12,x, algorithm="fricas")
```

```
[Out] 1/13*(b^12*x^13*x^(156*m) + 13*a*b^11*x^12*x^(144*m) + 78*a^2*b^10*x^11*x^(
132*m) + 286*a^3*b^9*x^10*x^(120*m) + 715*a^4*b^8*x^9*x^(108*m) + 1287*a^5*
b^7*x^8*x^(96*m) + 1716*a^6*b^6*x^7*x^(84*m) + 1716*a^7*b^5*x^6*x^(72*m) +
1287*a^8*b^4*x^5*x^(60*m) + 715*a^9*b^3*x^4*x^(48*m) + 286*a^10*b^2*x^3*x^(
36*m) + 78*a^11*b*x^2*x^(24*m) + 13*a^12*x*x^(12*m))/(12*m + 1)
```

giac [B] time = 0.27, size = 205, normalized size = 7.59

$$\frac{b^{12}x^{13}x^{156\,m}+13\,ab^{11}x^{12}x^{144\,m}+78\,a^{2}b^{10}x^{11}x^{132\,m}+286\,a^{3}b^{9}x^{10}x^{120\,m}+715\,a^{4}b^{8}x^{9}x^{108\,m}+1287\,a^{5}b^{7}x^{8}x^{96\,m}+1716\,a^{6}b^{6}x^{7}x^{84\,m}+1716\,a^{7}b^{5}x^{6}x^{72\,m}+1287\,a^{8}b^{4}x^{5}x^{60\,m}+715\,a^{9}b^{3}x^{4}x^{48\,m}+286\,a^{10}b^{2}x^{3}x^{36\,m}+78\,a^{11}bx^{2}x^{24\,m}+13\,a^{12}xx^{12\,m}}{13\,(12\,m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(1+13*m))^12,x, algorithm="giac")
```

```
[Out] 1/13*(b^12*x^13*x^(156*m) + 13*a*b^11*x^12*x^(144*m) + 78*a^2*b^10*x^11*x^(
132*m) + 286*a^3*b^9*x^10*x^(120*m) + 715*a^4*b^8*x^9*x^(108*m) + 1287*a^5*
b^7*x^8*x^(96*m) + 1716*a^6*b^6*x^7*x^(84*m) + 1716*a^7*b^5*x^6*x^(72*m) +
1287*a^8*b^4*x^5*x^(60*m) + 715*a^9*b^3*x^4*x^(48*m) + 286*a^10*b^2*x^3*x^(
36*m) + 78*a^11*b*x^2*x^(24*m) + 13*a^12*x*x^(12*m))/(12*m + 1)
```

maple [B] time = 0.08, size = 287, normalized size = 10.63

$$\frac{b^{12}x^{13}x^{156m}}{13+156m}+\frac{a\,b^{11}x^{12}x^{144m}}{12m+1}+\frac{6a^{2}b^{10}x^{11}x^{132m}}{12m+1}+\frac{22a^{3}b^{9}x^{10}x^{120m}}{12m+1}+\frac{55a^{4}b^{8}x^{9}x^{108m}}{12m+1}+\frac{99a^{5}b^{7}x^{8}x^{96m}}{12m+1}+\frac{132a^{6}b^{6}x^{7}x^{84m}}{12m+1}+\frac{132a^{7}b^{5}x^{6}x^{72m}}{12m+1}+\frac{99a^{8}b^{4}x^{5}x^{60m}}{12m+1}+\frac{55a^{9}b^{3}x^{4}x^{48m}}{12m+1}+\frac{22a^{10}b^{2}x^{3}x^{36m}}{12m+1}+\frac{6a^{11}b\,x^{2}x^{24m}}{12m+1}+\frac{a^{12}x\,x^{12m}}{12m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m+b*x^(1+13*m))^12,x)
```

```
[Out] 1/13*b^12*x^13/(12*m+1)*(x^m)^156+a*b^11*x^12/(12*m+1)*(x^m)^144+6*a^2*b^10
*x^11/(12*m+1)*(x^m)^132+22*a^3*b^9*x^10/(12*m+1)*(x^m)^120+55*a^4*b^8*x^9/
(12*m+1)*(x^m)^108+99*a^5*b^7*x^8/(12*m+1)*(x^m)^96+132*a^6*b^6*x^7/(12*m+1
)*(x^m)^84+132*a^7*b^5*x^6/(12*m+1)*(x^m)^72+99*a^8*b^4*x^5/(12*m+1)*(x^m)^
60+55*a^9*b^3*x^4/(12*m+1)*(x^m)^48+22*a^10*b^2*x^3/(12*m+1)*(x^m)^36+6*a^1
1*b*x^2/(12*m+1)*(x^m)^24+a^12/(12*m+1)*x*(x^m)^12
```

maxima [B] time = 1.48, size = 275, normalized size = 10.19

$$\frac{b^{12}x^{156\,m+13}}{13\,(12\,m+1)}+\frac{ab^{11}x^{144\,m+12}}{12\,m+1}+\frac{6\,a^{2}b^{10}x^{132\,m+11}}{12\,m+1}+\frac{22\,a^{3}b^{9}x^{120\,m+10}}{12\,m+1}+\frac{55\,a^{4}b^{8}x^{108\,m+9}}{12\,m+1}+\frac{99\,a^{5}b^{7}x^{96\,m+8}}{12\,m+1}+\frac{132\,a^{6}b^{6}x^{84\,m+7}}{12\,m+1}+\frac{132\,a^{7}b^{5}x^{72\,m+6}}{12\,m+1}+\frac{99\,a^{8}b^{4}x^{60\,m+5}}{12\,m+1}+\frac{55\,a^{9}b^{3}x^{48\,m+4}}{12\,m+1}+\frac{22\,a^{10}b^{2}x^{36\,m+3}}{12\,m+1}+\frac{6\,a^{11}bx^{24\,m+2}}{12\,m+1}+\frac{a^{12}x^{12\,m+1}}{12\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(1+13*m))^12,x, algorithm="maxima")
```

```
[Out] 1/13*b^12*x^(156*m + 13)/(12*m + 1) + a*b^11*x^(144*m + 12)/(12*m + 1) + 6*
a^2*b^10*x^(132*m + 11)/(12*m + 1) + 22*a^3*b^9*x^(120*m + 10)/(12*m + 1) +
 55*a^4*b^8*x^(108*m + 9)/(12*m + 1) + 99*a^5*b^7*x^(96*m + 8)/(12*m + 1) +
 132*a^6*b^6*x^(84*m + 7)/(12*m + 1) + 132*a^7*b^5*x^(72*m + 6)/(12*m + 1)
+ 99*a^8*b^4*x^(60*m + 5)/(12*m + 1) + 55*a^9*b^3*x^(48*m + 4)/(12*m + 1) +
```

```
 22*a^10*b^2*x^(36*m + 3)/(12*m + 1) + 6*a^11*b*x^(24*m + 2)/(12*m + 1) + a
^12*x^(12*m + 1)/(12*m + 1)
```

mupad [**B**] time = 5.95, size = 285, normalized size = 10.56

$$\frac{b^{12}\,x^{156\,m}\,x^{13}}{156\,m+13}+\frac{a^{12}\,x\,x^{12\,m}}{12\,m+1}+\frac{6\,a^{11}\,b\,x^{24\,m}\,x^2}{12\,m+1}+\frac{a\,b^{11}\,x^{144\,m}\,x^{12}}{12\,m+1}+\frac{22\,a^{10}\,b^2\,x^{36\,m}\,x^3}{12\,m+1}+\frac{55\,a^9\,b^3\,x^{48\,m}\,x^4}{12\,m+1}+\frac{99\,a^8\,b^4\,x^{60\,m}\,x^5}{12\,m+1}+\frac{132\,a^7\,b^5\,x^{72\,m}\,x^6}{12\,m+1}+\frac{132\,a^6\,b^6\,x^{84\,m}\,x^7}{12\,m+1}+\frac{99\,a^5\,b^7\,x^{96\,m}\,x^8}{12\,m+1}+\frac{55\,a^4\,b^8\,x^{108\,m}\,x^9}{12\,m+1}+\frac{22\,a^3\,b^9\,x^{120\,m}\,x^{10}}{12\,m+1}+\frac{6\,a^2\,b^{10}\,x^{132\,m}\,x^{11}}{12\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m + b*x^(13*m + 1))^12,x)
```

```
[Out] (b^12*x^(156*m)*x^13)/(156*m + 13) + (a^12*x*x^(12*m))/(12*m + 1) + (6*a^11
*b*x^(24*m)*x^2)/(12*m + 1) + (a*b^11*x^(144*m)*x^12)/(12*m + 1) + (22*a^10
*b^2*x^(36*m)*x^3)/(12*m + 1) + (55*a^9*b^3*x^(48*m)*x^4)/(12*m + 1) + (99*
a^8*b^4*x^(60*m)*x^5)/(12*m + 1) + (132*a^7*b^5*x^(72*m)*x^6)/(12*m + 1) +
(132*a^6*b^6*x^(84*m)*x^7)/(12*m + 1) + (99*a^5*b^7*x^(96*m)*x^8)/(12*m + 1
) + (55*a^4*b^8*x^(108*m)*x^9)/(12*m + 1) + (22*a^3*b^9*x^(120*m)*x^10)/(12
*m + 1) + (6*a^2*b^10*x^(132*m)*x^11)/(12*m + 1)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m+b*x**(1+13*m))**12,x)
```

```
[Out] Timed out
```

3.239 $\int \left(ax^m+bx^{1+6m}\right)^5\,dx$

Optimal. Leaf size=27

$$\frac{\left(a+bx^{5m+1}\right)^6}{6b(5m+1)}$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1593, 261}

$$\frac{\left(a+bx^{5m+1}\right)^6}{6b(5m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m + b*x^(1 + 6*m))^5,x]
[Out] (a + b*x^(1 + 5*m))^6/(6*b*(1 + 5*m))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(ax^m+bx^{1+6m}\right)^5\,dx &= \int x^{5m}\left(a+bx^{1+5m}\right)^5\,dx\\ &= \frac{\left(a+bx^{1+5m}\right)^6}{6b(1+5m)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{\left(a+bx^{5m+1}\right)^6}{6b(5m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m + b*x^(1 + 6*m))^5,x]
[Out] (a + b*x^(1 + 5*m))^6/(6*b*(1 + 5*m))
```

IntegrateAlgebraic [B] time = 0.05, size = 91, normalized size = 3.37

$$\frac{x^{5m+1}\left(6a^5+15a^4bx^{5m+1}+20a^3b^2x^{10m+2}+15a^2b^3x^{15m+3}+6ab^4x^{20m+4}+b^5x^{25m+5}\right)}{6(5m+1)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x^m + b*x^(1 + 6*m))^5,x]
```

```
[Out] (x^(1 + 5*m)*(6*a^5 + 15*a^4*b*x^(1 + 5*m) + 20*a^3*b^2*x^(2 + 10*m) + 15*a
^2*b^3*x^(3 + 15*m) + 6*a*b^4*x^(4 + 20*m) + b^5*x^(5 + 25*m)))/(6*(1 + 5*m
))
```

fricas [B] time = 0.41, size = 93, normalized size = 3.44

$$\frac{b^5x^6x^{30\,m} + 6\,ab^4x^5x^{25\,m} + 15\,a^2b^3x^4x^{20\,m} + 20\,a^3b^2x^3x^{15\,m} + 15\,a^4bx^2x^{10\,m} + 6\,a^5xx^{5\,m}}{6\,(5\,m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(1+6*m))^5,x, algorithm="fricas")
```

```
[Out] 1/6*(b^5*x^6*x^(30*m) + 6*a*b^4*x^5*x^(25*m) + 15*a^2*b^3*x^4*x^(20*m) + 20
*a^3*b^2*x^3*x^(15*m) + 15*a^4*b*x^2*x^(10*m) + 6*a^5*x*x^(5*m))/(5*m + 1)
```

giac [B] time = 0.20, size = 93, normalized size = 3.44

$$\frac{b^5x^6x^{30\,m} + 6\,ab^4x^5x^{25\,m} + 15\,a^2b^3x^4x^{20\,m} + 20\,a^3b^2x^3x^{15\,m} + 15\,a^4bx^2x^{10\,m} + 6\,a^5xx^{5\,m}}{6\,(5\,m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(1+6*m))^5,x, algorithm="giac")
```

```
[Out] 1/6*(b^5*x^6*x^(30*m) + 6*a*b^4*x^5*x^(25*m) + 15*a^2*b^3*x^4*x^(20*m) + 20
*a^3*b^2*x^3*x^(15*m) + 15*a^4*b*x^2*x^(10*m) + 6*a^5*x*x^(5*m))/(5*m + 1)
```

maple [B] time = 0.06, size = 126, normalized size = 4.67

$$\frac{b^5x^6x^{30m}}{6+30m} + \frac{a\,b^4x^5x^{25m}}{5m+1} + \frac{5a^2b^3x^4x^{20m}}{2\,(5m+1)} + \frac{10a^3b^2x^3x^{15m}}{3\,(5m+1)} + \frac{5a^4b\,x^2x^{10m}}{2\,(5m+1)} + \frac{a^5x\,x^{5m}}{5m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m+b*x^(1+6*m))^5,x)
```

```
[Out] 1/6*b^5*x^6/(1+5*m)*(x^m)^30+a*b^4*x^5/(1+5*m)*(x^m)^25+5/2*a^2*b^3*x^4/(1+
5*m)*(x^m)^20+10/3*a^3*b^2*x^3/(1+5*m)*(x^m)^15+5/2*a^4*b*x^2/(1+5*m)*(x^m)
^10+a^5/(1+5*m)*x*(x^m)^5
```

maxima [B] time = 1.41, size = 121, normalized size = 4.48

$$\frac{b^5x^{30\,m+6}}{6\,(5\,m+1)} + \frac{ab^4x^{25\,m+5}}{5\,m+1} + \frac{5\,a^2b^3x^{20\,m+4}}{2\,(5\,m+1)} + \frac{10\,a^3b^2x^{15\,m+3}}{3\,(5\,m+1)} + \frac{5\,a^4bx^{10\,m+2}}{2\,(5\,m+1)} + \frac{a^5x^{5\,m+1}}{5\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(1+6*m))^5,x, algorithm="maxima")
```

```
[Out] 1/6*b^5*x^(30*m + 6)/(5*m + 1) + a*b^4*x^(25*m + 5)/(5*m + 1) + 5/2*a^2*b^3
*x^(20*m + 4)/(5*m + 1) + 10/3*a^3*b^2*x^(15*m + 3)/(5*m + 1) + 5/2*a^4*b*x
^(10*m + 2)/(5*m + 1) + a^5*x^(5*m + 1)/(5*m + 1)
```

mupad [B] time = 5.44, size = 124, normalized size = 4.59

$$\frac{b^5\,x^{30\,m}\,x^6}{30\,m+6} + \frac{a^5\,x\,x^{5\,m}}{5\,m+1} + \frac{5\,a^4\,b\,x^{10\,m}\,x^2}{10\,m+2} + \frac{a\,b^4\,x^{25\,m}\,x^5}{5\,m+1} + \frac{5\,a^2\,b^3\,x^{20\,m}\,x^4}{10\,m+2} + \frac{10\,a^3\,b^2\,x^{15\,m}\,x^3}{15\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m + b*x^(6*m + 1))^5,x)
```

```
[Out] (b^5*x^(30*m)*x^6)/(30*m + 6) + (a^5*x*x^(5*m))/(5*m + 1) + (5*a^4*b*x^(10*
m)*x^2)/(10*m + 2) + (a*b^4*x^(25*m)*x^5)/(5*m + 1) + (5*a^2*b^3*x^(20*m)*x
^4)/(10*m + 2) + (10*a^3*b^2*x^(15*m)*x^3)/(15*m + 3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m+b*x**(1+6*m))**5,x)
```

```
[Out] Timed out
```

3.240 $\int \frac{1}{\left(bx^{1-2m}+ax^m\right)^3}\,dx$

Optimal. Leaf size=27

$$-\frac{1}{2b(1-3m)\left(a+bx^{1-3m}\right)^2}$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1593, 261}

$$-\frac{1}{2b(1-3m)\left(a+bx^{1-3m}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1 - 2*m) + a*x^m)^(-3),x]
```

```
[Out] -1/(2*b*(1 - 3*m)*(a + b*x^(1 - 3*m))^2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(bx^{1-2m}+ax^m\right)^3}\,dx &= \int \frac{x^{-3m}}{\left(a+bx^{1-3m}\right)^3}\,dx\\ &= -\frac{1}{2b(1-3m)\left(a+bx^{1-3m}\right)^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 1.00

$$-\frac{1}{2b(1-3m)\left(a+bx^{1-3m}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1 - 2*m) + a*x^m)^(-3),x]
```

```
[Out] -1/2*1/(b*(1 - 3*m)*(a + b*x^(1 - 3*m))^2)
```

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{1-2m}+ax^m\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^(1 - 2*m) + a*x^m)^(-3),x]
```

```
[Out] Defer[IntegrateAlgebraic][(b*x^(1 - 2*m) + a*x^m)^(-3), x]
```

fricas [B] time = 0.42, size = 82, normalized size = 3.04

$$-\frac{2\,axx^{3\,m}+bx^{2}}{2\left(2\left(3\,a^{3}bm-a^{3}b\right)xx^{3\,m}+\left(3\,a^{2}b^{2}m-a^{2}b^{2}\right)x^{2}+\left(3\,a^{4}m-a^{4}\right)x^{6\,m}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1-2*m)+a*x^m)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*x*x^(3*m) + b*x^2)/(2*(3*a^3*b*m - a^3*b)*x*x^(3*m) + (3*a^2*b^2*
m - a^2*b^2)*x^2 + (3*a^4*m - a^4)*x^(6*m))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^{m}+bx^{-2\,m+1}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1-2*m)+a*x^m)^3,x, algorithm="giac")
```

```
[Out] integrate((a*x^m + b*x^(-2*m + 1))^(-3), x)
```

maple [A] time = 0.07, size = 39, normalized size = 1.44

$$-\frac{\left(2a\,x^{3m}+bx\right)x}{2\left(3m-1\right)\left(a\,x^{3m}+bx\right)^{2}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1-2*m)+a*x^m)^3,x)
```

```
[Out] -1/2*x*(2*a*(x^m)^3+b*x)/(-1+3*m)/a^2/(a*(x^m)^3+b*x)^2
```

maxima [B] time = 1.40, size = 66, normalized size = 2.44

$$-\frac{2\,axx^{3\,m}+bx^{2}}{2\left(2\,a^{3}b(3\,m-1)xx^{3\,m}+a^{2}b^{2}(3\,m-1)x^{2}+a^{4}(3\,m-1)x^{6\,m}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1-2*m)+a*x^m)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*x*x^(3*m) + b*x^2)/(2*a^3*b*(3*m - 1)*x*x^(3*m) + a^2*b^2*(3*m -
1)*x^2 + a^4*(3*m - 1)*x^(6*m))
```

mupad [B] time = 5.21, size = 38, normalized size = 1.41

$$-\frac{x\,\left(b\,x+2\,a\,x^{3\,m}\right)}{2\,a^{2}\,\left(3\,m-1\right)\,{\left(b\,x+a\,x^{3\,m}\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^m + b*x^(1 - 2*m))^3,x)
```

```
[Out] -(x*(b*x + 2*a*x^(3*m)))/(2*a^2*(3*m - 1)*(b*x + a*x^(3*m))^2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(1-2*m)+a*x**m)**3,x)
```

```
[Out] Timed out
```

3.241 $\int \frac{1}{\frac{b}{x}+ax}\,dx$

Optimal. Leaf size=15

$$\frac{\log\left(ax^2+b\right)}{2a}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 260}

$$\frac{\log\left(ax^2+b\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x + a*x)^(-1),x]
```

```
[Out] Log[b + a*x^2]/(2*a)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\frac{b}{x}+ax}\,dx &= \int \frac{x}{b+ax^2}\,dx\\ &= \frac{\log\left(b+ax^2\right)}{2a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log\left(ax^2+b\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x + a*x)^(-1),x]
```

```
[Out] Log[b + a*x^2]/(2*a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\frac{b}{x}+ax}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b/x + a*x)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(b/x + a*x)^(-1), x]
```

fricas [A] time = 0.37, size = 13, normalized size = 0.87

$$\frac{\log\left(ax^2+b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x),x, algorithm="fricas")
[Out] 1/2*log(a*x^2 + b)/a
```

giac [A] time = 0.15, size = 14, normalized size = 0.93

$$\frac{\log\left(\left|ax^2+b\right|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x),x, algorithm="giac")
[Out] 1/2*log(abs(a*x^2 + b))/a
```

maple [A] time = 0.04, size = 14, normalized size = 0.93

$$\frac{\ln\left(a\,x^2+b\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x+a*x),x)
[Out] 1/2/a*ln(a*x^2+b)
```

maxima [A] time = 1.29, size = 13, normalized size = 0.87

$$\frac{\log\left(ax^2+b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x),x, algorithm="maxima")
[Out] 1/2*log(a*x^2 + b)/a
```

mupad [B] time = 0.05, size = 13, normalized size = 0.87

$$\frac{\ln\left(a\,x^2+b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b/x),x)
[Out] log(b + a*x^2)/(2*a)
```

sympy [A] time = 0.12, size = 10, normalized size = 0.67

$$\frac{\log\left(ax^2+b\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x),x)
[Out] log(a*x**2 + b)/(2*a)
```

3.242 $\int \frac{1}{\frac{b}{x^2}+ax}\,dx$

Optimal. Leaf size=15

$$\frac{\log\left(ax^3+b\right)}{3a}$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 260}

$$\frac{\log\left(ax^3+b\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^2 + a*x)^(-1),x]
[Out] Log[b + a*x^3]/(3*a)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\frac{b}{x^2}+ax}\,dx &= \int \frac{x^2}{b+ax^3}\,dx\\ &= \frac{\log\left(b+ax^3\right)}{3a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log\left(ax^3+b\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^2 + a*x)^(-1),x]
[Out] Log[b + a*x^3]/(3*a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\frac{b}{x^2}+ax}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b/x^2 + a*x)^(-1),x]
[Out] IntegrateAlgebraic[(b/x^2 + a*x)^(-1), x]
```

fricas [A] time = 0.39, size = 13, normalized size = 0.87

$$\frac{\log\left(ax^3 + b\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^2+a*x),x, algorithm="fricas")
[Out] 1/3*log(a*x^3 + b)/a
```

giac [A] time = 0.15, size = 14, normalized size = 0.93

$$\frac{\log\left(\left|ax^3 + b\right|\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^2+a*x),x, algorithm="giac")
[Out] 1/3*log(abs(a*x^3 + b))/a
```

maple [A] time = 0.04, size = 14, normalized size = 0.93

$$\frac{\ln\left(a\,x^3 + b\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^2+a*x),x)
[Out] 1/3/a*ln(a*x^3+b)
```

maxima [A] time = 1.27, size = 13, normalized size = 0.87

$$\frac{\log\left(ax^3 + b\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^2+a*x),x, algorithm="maxima")
[Out] 1/3*log(a*x^3 + b)/a
```

mupad [B] time = 5.12, size = 13, normalized size = 0.87

$$\frac{\ln\left(a\,x^3 + b\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b/x^2),x)
[Out] log(b + a*x^3)/(3*a)
```

sympy [A] time = 0.15, size = 10, normalized size = 0.67

$$\frac{\log\left(ax^3 + b\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x**2+a*x),x)
[Out] log(a*x**3 + b)/(3*a)
```

3.243 $\int \frac{1}{\frac{b}{x^3}+ax}\,dx$

Optimal. Leaf size=15

$$\frac{\log\left(ax^4+b\right)}{4a}$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 260}

$$\frac{\log\left(ax^4+b\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^3 + a*x)^(-1),x]
```

```
[Out] Log[b + a*x^4]/(4*a)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\frac{b}{x^3}+ax}\,dx &= \int \frac{x^3}{b+ax^4}\,dx\\ &= \frac{\log\left(b+ax^4\right)}{4a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log\left(ax^4+b\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^3 + a*x)^(-1),x]
```

```
[Out] Log[b + a*x^4]/(4*a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\frac{b}{x^3}+ax}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b/x^3 + a*x)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(b/x^3 + a*x)^(-1), x]
```

fricas [A] time = 0.38, size = 13, normalized size = 0.87

$$\frac{\log\left(ax^4+b\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/4*log(a*x^4 + b)/a
```

giac [A] time = 0.15, size = 14, normalized size = 0.93

$$\frac{\log\left(|ax^4+b|\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(a*x^4 + b))/a
```

maple [A] time = 0.04, size = 14, normalized size = 0.93

$$\frac{\ln\left(a\,x^4+b\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^3+a*x),x)
```

```
[Out] 1/4*ln(a*x^4+b)/a
```

maxima [A] time = 1.37, size = 13, normalized size = 0.87

$$\frac{\log\left(ax^4+b\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/4*log(a*x^4 + b)/a
```

mupad [B] time = 0.05, size = 13, normalized size = 0.87

$$\frac{\ln\left(a\,x^4+b\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b/x^3),x)
```

```
[Out] log(b + a*x^4)/(4*a)
```

sympy [A] time = 0.16, size = 10, normalized size = 0.67

$$\frac{\log\left(ax^4+b\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x**3+a*x),x)
```

```
[Out] log(a*x**4 + b)/(4*a)
```

3.244 $\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx$

Optimal. Leaf size=19

$$\frac{x^4}{4b\left(ax^2+b\right)^2}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 264}

$$\frac{x^4}{4b\left(ax^2+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x + a*x)^(-3),x]
```

```
[Out] x^4/(4*b*(b + a*x^2)^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx &= \int \frac{x^3}{\left(b+ax^2\right)^3}\,dx\\ &= \frac{x^4}{4b\left(b+ax^2\right)^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.26

$$-\frac{2ax^2+b}{4a^2\left(ax^2+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x + a*x)^(-3),x]
```

```
[Out] -1/4*(b + 2*a*x^2)/(a^2*(b + a*x^2)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b/x + a*x)^(-3),x]
[Out] IntegrateAlgebraic[(b/x + a*x)^(-3), x]
```

fricas [B] time = 0.37, size = 36, normalized size = 1.89

$$-\frac{2\,ax^2+b}{4\left(a^4x^4+2\,a^3bx^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x)^3,x, algorithm="fricas")
[Out] -1/4*(2*a*x^2 + b)/(a^4*x^4 + 2*a^3*b*x^2 + a^2*b^2)
```

giac [A] time = 0.19, size = 22, normalized size = 1.16

$$-\frac{2\,ax^2+b}{4\left(ax^2+b\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x)^3,x, algorithm="giac")
[Out] -1/4*(2*a*x^2 + b)/((a*x^2 + b)^2*a^2)
```

maple [A] time = 0.05, size = 31, normalized size = 1.63

$$\frac{b}{4\left(a\,x^2+b\right)^2a^2}-\frac{1}{2\left(a\,x^2+b\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x+a*x)^3,x)
[Out] -1/2/a^2/(a*x^2+b)+1/4/a^2*b/(a*x^2+b)^2
```

maxima [B] time = 1.33, size = 36, normalized size = 1.89

$$-\frac{2\,ax^2+b}{4\left(a^4x^4+2\,a^3bx^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x)^3,x, algorithm="maxima")
[Out] -1/4*(2*a*x^2 + b)/(a^4*x^4 + 2*a^3*b*x^2 + a^2*b^2)
```

mupad [B] time = 0.04, size = 37, normalized size = 1.95

$$-\frac{\frac{b}{4\,a^2}+\frac{x^2}{2\,a}}{a^2\,x^4+2\,a\,b\,x^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b/x)^3,x)
[Out] -(b/(4*a^2) + x^2/(2*a))/(b^2 + a^2*x^4 + 2*a*b*x^2)
```

sympy [B] time = 0.28, size = 36, normalized size = 1.89

$$\frac{-2ax^2 - b}{4a^4x^4 + 8a^3bx^2 + 4a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x)**3,x)
```

```
[Out] (-2*a*x**2 - b)/(4*a**4*x**4 + 8*a**3*b*x**2 + 4*a**2*b**2)
```

3.245 $\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3} dx$

Optimal. Leaf size=19

$$\frac{x^{10}}{10b\left(ax^5+b\right)^2}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 264}

$$\frac{x^{10}}{10b\left(ax^5+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^3 + a*x^2)^(-3),x]
```

```
[Out] x^10/(10*b*(b + a*x^5)^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3} dx &= \int \frac{x^9}{\left(b+ax^5\right)^3} dx\\ &= \frac{x^{10}}{10b\left(b+ax^5\right)^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 1.26

$$-\frac{2ax^5+b}{10a^2\left(ax^5+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^3 + a*x^2)^(-3),x]
```

```
[Out] -1/10*(b + 2*a*x^5)/(a^2*(b + a*x^5)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b/x^3 + a*x^2)^(-3),x]
```

```
[Out] IntegrateAlgebraic[(b/x^3 + a*x^2)^(-3), x]
```

fricas [B] time = 0.37, size = 36, normalized size = 1.89

$$-\frac{2\,ax^5+b}{10\left(a^4x^{10}+2\,a^3bx^5+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3+a*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/10*(2*a*x^5 + b)/(a^4*x^10 + 2*a^3*b*x^5 + a^2*b^2)
```

giac [A] time = 0.17, size = 22, normalized size = 1.16

$$-\frac{2\,ax^5+b}{10\left(ax^5+b\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3+a*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/10*(2*a*x^5 + b)/((a*x^5 + b)^2*a^2)
```

maple [A] time = 0.05, size = 31, normalized size = 1.63

$$\frac{b}{10\left(a\,x^5+b\right)^2a^2}-\frac{1}{5\left(a\,x^5+b\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^3+a*x^2)^3,x)
```

```
[Out] -1/5/a^2/(a*x^5+b)+1/10/a^2*b/(a*x^5+b)^2
```

maxima [B] time = 1.29, size = 36, normalized size = 1.89

$$-\frac{2\,ax^5+b}{10\left(a^4x^{10}+2\,a^3bx^5+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3+a*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/10*(2*a*x^5 + b)/(a^4*x^10 + 2*a^3*b*x^5 + a^2*b^2)
```

mupad [B] time = 0.06, size = 37, normalized size = 1.95

$$-\frac{\frac{b}{10\,a^2}+\frac{x^5}{5\,a}}{a^2\,x^{10}+2\,a\,b\,x^5+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b/x^3)^3,x)
```

```
[Out] -(b/(10*a^2) + x^5/(5*a))/(b^2 + a^2*x^10 + 2*a*b*x^5)
```

sympy [**B**] time = 0.50, size = 36, normalized size = 1.89

$$\frac{-2ax^5 - b}{10a^4x^{10} + 20a^3bx^5 + 10a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x**3+a*x**2)**3,x)
```

```
[Out] (-2*a*x**5 - b)/(10*a**4*x**10 + 20*a**3*b*x**5 + 10*a**2*b**2)
```

3.246 $\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx$

Optimal. Leaf size=19

$$\frac{x^{16}}{16b\left(ax^8+b\right)^2}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1593, 264}

$$\frac{x^{16}}{16b\left(ax^8+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^5 + a*x^3)^(-3),x]
```

```
[Out] x^16/(16*b*(b + a*x^8)^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx &= \int \frac{x^{15}}{\left(b+ax^8\right)^3}\,dx\\ &= \frac{x^{16}}{16b\left(b+ax^8\right)^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.26

$$-\frac{2ax^8+b}{16a^2\left(ax^8+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^5 + a*x^3)^(-3),x]
```

```
[Out] -1/16*(b + 2*a*x^8)/(a^2*(b + a*x^8)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b/x^5 + a*x^3)^(-3),x]
[Out] IntegrateAlgebraic[(b/x^5 + a*x^3)^(-3), x]
```

fricas [B] time = 0.38, size = 36, normalized size = 1.89

$$-\frac{2\,ax^8+b}{16\,\left(a^4x^{16}+2\,a^3bx^8+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^5+a*x^3)^3,x, algorithm="fricas")
[Out] -1/16*(2*a*x^8 + b)/(a^4*x^16 + 2*a^3*b*x^8 + a^2*b^2)
```

giac [A] time = 0.15, size = 22, normalized size = 1.16

$$-\frac{2\,ax^8+b}{16\,\left(ax^8+b\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^5+a*x^3)^3,x, algorithm="giac")
[Out] -1/16*(2*a*x^8 + b)/((a*x^8 + b)^2*a^2)
```

maple [A] time = 0.06, size = 31, normalized size = 1.63

$$\frac{b}{16\left(a\,x^8+b\right)^2a^2}-\frac{1}{8\left(a\,x^8+b\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^5+a*x^3)^3,x)
[Out] -1/8/a^2/(a*x^8+b)+1/16/a^2*b/(a*x^8+b)^2
```

maxima [B] time = 1.31, size = 36, normalized size = 1.89

$$-\frac{2\,ax^8+b}{16\,\left(a^4x^{16}+2\,a^3bx^8+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^5+a*x^3)^3,x, algorithm="maxima")
[Out] -1/16*(2*a*x^8 + b)/(a^4*x^16 + 2*a^3*b*x^8 + a^2*b^2)
```

mupad [B] time = 5.18, size = 37, normalized size = 1.95

$$-\frac{\frac{b}{16\,a^2}+\frac{x^8}{8\,a}}{a^2\,x^{16}+2\,a\,b\,x^8+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^3 + b/x^5)^3,x)
[Out] -(b/(16*a^2) + x^8/(8*a))/(b^2 + a^2*x^16 + 2*a*b*x^8)
```

sympy [B] time = 0.74, size = 36, normalized size = 1.89

$$\frac{-2ax^8 - b}{16a^4x^{16} + 32a^3bx^8 + 16a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x**5+a*x**3)**3,x)
```

```
[Out] (-2*a*x**8 - b)/(16*a**4*x**16 + 32*a**3*b*x**8 + 16*a**2*b**2)
```

3.247 $\int \left(\frac{a}{x}+bx\right)^2\,dx$

Optimal. Leaf size=24

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 270}

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a/x + b*x)^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{a}{x}+bx\right)^2\,dx &= \int \frac{\left(a+bx^2\right)^2}{x^2}\,dx\\ &= \int \left(2ab+\frac{a^2}{x^2}+b^2x^2\right)dx\\ &= -\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a/x + b*x)^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{a}{x}+bx\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a/x + b*x)^2,x]
[Out] IntegrateAlgebraic[(a/x + b*x)^2, x]
```

fricas [**A**] time = 0.38, size = 25, normalized size = 1.04

$$\frac{b^2x^4 + 6\,abx^2 - 3\,a^2}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^2,x, algorithm="fricas")
[Out] 1/3*(b^2*x^4 + 6*a*b*x^2 - 3*a^2)/x
```

giac [**A**] time = 0.15, size = 22, normalized size = 0.92

$$\frac{1}{3}\,b^2x^3 + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^2,x, algorithm="giac")
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

maple [**A**] time = 0.04, size = 23, normalized size = 0.96

$$\frac{b^2x^3}{3} + 2abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x+b*x)^2,x)
[Out] -a^2/x+2*a*b*x+1/3*b^2*x^3
```

maxima [**A**] time = 1.36, size = 22, normalized size = 0.92

$$\frac{1}{3}\,b^2x^3 + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^2,x, algorithm="maxima")
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

mupad [**B**] time = 0.04, size = 22, normalized size = 0.92

$$\frac{b^2\,x^3}{3} - \frac{a^2}{x} + 2\,a\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + a/x)^2,x)
[Out] (b^2*x^3)/3 - a^2/x + 2*a*b*x
```

sympy [**A**] time = 0.10, size = 19, normalized size = 0.79

$$-\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)**2,x)
[Out] -a**2/x + 2*a*b*x + b**2*x**3/3
```

3.248 $\int \left(\frac{a}{x}+bx\right)^3 \, dx$

Optimal. Leaf size=40

$$-\frac{a^3}{2x^2}+3a^2b\log(x)+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}$$

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1593, 266, 43}

$$3a^2b\log(x)-\frac{a^3}{2x^2}+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a/x + b*x)^3,x]
```

```
[Out] -a^3/(2*x^2) + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{a}{x}+bx\right)^3 \, dx &= \int \frac{\left(a+bx^2\right)^3}{x^3} \, dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^3}{x^2} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(3ab^2+\frac{a^3}{x^2}+\frac{3a^2b}{x}+b^3x\right) dx, x, x^2\right) \\
&= -\frac{a^3}{2x^2}+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}+3a^2b\log(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$-\frac{a^3}{2x^2}+3a^2b\log(x)+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a/x + b*x)^3,x]
[Out] -1/2*a^3/x^2 + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{a}{x}+bx\right)^3 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a/x + b*x)^3,x]
[Out] IntegrateAlgebraic[(a/x + b*x)^3, x]
```

fricas [A] time = 0.37, size = 38, normalized size = 0.95

$$\frac{b^3x^6+6\,ab^2x^4+12\,a^2bx^2\log(x)-2\,a^3}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^3,x, algorithm="fricas")
[Out] 1/4*(b^3*x^6 + 6*a*b^2*x^4 + 12*a^2*b*x^2*log(x) - 2*a^3)/x^2
```

giac [A] time = 0.17, size = 46, normalized size = 1.15

$$\frac{1}{4}\,b^3x^4+\frac{3}{2}\,ab^2x^2+\frac{3}{2}\,a^2b\log\left(x^2\right)-\frac{3\,a^2bx^2+a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^3,x, algorithm="giac")
[Out] 1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*b*log(x^2) - 1/2*(3*a^2*b*x^2 + a^3)/
x^2
```

maple [A] time = 0.04, size = 35, normalized size = 0.88

$$\frac{b^3x^4}{4}+\frac{3a\,b^2x^2}{2}+3a^2b\ln(x)-\frac{a^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x+b*x)^3,x)
[Out] -1/2*a^3/x^2+3/2*a*b^2*x^2+1/4*b^3*x^4+3*a^2*b*ln(x)
```

maxima [A] time = 1.32, size = 34, normalized size = 0.85

$$\frac{1}{4}\,b^3x^4+\frac{3}{2}\,ab^2x^2+3\,a^2b\log(x)-\frac{a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^3,x, algorithm="maxima")
[Out] 1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3*a^2*b*log(x) - 1/2*a^3/x^2
```

mupad [B] time = 0.04, size = 34, normalized size = 0.85

$$\frac{b^3\,x^4}{4}-\frac{a^3}{2\,x^2}+\frac{3\,a\,b^2\,x^2}{2}+3\,a^2\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + a/x)^3,x)
```

```
[Out] (b^3*x^4)/4 - a^3/(2*x^2) + (3*a*b^2*x^2)/2 + 3*a^2*b*log(x)
```

sympy **[A]** time = 0.15, size = 37, normalized size = 0.92

$$-\frac{a^3}{2x^2} + 3a^2b\log(x) + \frac{3ab^2x^2}{2} + \frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)**3,x)
```

```
[Out] -a**3/(2*x**2) + 3*a**2*b*log(x) + 3*a*b**2*x**2/2 + b**3*x**4/4
```

3.249 $\int \left(\frac{a}{x}+bx\right)^4 dx$

Optimal. Leaf size=50

$$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 270}

$$6a^2b^2x-\frac{4a^3b}{x}-\frac{a^4}{3x^3}+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a/x + b*x)^4,x]
```

```
[Out] -a^4/(3*x^3) - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{a}{x}+bx\right)^4 dx &= \int \frac{\left(a+bx^2\right)^4}{x^4}\,dx\\ &= \int \left(6a^2b^2+\frac{a^4}{x^4}+\frac{4a^3b}{x^2}+4ab^3x^2+b^4x^4\right)dx\\ &= -\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.00

$$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a/x + b*x)^4,x]
```

```
[Out] -1/3*a^4/x^3 - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{a}{x}+bx\right)^4 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a/x + b*x)^4,x]
[Out] IntegrateAlgebraic[(a/x + b*x)^4, x]
```

fricas [A] time = 0.38, size = 48, normalized size = 0.96

$$\frac{3\,b^4x^8+20\,ab^3x^6+90\,a^2b^2x^4-60\,a^3bx^2-5\,a^4}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^4,x, algorithm="fricas")
[Out] 1/15*(3*b^4*x^8 + 20*a*b^3*x^6 + 90*a^2*b^2*x^4 - 60*a^3*b*x^2 - 5*a^4)/x^3
```

giac [A] time = 0.15, size = 45, normalized size = 0.90

$$\frac{1}{5}\,b^4x^5+\frac{4}{3}\,ab^3x^3+6\,a^2b^2x-\frac{12\,a^3bx^2+a^4}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^4,x, algorithm="giac")
[Out] 1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 1/3*(12*a^3*b*x^2 + a^4)/x^3
```

maple [A] time = 0.05, size = 45, normalized size = 0.90

$$\frac{b^4x^5}{5}+\frac{4a\,b^3x^3}{3}+6a^2b^2x-\frac{4a^3b}{x}-\frac{a^4}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x+b*x)^4,x)
[Out] -1/3*a^4/x^3-4*a^3*b/x+6*a^2*b^2*x+4/3*a*b^3*x^3+1/5*b^4*x^5
```

maxima [A] time = 1.32, size = 44, normalized size = 0.88

$$\frac{1}{5}\,b^4x^5+\frac{4}{3}\,ab^3x^3+6\,a^2b^2x-\frac{4\,a^3b}{x}-\frac{a^4}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^4,x, algorithm="maxima")
[Out] 1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 4*a^3*b/x - 1/3*a^4/x^3
```

mupad [B] time = 0.05, size = 47, normalized size = 0.94

$$\frac{b^4\,x^5}{5}-\frac{\frac{a^4}{3}+4\,b\,a^3\,x^2}{x^3}+6\,a^2\,b^2\,x+\frac{4\,a\,b^3\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + a/x)^4,x)
[Out] (b^4*x^5)/5 - (a^4/3 + 4*a^3*b*x^2)/x^3 + 6*a^2*b^2*x + (4*a*b^3*x^3)/3
```

sympy [A] time = 0.19, size = 49, normalized size = 0.98

$$6a^2b^2x+\frac{4ab^3x^3}{3}+\frac{b^4x^5}{5}+\frac{-a^4-12a^3bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)**4,x)
```

```
[Out] 6*a**2*b**2*x + 4*a*b**3*x**3/3 + b**4*x**5/5 + (-a**4 - 12*a**3*b*x**2)/(3
*x**3)
```

3.250 $\int \frac{1}{\frac{1}{x^2}+x^3}\,dx$

Optimal. Leaf size=185

$$-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)+\frac{1}{5}\log(x+1)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\right.}$$

Rubi [A] time = 0.35, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {1593, 293, 634, 618, 204, 628, 31}

$$-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)+\frac{1}{5}\log(x+1)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}x+\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{5}\left(5+2\sqrt{5}\right)}-\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-2) + x^3)^(-1),x]
```

```
[Out] -(Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] + 2*Sqrt[2/(5 + Sqrt
[5])]*x])/5 - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 + 2*Sqrt[5])/5] - Sqrt[
(2*(5 + Sqrt[5]))/5]*x])/5 + Log[1 + x]/5 - ((1 + Sqrt[5])*Log[1 - ((1 - Sq
rt[5])*x)/2 + x^2])/20 - ((1 - Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])
/20
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 293

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x]; -(((-r)^(m + 1)*Int[1/(r + s*x), x])/(a*n*s^m
)) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Fr
eeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ
[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\frac{1}{x^2}+x^3}\,dx &= \int \frac{x^2}{1+x^5}\,dx \\
&= \frac{2}{5}\int \frac{\frac{1}{4}\left(-1-\sqrt{5}\right)-\frac{1}{4}\left(1+\sqrt{5}\right)x}{1-\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx+\frac{2}{5}\int \frac{\frac{1}{4}\left(-1+\sqrt{5}\right)-\frac{1}{4}\left(1-\sqrt{5}\right)x}{1-\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx+\frac{1}{5}\int \frac{1}{1+x}\,dx \\
&= \frac{1}{5}\log(1+x)+\frac{\int \frac{1}{1+\frac{1}{2}\left(-1-\sqrt{5}\right)x+x^2}\,dx}{2\sqrt{5}}-\frac{\int \frac{1}{1+\frac{1}{2}\left(-1+\sqrt{5}\right)x+x^2}\,dx}{2\sqrt{5}}+\frac{1}{20}\left(-1-\sqrt{5}\right)\int \frac{\frac{1}{2}\left(-1+\sqrt{5}\right)}{1+\frac{1}{2}\left(-1+\sqrt{5}\right)} \\
&= \frac{1}{5}\log(1+x)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(2-x-\sqrt{5}x+2x^2\right)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(2-x+\sqrt{5}x+2x^2\right)+ \\
&= \sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\tan^{-1}\left(\frac{1-\sqrt{5}-4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}-4x\right)\right.
\end{aligned}$$

Mathematica [A] time = 0.14, size = 144, normalized size = 0.78

$$\frac{1}{20}\left(-\left(1+\sqrt{5}\right)\log\left(x^2+\frac{1}{2}\left(\sqrt{5}-1\right)x+1\right)+\left(\sqrt{5}-1\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)+4\log(x+1)-2\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{-4x+\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}\right)-2\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^(-2) + x^3)^(-1),x]
```

```
[Out] (-2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 + Sqrt[5] - 4*x)/Sqrt[10 - 2*Sqrt[5]]]
- 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-1 + Sqrt[5] + 4*x)/Sqrt[2*(5 + Sqrt[5])]]
 + 4*Log[1 + x] - (1 + Sqrt[5])*Log[1 + ((-1 + Sqrt[5])*x)/2 + x^2] + (-1 +
 Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])/20
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\frac{1}{x^2}+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^(-2) + x^3)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(x^(-2) + x^3)^(-1), x]
```

fricas [B] time = 1.28, size = 637, normalized size = 3.44

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^2+x^3),x, algorithm="fricas")
```

```
[Out] -1/20*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)*log(1/16*(2*sqrt(1/2)*s
qrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + x) + 1/20*(sqrt(5) + 2*sqrt(-3/16*(2*sq
rt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqrt(1/2)*sqrt(sqrt(5)
- 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 3/16*(2
*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/2)*sqrt(sqrt(5) - 5)
 + 1/2*sqrt(5) - 5/2) - 1)*log(-1/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(
5) + 1)^2 - 1/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 1/2*sqrt
(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqrt(1/2)*s
qrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) -
1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/2)*sqrt(
sqrt(5) - 5) + 1/2*sqrt(5) - 5/2)*(sqrt(5) - 1) + 2*x - 1) + 1/20*(sqrt(5)
- 2*sqrt(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqr
t(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sq
rt(5) - 1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/
2)*sqrt(sqrt(5) - 5) + 1/2*sqrt(5) - 5/2) - 1)*log(-1/16*(2*sqrt(1/2)*sqrt(
sqrt(5) - 5) + sqrt(5) + 1)^2 - 1/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(
5) - 1)^2 - 1/2*sqrt(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2
+ 1/8*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(
5) - 5) - sqrt(5) - 1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)
^2 + sqrt(1/2)*sqrt(sqrt(5) - 5) + 1/2*sqrt(5) - 5/2)*(sqrt(5) - 1) + 2*x -
 1) + 1/20*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)*log(1/16*(2*sqrt(1
/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + x) + 1/5*log(x + 1)
```

giac [A] time = 0.18, size = 112, normalized size = 0.61

$$\frac{1}{20}\left(\sqrt{5}-1\right)\log\left(x^{2}-\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right)-\frac{1}{20}\left(\sqrt{5}+1\right)\log\left(x^{2}+\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right)-\frac{1}{10}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)+\frac{1}{10}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)+\frac{1}{5}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^2+x^3),x, algorithm="giac")
```

```
[Out] 1/20*(sqrt(5) - 1)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) - 1/20*(sqrt(5) + 1)*
log(x^2 + 1/2*x*(sqrt(5) - 1) + 1) - 1/10*sqrt(-2*sqrt(5) + 10)*arctan((4*x
 + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10)) + 1/10*sqrt(2*sqrt(5) + 10)*arctan((4
*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10)) + 1/5*log(abs(x + 1))
```

maple [A] time = 0.11, size = 156, normalized size = 0.84

$$\frac{2\sqrt{5}\,\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}-\frac{2\sqrt{5}\,\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}+\frac{\ln\left(x+1\right)}{5}+\frac{\sqrt{5}\,\ln\left(2x^{2}-\sqrt{5}\,x-x+2\right)}{20}-\frac{\ln\left(2x^{2}-\sqrt{5}\,x-x+2\right)}{20}-\frac{\sqrt{5}\,\ln\left(2x^{2}+\sqrt{5}\,x-x+2\right)}{20}-\frac{\ln\left(2x^{2}+\sqrt{5}\,x-x+2\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^2+x^3),x)
```

```
[Out] -1/20*ln(2*x^2+5^(1/2)*x-x+2)*5^(1/2)-1/20*ln(2*x^2+5^(1/2)*x-x+2)-2/5/(10+
2*5^(1/2))^(1/2)*arctan((4*x+5^(1/2)-1)/(10+2*5^(1/2))^(1/2))*5^(1/2)+1/20*
ln(2*x^2-5^(1/2)*x-x+2)*5^(1/2)-1/20*ln(2*x^2-5^(1/2)*x-x+2)+2/5/(10-2*5^(1
/2))^(1/2)*arctan((4*x-5^(1/2)-1)/(10-2*5^(1/2))^(1/2))*5^(1/2)+1/5*ln(x+1)
```

maxima [A] time = 2.90, size = 124, normalized size = 0.67

$$-\frac{2\sqrt{5}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)}{5\sqrt{2\sqrt{5}+10}}+\frac{2\sqrt{5}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)}{5\sqrt{-2\sqrt{5}+10}}+\frac{\log\left(2x^{2}-x\left(\sqrt{5}+1\right)+2\right)}{5\left(\sqrt{5}+1\right)}-\frac{\log\left(2x^{2}+x\left(\sqrt{5}-1\right)+2\right)}{5\left(\sqrt{5}-1\right)}+\frac{1}{5}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^2+x^3),x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(5)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10))/sqrt(2*sqrt(5
) + 10) + 2/5*sqrt(5)*arctan((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10))/sqr
t(-2*sqrt(5) + 10) + 1/5*log(2*x^2 - x*(sqrt(5) + 1) + 2)/(sqrt(5) + 1) - 1
/5*log(2*x^2 + x*(sqrt(5) - 1) + 2)/(sqrt(5) - 1) + 1/5*log(x + 1)
```

mupad [B] time = 5.91, size = 197, normalized size = 1.06

$$\frac{\ln(x+1)}{5} - \ln\left(1 - \frac{x\left(\sqrt{2}\,\sqrt{-\sqrt{5}-5} - \sqrt{5}+1\right)^3}{64}\right)\left(\frac{\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{20} - \frac{\sqrt{5}}{20} + \frac{1}{20}\right) + \ln\left(\frac{x\left(\sqrt{2}\,\sqrt{-\sqrt{5}-5} + \sqrt{5}-1\right)^3}{64} + 1\right)\left(\frac{\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{20} + \frac{\sqrt{5}}{20} - \frac{1}{20}\right) - \ln\left(1 - \frac{x\left(\sqrt{5} + \sqrt{2}\,\sqrt{\sqrt{5}-5}+1\right)^3}{64}\right)\left(\frac{\sqrt{5}}{20} + \frac{\sqrt{2}\,\sqrt{\sqrt{5}-5}}{20} + \frac{1}{20}\right) - \ln\left(1 - \frac{x\left(\sqrt{5} - \sqrt{2}\,\sqrt{\sqrt{5}-5}+1\right)^3}{64}\right)\left(\frac{\sqrt{5}}{20} - \frac{\sqrt{2}\,\sqrt{\sqrt{5}-5}}{20} + \frac{1}{20}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^2 + x^3),x)
```

```
[Out] log(x + 1)/5 - log(1 - (x*(2^(1/2)*(- 5^(1/2) - 5)^(1/2) - 5^(1/2) + 1)^3)/
64)*((2^(1/2)*(- 5^(1/2) - 5)^(1/2))/20 - 5^(1/2)/20 + 1/20) + log((x*(2^(1
/2)*(- 5^(1/2) - 5)^(1/2) + 5^(1/2) - 1)^3)/64 + 1)*((2^(1/2)*(- 5^(1/2) -
5)^(1/2))/20 + 5^(1/2)/20 - 1/20) - log(1 - (x*(5^(1/2) + 2^(1/2)*(5^(1/2)
- 5)^(1/2) + 1)^3)/64)*(5^(1/2)/20 + (2^(1/2)*(5^(1/2) - 5)^(1/2))/20 + 1/2
0) - log(1 - (x*(5^(1/2) - 2^(1/2)*(5^(1/2) - 5)^(1/2) + 1)^3)/64)*(5^(1/2)
/20 - (2^(1/2)*(5^(1/2) - 5)^(1/2))/20 + 1/20)
```

sympy [A] time = 1.54, size = 36, normalized size = 0.19

$$\frac{\log(x+1)}{5} + \operatorname{RootSum}\left(625t^4 + 125t^3 + 25t^2 + 5t + 1, \left(t \mapsto t\log\left(25t^2 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x**2+x**3),x)
```

```
[Out] log(x + 1)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambda(
_t, _t*log(25*_t**2 + x)))
```

3.251 $\int x^p \left(ax^n + bx^{1+13n+p}\right)^{12} \, dx$

Optimal. Leaf size=29

$$\frac{\left(a + bx^{12n+p+1}\right)^{13}}{13b(12n + p + 1)}$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1584, 261}

$$\frac{\left(a + bx^{12n+p+1}\right)^{13}}{13b(12n + p + 1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^p*(a*x^n + b*x^(1 + 13*n + p))^12,x]
```

```
[Out] (a + b*x^(1 + 12*n + p))^13/(13*b*(1 + 12*n + p))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int x^p \left(ax^n + bx^{1+13n+p}\right)^{12} \, dx &= \int x^{12n+p} \left(a + bx^{1+12n+p}\right)^{12} \, dx \\
&= \frac{\left(a + bx^{1+12n+p}\right)^{13}}{13b(1 + 12n + p)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 1.00

$$\frac{\left(a + bx^{12n+p+1}\right)^{13}}{13b(12n + p + 1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^p*(a*x^n + b*x^(1 + 13*n + p))^12,x]
```

```
[Out] (a + b*x^(1 + 12*n + p))^13/(13*b*(1 + 12*n + p))
```

IntegrateAlgebraic [B] time = 0.09, size = 232, normalized size = 8.00

$$\frac{x^{12n+p+1} \left(13a^{12} + 78a^{11}bx^{12n+p+1} + 286a^{10}b^2x^{24n+2p+2} + 715a^9b^3x^{36n+3p+3} + 1287a^8b^4x^{48n+4p+4} + 1716a^7b^5x^{60n+5p+5} + 1716a^6b^6x^{72n+6p+6} + 1287a^5b^7x^{84n+7p+7} + 715a^4b^8x^{96n+8p+8} + 286a^3b^9x^{108n+9p+9} + 78a^2b^{10}x^{120n+10p+10} + 13ab^{11}x^{132n+11p+11} + b^{12}x^{144n+12p+12}\right)}{13(12n + p + 1)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^p*(a*x^n + b*x^(1 + 13*n + p))^12,x]
```

```
[Out] (x^(1 + 12*n + p)*(13*a^12 + 78*a^11*b*x^(1 + 12*n + p) + 286*a^10*b^2*x^(2
 + 24*n + 2*p) + 715*a^9*b^3*x^(3 + 36*n + 3*p) + 1287*a^8*b^4*x^(4 + 48*n
+ 4*p) + 1716*a^7*b^5*x^(5 + 60*n + 5*p) + 1716*a^6*b^6*x^(6 + 72*n + 6*p)
+ 1287*a^5*b^7*x^(7 + 84*n + 7*p) + 715*a^4*b^8*x^(8 + 96*n + 8*p) + 286*a^
3*b^9*x^(9 + 108*n + 9*p) + 78*a^2*b^10*x^(10 + 120*n + 10*p) + 13*a*b^11*x
^(11 + 132*n + 11*p) + b^12*x^(12 + 144*n + 12*p)))/(13*(1 + 12*n + p))
```

fricas [**B**] time = 0.43, size = 297, normalized size = 10.24

$$\frac{78\,a^{2}b^{10}x^{2\,n}x^{143\,n+11\,p+11}+286\,a^{3}b^{9}x^{3\,n}x^{130\,n+10\,p+10}+715\,a^{4}b^{8}x^{4\,n}x^{117\,n+9\,p+9}+1287\,a^{5}b^{7}x^{5\,n}x^{104\,n+8\,p+8}+1716\,a^{6}b^{6}x^{6\,n}x^{91\,n+7\,p+7}+1716\,a^{7}b^{5}x^{7\,n}x^{78\,n+6\,p+6}+1287\,a^{8}b^{4}x^{8\,n}x^{65\,n+5\,p+5}+715\,a^{9}b^{3}x^{9\,n}x^{52\,n+4\,p+4}+286\,a^{10}b^{2}x^{10\,n}x^{39\,n+3\,p+3}+78\,a^{11}bx^{11\,n}x^{26\,n+2\,p+2}+13\,a^{12}x^{12\,n}x^{13\,n+p+1}+13\,ab^{11}x^{156\,n+12\,p+12}x^{n}+b^{12}x^{169\,n+13\,p+13}}{13\,(12\,n+p+1)x^{13\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*(a*x^n+b*x^(1+13*n+p))^12,x, algorithm="fricas")
```

```
[Out] 1/13*(78*a^2*b^10*x^(2*n)*x^(143*n + 11*p + 11) + 286*a^3*b^9*x^(3*n)*x^(13
0*n + 10*p + 10) + 715*a^4*b^8*x^(4*n)*x^(117*n + 9*p + 9) + 1287*a^5*b^7*x
^(5*n)*x^(104*n + 8*p + 8) + 1716*a^6*b^6*x^(6*n)*x^(91*n + 7*p + 7) + 1716
*a^7*b^5*x^(7*n)*x^(78*n + 6*p + 6) + 1287*a^8*b^4*x^(8*n)*x^(65*n + 5*p +
5) + 715*a^9*b^3*x^(9*n)*x^(52*n + 4*p + 4) + 286*a^10*b^2*x^(10*n)*x^(39*n
 + 3*p + 3) + 78*a^11*b*x^(11*n)*x^(26*n + 2*p + 2) + 13*a^12*x^(12*n)*x^(1
3*n + p + 1) + 13*a*b^11*x^(156*n + 12*p + 12)*x^n + b^12*x^(169*n + 13*p +
 13))/((12*n + p + 1)*x^(13*n))
```

giac [**B**] time = 2.47, size = 269, normalized size = 9.28

$$\frac{b^{12}x^{13}x^{156\,n}x^{13\,p}+13\,ab^{11}x^{12}x^{144\,n}x^{12\,p}+78\,a^{2}b^{10}x^{11}x^{132\,n}x^{11\,p}+286\,a^{3}b^{9}x^{10}x^{120\,n}x^{10\,p}+715\,a^{4}b^{8}x^{9}x^{108\,n}x^{9\,p}+1287\,a^{5}b^{7}x^{8}x^{96\,n}x^{8\,p}+1716\,a^{6}b^{6}x^{7}x^{84\,n}x^{7\,p}+1716\,a^{7}b^{5}x^{6}x^{72\,n}x^{6\,p}+1287\,a^{8}b^{4}x^{5}x^{60\,n}x^{5\,p}+715\,a^{9}b^{3}x^{4}x^{48\,n}x^{4\,p}+286\,a^{10}b^{2}x^{3}x^{36\,n}x^{3\,p}+78\,a^{11}bx^{2}x^{24\,n}x^{2\,p}+13\,a^{12}xx^{12\,n}x^{p}}{13\,(12\,n+p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*(a*x^n+b*x^(1+13*n+p))^12,x, algorithm="giac")
```

```
[Out] 1/13*(b^12*x^13*x^(156*n)*x^(13*p) + 13*a*b^11*x^12*x^(144*n)*x^(12*p) + 78
*a^2*b^10*x^11*x^(132*n)*x^(11*p) + 286*a^3*b^9*x^10*x^(120*n)*x^(10*p) + 7
15*a^4*b^8*x^9*x^(108*n)*x^(9*p) + 1287*a^5*b^7*x^8*x^(96*n)*x^(8*p) + 1716
*a^6*b^6*x^7*x^(84*n)*x^(7*p) + 1716*a^7*b^5*x^6*x^(72*n)*x^(6*p) + 1287*a^
8*b^4*x^5*x^(60*n)*x^(5*p) + 715*a^9*b^3*x^4*x^(48*n)*x^(4*p) + 286*a^10*b^
2*x^3*x^(36*n)*x^(3*p) + 78*a^11*b*x^2*x^(24*n)*x^(2*p) + 13*a^12*x*x^(12*n
)*x^p)/(12*n + p + 1)
```

maple [**B**] time = 0.21, size = 363, normalized size = 12.52

$$\frac{b^{12}x^{13}x^{156n}x^{13p}}{13+156n+13p}+\frac{a\,b^{11}x^{12}x^{144n}x^{12p}}{12n+p+1}+\frac{6a^{2}b^{10}x^{11}x^{132n}x^{11p}}{12n+p+1}+\frac{22a^{3}b^{9}x^{10}x^{120n}x^{10p}}{12n+p+1}+\frac{55a^{4}b^{8}x^{9}x^{108n}x^{9p}}{12n+p+1}+\frac{99a^{5}b^{7}x^{8}x^{96n}x^{8p}}{12n+p+1}+\frac{132a^{6}b^{6}x^{7}x^{84n}x^{7p}}{12n+p+1}+\frac{132a^{7}b^{5}x^{6}x^{72n}x^{6p}}{12n+p+1}+\frac{99a^{8}b^{4}x^{5}x^{60n}x^{5p}}{12n+p+1}+\frac{55a^{9}b^{3}x^{4}x^{48n}x^{4p}}{12n+p+1}+\frac{22a^{10}b^{2}x^{3}x^{36n}x^{3p}}{12n+p+1}+\frac{6a^{11}b\,x^{2}x^{24n}x^{2p}}{12n+p+1}+\frac{a^{12}x\,x^{p}x^{12n}}{12n+p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^p*(a*x^n+b*x^(1+13*n+p))^12,x)
```

```
[Out] 1/13*b^12*x^13*(x^n)^156/(1+12*n+p)*(x^p)^13+a*b^11*x^12*(x^n)^144/(1+12*n+
p)*(x^p)^12+6*a^2*b^10*x^11*(x^n)^132/(1+12*n+p)*(x^p)^11+22*a^3*b^9*x^10*(
x^n)^120/(1+12*n+p)*(x^p)^10+55*a^4*b^8*x^9*(x^n)^108/(1+12*n+p)*(x^p)^9+99
*a^5*b^7*x^8*(x^n)^96/(1+12*n+p)*(x^p)^8+132*a^6*b^6*x^7*(x^n)^84/(1+12*n+p
)*(x^p)^7+132*a^7*b^5*x^6*(x^n)^72/(1+12*n+p)*(x^p)^6+99*a^8*b^4*x^5*(x^n)^
60/(1+12*n+p)*(x^p)^5+55*a^9*b^3*x^4*(x^n)^48/(1+12*n+p)*(x^p)^4+22*a^10*b^
2*x^3*(x^n)^36/(1+12*n+p)*(x^p)^3+6*a^11*b*x^2*(x^n)^24/(1+12*n+p)*(x^p)^2+
a^12/(1+12*n+p)*x*(x^n)^12*x^p
```

maxima [**B**] time = 1.48, size = 325, normalized size = 11.21

$$\frac{b^{12}x^{156\,n+13\,p+13}}{13\,(12\,n+p+1)}+\frac{ab^{11}x^{144\,n+12\,p+12}}{12\,n+p+1}+\frac{6\,a^{2}b^{10}x^{132\,n+11\,p+11}}{12\,n+p+1}+\frac{22\,a^{3}b^{9}x^{120\,n+10\,p+10}}{12\,n+p+1}+\frac{55\,a^{4}b^{8}x^{108\,n+9\,p+9}}{12\,n+p+1}+\frac{99\,a^{5}b^{7}x^{96\,n+8\,p+8}}{12\,n+p+1}+\frac{132\,a^{6}b^{6}x^{84\,n+7\,p+7}}{12\,n+p+1}+\frac{132\,a^{7}b^{5}x^{72\,n+6\,p+6}}{12\,n+p+1}+\frac{99\,a^{8}b^{4}x^{60\,n+5\,p+5}}{12\,n+p+1}+\frac{55\,a^{9}b^{3}x^{48\,n+4\,p+4}}{12\,n+p+1}+\frac{22\,a^{10}b^{2}x^{36\,n+3\,p+3}}{12\,n+p+1}+\frac{6\,a^{11}bx^{24\,n+2\,p+2}}{12\,n+p+1}+\frac{a^{12}x^{12\,n+p+1}}{12\,n+p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*(a*x^n+b*x^(1+13*n+p))^12,x, algorithm="maxima")
```

```
[Out] 1/13*b^12*x^(156*n + 13*p + 13)/(12*n + p + 1) + a*b^11*x^(144*n + 12*p + 1
2)/(12*n + p + 1) + 6*a^2*b^10*x^(132*n + 11*p + 11)/(12*n + p + 1) + 22*a^
3*b^9*x^(120*n + 10*p + 10)/(12*n + p + 1) + 55*a^4*b^8*x^(108*n + 9*p + 9)
/(12*n + p + 1) + 99*a^5*b^7*x^(96*n + 8*p + 8)/(12*n + p + 1) + 132*a^6*b^
6*x^(84*n + 7*p + 7)/(12*n + p + 1) + 132*a^7*b^5*x^(72*n + 6*p + 6)/(12*n
+ p + 1) + 99*a^8*b^4*x^(60*n + 5*p + 5)/(12*n + p + 1) + 55*a^9*b^3*x^(48*
n + 4*p + 4)/(12*n + p + 1) + 22*a^10*b^2*x^(36*n + 3*p + 3)/(12*n + p + 1)
 + 6*a^11*b*x^(24*n + 2*p + 2)/(12*n + p + 1) + a^12*x^(12*n + p + 1)/(12*n
 + p + 1)
```

mupad [B] time = 6.78, size = 363, normalized size = 12.52

$$\frac{a^{12}\,x\,x^p\,x^{12\,n}}{12\,n+p+1}+\frac{b^{12}\,x^{156\,n}\,x^{13\,p}\,x^{13}}{156\,n+13\,p+13}+\frac{22\,a^{10}\,b^2\,x^{36\,n}\,x^{3\,p}\,x^3}{12\,n+p+1}+\frac{55\,a^9\,b^3\,x^{48\,n}\,x^{4\,p}\,x^4}{12\,n+p+1}+\frac{99\,a^8\,b^4\,x^{60\,n}\,x^{5\,p}\,x^5}{12\,n+p+1}+\frac{132\,a^7\,b^5\,x^{72\,n}\,x^{6\,p}\,x^6}{12\,n+p+1}+\frac{132\,a^6\,b^6\,x^{84\,n}\,x^{7\,p}\,x^7}{12\,n+p+1}+\frac{99\,a^5\,b^7\,x^{96\,n}\,x^{8\,p}\,x^8}{12\,n+p+1}+\frac{55\,a^4\,b^8\,x^{108\,n}\,x^{9\,p}\,x^9}{12\,n+p+1}+\frac{22\,a^3\,b^9\,x^{120\,n}\,x^{10\,p}\,x^{10}}{12\,n+p+1}+\frac{6\,a^2\,b^{10}\,x^{132\,n}\,x^{11\,p}\,x^{11}}{12\,n+p+1}+\frac{6\,a^{11}\,b\,x^{24\,n}\,x^{2\,p}\,x^2}{12\,n+p+1}+\frac{a\,b^{11}\,x^{144\,n}\,x^{12\,p}\,x^{12}}{12\,n+p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^p*(a*x^n + b*x^(13*n + p + 1))^12,x)
```

```
[Out] (a^12*x*x^p*x^(12*n))/(12*n + p + 1) + (b^12*x^(156*n)*x^(13*p)*x^13)/(156*
n + 13*p + 13) + (22*a^10*b^2*x^(36*n)*x^(3*p)*x^3)/(12*n + p + 1) + (55*a^
9*b^3*x^(48*n)*x^(4*p)*x^4)/(12*n + p + 1) + (99*a^8*b^4*x^(60*n)*x^(5*p)*x
^5)/(12*n + p + 1) + (132*a^7*b^5*x^(72*n)*x^(6*p)*x^6)/(12*n + p + 1) + (1
32*a^6*b^6*x^(84*n)*x^(7*p)*x^7)/(12*n + p + 1) + (99*a^5*b^7*x^(96*n)*x^(8
*p)*x^8)/(12*n + p + 1) + (55*a^4*b^8*x^(108*n)*x^(9*p)*x^9)/(12*n + p + 1)
 + (22*a^3*b^9*x^(120*n)*x^(10*p)*x^10)/(12*n + p + 1) + (6*a^2*b^10*x^(132
*n)*x^(11*p)*x^11)/(12*n + p + 1) + (6*a^11*b*x^(24*n)*x^(2*p)*x^2)/(12*n +
 p + 1) + (a*b^11*x^(144*n)*x^(12*p)*x^12)/(12*n + p + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**p*(a*x**n+b*x**(1+13*n+p))**12,x)
```

```
[Out] Timed out
```

3.252 $\int x^{12} \left(a+bx^{13}\right)^{12} \, dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a + b*x^13)^12,x]
```

```
[Out] (a + b*x^13)^13/(169*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{12} \left(a+bx^{13}\right)^{12} \, dx = \frac{\left(a+bx^{13}\right)^{13}}{169b}$$

Mathematica [B] time = 0.01, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{13}}{13}+\frac{6}{13}a^{11}bx^{26}+\frac{22}{13}a^{10}b^2x^{39}+\frac{55}{13}a^9b^3x^{52}+\frac{99}{13}a^8b^4x^{65}+\frac{132}{13}a^7b^5x^{78}+\frac{132}{13}a^6b^6x^{91}+\frac{99}{13}a^5b^7x^{104}+\frac{55}{13}a^4b^8x^{117}+\frac{22}{13}a^3b^9x^{130}+\frac{6}{13}a^2b^{10}x^{143}+\frac{1}{13}ab^{11}x^{156}+\frac{b^{12}x^{169}}{169}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a + b*x^13)^12,x]
```

```
[Out] (a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x
^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)
/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13
 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{12} \left(a+bx^{13}\right)^{12} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^12*(a + b*x^13)^12,x]
```

```
[Out] IntegrateAlgebraic[x^12*(a + b*x^13)^12, x]
```

fricas [B] time = 0.35, size = 134, normalized size = 8.38

$$\frac{1}{169}x^{169}b^{12}+\frac{1}{13}x^{156}b^{11}a+\frac{6}{13}x^{143}b^{10}a^2+\frac{22}{13}x^{130}b^9a^3+\frac{55}{13}x^{117}b^8a^4+\frac{99}{13}x^{104}b^7a^5+\frac{132}{13}x^{91}b^6a^6+\frac{132}{13}x^{78}b^5a^7+\frac{99}{13}x^{65}b^4a^8+\frac{55}{13}x^{52}b^3a^9+\frac{22}{13}x^{39}b^2a^{10}+\frac{6}{13}x^{26}ba^{11}+\frac{1}{13}x^{13}a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^13+a)^12,x, algorithm="fricas")
```

```
[Out] 1/169*x^169*b^12 + 1/13*x^156*b^11*a + 6/13*x^143*b^10*a^2 + 22/13*x^130*b^
9*a^3 + 55/13*x^117*b^8*a^4 + 99/13*x^104*b^7*a^5 + 132/13*x^91*b^6*a^6 + 1
32/13*x^78*b^5*a^7 + 99/13*x^65*b^4*a^8 + 55/13*x^52*b^3*a^9 + 22/13*x^39*b
^2*a^10 + 6/13*x^26*b*a^11 + 1/13*x^13*a^12
```

giac [A] time = 0.15, size = 14, normalized size = 0.88

$$\frac{\left(bx^{13}+a\right)^{13}}{169\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^13+a)^12,x, algorithm="giac")
```

```
[Out] 1/169*(b*x^13 + a)^13/b
```

maple [B] time = 0.04, size = 135, normalized size = 8.44

$$\frac{1}{169}b^{12}x^{169}+\frac{1}{13}a\,b^{11}x^{156}+\frac{6}{13}a^{2}b^{10}x^{143}+\frac{22}{13}a^{3}b^{9}x^{130}+\frac{55}{13}a^{4}b^{8}x^{117}+\frac{99}{13}a^{5}b^{7}x^{104}+\frac{132}{13}a^{6}b^{6}x^{91}+\frac{132}{13}a^{7}b^{5}x^{78}+\frac{99}{13}a^{8}b^{4}x^{65}+\frac{55}{13}a^{9}b^{3}x^{52}+\frac{22}{13}a^{10}b^{2}x^{39}+\frac{6}{13}a^{11}b\,x^{26}+\frac{1}{13}a^{12}x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b*x^13+a)^12,x)
```

```
[Out] 1/169*b^12*x^169+1/13*a*b^11*x^156+6/13*a^2*b^10*x^143+22/13*a^3*b^9*x^130+
55/13*a^4*b^8*x^117+99/13*a^5*b^7*x^104+132/13*a^6*b^6*x^91+132/13*a^7*b^5*
x^78+99/13*a^8*b^4*x^65+55/13*a^9*b^3*x^52+22/13*a^10*b^2*x^39+6/13*a^11*b*
x^26+1/13*a^12*x^13
```

maxima [A] time = 1.33, size = 14, normalized size = 0.88

$$\frac{\left(bx^{13}+a\right)^{13}}{169\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^13+a)^12,x, algorithm="maxima")
```

```
[Out] 1/169*(b*x^13 + a)^13/b
```

mupad [B] time = 5.33, size = 14, normalized size = 0.88

$$\frac{\left(b\,x^{13}+a\right)^{13}}{169\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a + b*x^13)^12,x)
```

```
[Out] (a + b*x^13)^13/(169*b)
```

sympy [B] time = 0.11, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{13}}{13}+\frac{6a^{11}bx^{26}}{13}+\frac{22a^{10}b^{2}x^{39}}{13}+\frac{55a^{9}b^{3}x^{52}}{13}+\frac{99a^{8}b^{4}x^{65}}{13}+\frac{132a^{7}b^{5}x^{78}}{13}+\frac{132a^{6}b^{6}x^{91}}{13}+\frac{99a^{5}b^{7}x^{104}}{13}+\frac{55a^{4}b^{8}x^{117}}{13}+\frac{22a^{3}b^{9}x^{130}}{13}+\frac{6a^{2}b^{10}x^{143}}{13}+\frac{ab^{11}x^{156}}{13}+\frac{b^{12}x^{169}}{169}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b*x**13+a)**12,x)
```

```
[Out] a**12*x**13/13 + 6*a**11*b*x**26/13 + 22*a**10*b**2*x**39/13 + 55*a**9*b**3
*x**52/13 + 99*a**8*b**4*x**65/13 + 132*a**7*b**5*x**78/13 + 132*a**6*b**6*
x**91/13 + 99*a**5*b**7*x**104/13 + 55*a**4*b**8*x**117/13 + 22*a**3*b**9*x
**130/13 + 6*a**2*b**10*x**143/13 + a*b**11*x**156/13 + b**12*x**169/169
```

3.253 $\int x^{12} \left(ax + bx^{26}\right)^{12} dx$

Optimal. Leaf size=16

$$\frac{\left(a + bx^{25}\right)^{13}}{325b}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 261}

$$\frac{\left(a + bx^{25}\right)^{13}}{325b}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a*x + b*x^26)^12,x]
```

```
[Out] (a + b*x^25)^13/(325*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int x^{12} \left(ax + bx^{26}\right)^{12} dx &= \int x^{24} \left(a + bx^{25}\right)^{12} dx \\ &= \frac{\left(a + bx^{25}\right)^{13}}{325b} \end{aligned}$$

Mathematica [B] time = 0.01, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25} + \frac{6}{25}a^{11}bx^{50} + \frac{22}{25}a^{10}b^2x^{75} + \frac{11}{5}a^9b^3x^{100} + \frac{99}{25}a^8b^4x^{125} + \frac{132}{25}a^7b^5x^{150} + \frac{132}{25}a^6b^6x^{175} + \frac{99}{25}a^5b^7x^{200} + \frac{11}{5}a^4b^8x^{225} + \frac{22}{25}a^3b^9x^{250} + \frac{6}{25}a^2b^{10}x^{275} + \frac{1}{25}ab^{11}x^{300} + \frac{b^{12}x^{325}}{325}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a*x + b*x^26)^12,x]
```

```
[Out] (a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x
^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^1
75)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/
25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{12} \left(ax + bx^{26}\right)^{12} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^12*(a*x + b*x^26)^12,x]
```

```
[Out] IntegrateAlgebraic[x^12*(a*x + b*x^26)^12, x]
```

fricas [B] time = 0.35, size = 134, normalized size = 8.38

$$\frac{1}{325}x^{325}b^{12}+\frac{1}{25}x^{300}b^{11}a+\frac{6}{25}x^{275}b^{10}a^{2}+\frac{22}{25}x^{250}b^{9}a^{3}+\frac{11}{5}x^{225}b^{8}a^{4}+\frac{99}{25}x^{200}b^{7}a^{5}+\frac{132}{25}x^{175}b^{6}a^{6}+\frac{132}{25}x^{150}b^{5}a^{7}+\frac{99}{25}x^{125}b^{4}a^{8}+\frac{11}{5}x^{100}b^{3}a^{9}+\frac{22}{25}x^{75}b^{2}a^{10}+\frac{6}{25}x^{50}ba^{11}+\frac{1}{25}x^{25}a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/325*x^325*b^12 + 1/25*x^300*b^11*a + 6/25*x^275*b^10*a^2 + 22/25*x^250*b^
9*a^3 + 11/5*x^225*b^8*a^4 + 99/25*x^200*b^7*a^5 + 132/25*x^175*b^6*a^6 + 1
32/25*x^150*b^5*a^7 + 99/25*x^125*b^4*a^8 + 11/5*x^100*b^3*a^9 + 22/25*x^75
*b^2*a^10 + 6/25*x^50*b*a^11 + 1/25*x^25*a^12
```

giac [B] time = 0.16, size = 134, normalized size = 8.38

$$\frac{1}{325}b^{12}x^{325}+\frac{1}{25}ab^{11}x^{300}+\frac{6}{25}a^{2}b^{10}x^{275}+\frac{22}{25}a^{3}b^{9}x^{250}+\frac{11}{5}a^{4}b^{8}x^{225}+\frac{99}{25}a^{5}b^{7}x^{200}+\frac{132}{25}a^{6}b^{6}x^{175}+\frac{132}{25}a^{7}b^{5}x^{150}+\frac{99}{25}a^{8}b^{4}x^{125}+\frac{11}{5}a^{9}b^{3}x^{100}+\frac{22}{25}a^{10}b^{2}x^{75}+\frac{6}{25}a^{11}bx^{50}+\frac{1}{25}a^{12}x^{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

maple [B] time = 0.04, size = 135, normalized size = 8.44

$$\frac{1}{325}b^{12}x^{325}+\frac{1}{25}a\,b^{11}x^{300}+\frac{6}{25}a^{2}b^{10}x^{275}+\frac{22}{25}a^{3}b^{9}x^{250}+\frac{11}{5}a^{4}b^{8}x^{225}+\frac{99}{25}a^{5}b^{7}x^{200}+\frac{132}{25}a^{6}b^{6}x^{175}+\frac{132}{25}a^{7}b^{5}x^{150}+\frac{99}{25}a^{8}b^{4}x^{125}+\frac{11}{5}a^{9}b^{3}x^{100}+\frac{22}{25}a^{10}b^{2}x^{75}+\frac{6}{25}a^{11}b\,x^{50}+\frac{1}{25}a^{12}x^{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b*x^26+a*x)^12,x)
```

```
[Out] 1/325*b^12*x^325+1/25*a*b^11*x^300+6/25*a^2*b^10*x^275+22/25*a^3*b^9*x^250+
11/5*a^4*b^8*x^225+99/25*a^5*b^7*x^200+132/25*a^6*b^6*x^175+132/25*a^7*b^5*
x^150+99/25*a^8*b^4*x^125+11/5*a^9*b^3*x^100+22/25*a^10*b^2*x^75+6/25*a^11*
b*x^50+1/25*a^12*x^25
```

maxima [B] time = 1.34, size = 134, normalized size = 8.38

$$\frac{1}{325}b^{12}x^{325}+\frac{1}{25}ab^{11}x^{300}+\frac{6}{25}a^{2}b^{10}x^{275}+\frac{22}{25}a^{3}b^{9}x^{250}+\frac{11}{5}a^{4}b^{8}x^{225}+\frac{99}{25}a^{5}b^{7}x^{200}+\frac{132}{25}a^{6}b^{6}x^{175}+\frac{132}{25}a^{7}b^{5}x^{150}+\frac{99}{25}a^{8}b^{4}x^{125}+\frac{11}{5}a^{9}b^{3}x^{100}+\frac{22}{25}a^{10}b^{2}x^{75}+\frac{6}{25}a^{11}bx^{50}+\frac{1}{25}a^{12}x^{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

mupad [B] time = 0.00, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{25}}{25}+\frac{6\,a^{11}\,b\,x^{50}}{25}+\frac{22\,a^{10}\,b^2\,x^{75}}{25}+\frac{11\,a^9\,b^3\,x^{100}}{5}+\frac{99\,a^8\,b^4\,x^{125}}{25}+\frac{132\,a^7\,b^5\,x^{150}}{25}+\frac{132\,a^6\,b^6\,x^{175}}{25}+\frac{99\,a^5\,b^7\,x^{200}}{25}+\frac{11\,a^4\,b^8\,x^{225}}{5}+\frac{22\,a^3\,b^9\,x^{250}}{25}+\frac{6\,a^2\,b^{10}\,x^{275}}{25}+\frac{a\,b^{11}\,x^{300}}{25}+\frac{b^{12}\,x^{325}}{325}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a*x + b*x^26)^12,x)
```

```
[Out] (a^12*x^25)/25 + (b^12*x^325)/325 + (6*a^11*b*x^50)/25 + (a*b^11*x^300)/25
+ (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x^100)/5 + (99*a^8*b^4*x^125)/25 + (1
32*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^175)/25 + (99*a^5*b^7*x^200)/25 + (11
*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/25 + (6*a^2*b^10*x^275)/25
```

sympy [B] time = 0.13, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25}+\frac{6a^{11}bx^{50}}{25}+\frac{22a^{10}b^{2}x^{75}}{25}+\frac{11a^{9}b^{3}x^{100}}{5}+\frac{99a^{8}b^{4}x^{125}}{25}+\frac{132a^{7}b^{5}x^{150}}{25}+\frac{132a^{6}b^{6}x^{175}}{25}+\frac{99a^{5}b^{7}x^{200}}{25}+\frac{11a^{4}b^{8}x^{225}}{5}+\frac{22a^{3}b^{9}x^{250}}{25}+\frac{6a^{2}b^{10}x^{275}}{25}+\frac{ab^{11}x^{300}}{25}+\frac{b^{12}x^{325}}{325}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b*x**26+a*x)**12,x)
```

```
[Out] a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**3
*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b**
6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b**9
*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/325
```

3.254 $\int x^{12} \left(ax^2+bx^{39}\right)^{12} \, dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 261}

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a*x^2 + b*x^39)^12,x]
```

```
[Out] (a + b*x^37)^13/(481*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{12} \left(ax^2+bx^{39}\right)^{12} \, dx &= \int x^{36} \left(a+bx^{37}\right)^{12} \, dx\\ &= \frac{\left(a+bx^{37}\right)^{13}}{481b}\end{aligned}$$

Mathematica [B] time = 0.01, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{333}+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^2b^{10}x^{407}+\frac{1}{37}ab^{11}x^{444}+\frac{b^{12}x^{481}}{481}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a*x^2 + b*x^39)^12,x]
```

```
[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{12} \left(ax^2+bx^{39}\right)^{12} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^12*(a*x^2 + b*x^39)^12,x]
```

```
[Out] IntegrateAlgebraic[x^12*(a*x^2 + b*x^39)^12, x]
```

fricas [B] time = 0.35, size = 134, normalized size = 8.38

$$\frac{1}{481}x^{481}b^{12}+\frac{1}{37}x^{444}b^{11}a+\frac{6}{37}x^{407}b^{10}a^2+\frac{22}{37}x^{370}b^9a^3+\frac{55}{37}x^{333}b^8a^4+\frac{99}{37}x^{296}b^7a^5+\frac{132}{37}x^{259}b^6a^6+\frac{132}{37}x^{222}b^5a^7+\frac{99}{37}x^{185}b^4a^8+\frac{55}{37}x^{148}b^3a^9+\frac{22}{37}x^{111}b^2a^{10}+\frac{6}{37}x^{74}ba^{11}+\frac{1}{37}x^{37}a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^39+a*x^2)^12,x, algorithm="fricas")
```

```
[Out] 1/481*x^481*b^12 + 1/37*x^444*b^11*a + 6/37*x^407*b^10*a^2 + 22/37*x^370*b^
9*a^3 + 55/37*x^333*b^8*a^4 + 99/37*x^296*b^7*a^5 + 132/37*x^259*b^6*a^6 +
132/37*x^222*b^5*a^7 + 99/37*x^185*b^4*a^8 + 55/37*x^148*b^3*a^9 + 22/37*x^
111*b^2*a^10 + 6/37*x^74*b*a^11 + 1/37*x^37*a^12
```

giac [B] time = 0.15, size = 134, normalized size = 8.38

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}ab^{11}x^{444}+\frac{6}{37}a^2b^{10}x^{407}+\frac{22}{37}a^3b^9x^{370}+\frac{55}{37}a^4b^8x^{333}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^6b^6x^{259}+\frac{132}{37}a^7b^5x^{222}+\frac{99}{37}a^8b^4x^{185}+\frac{55}{37}a^9b^3x^{148}+\frac{22}{37}a^{10}b^2x^{111}+\frac{6}{37}a^{11}bx^{74}+\frac{1}{37}a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^39+a*x^2)^12,x, algorithm="giac")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

maple [B] time = 0.04, size = 135, normalized size = 8.44

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}a\,b^{11}x^{444}+\frac{6}{37}a^2b^{10}x^{407}+\frac{22}{37}a^3b^9x^{370}+\frac{55}{37}a^4b^8x^{333}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^6b^6x^{259}+\frac{132}{37}a^7b^5x^{222}+\frac{99}{37}a^8b^4x^{185}+\frac{55}{37}a^9b^3x^{148}+\frac{22}{37}a^{10}b^2x^{111}+\frac{6}{37}a^{11}b\,x^{74}+\frac{1}{37}a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b*x^39+a*x^2)^12,x)
```

```
[Out] 1/481*b^12*x^481+1/37*a*b^11*x^444+6/37*a^2*b^10*x^407+22/37*a^3*b^9*x^370+
55/37*a^4*b^8*x^333+99/37*a^5*b^7*x^296+132/37*a^6*b^6*x^259+132/37*a^7*b^5
*x^222+99/37*a^8*b^4*x^185+55/37*a^9*b^3*x^148+22/37*a^10*b^2*x^111+6/37*a^
11*b*x^74+1/37*a^12*x^37
```

maxima [B] time = 1.35, size = 134, normalized size = 8.38

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}ab^{11}x^{444}+\frac{6}{37}a^2b^{10}x^{407}+\frac{22}{37}a^3b^9x^{370}+\frac{55}{37}a^4b^8x^{333}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^6b^6x^{259}+\frac{132}{37}a^7b^5x^{222}+\frac{99}{37}a^8b^4x^{185}+\frac{55}{37}a^9b^3x^{148}+\frac{22}{37}a^{10}b^2x^{111}+\frac{6}{37}a^{11}bx^{74}+\frac{1}{37}a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^39+a*x^2)^12,x, algorithm="maxima")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

mupad [B] time = 5.22, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{37}}{37}+\frac{6\,a^{11}\,b\,x^{74}}{37}+\frac{22\,a^{10}\,b^2\,x^{111}}{37}+\frac{55\,a^9\,b^3\,x^{148}}{37}+\frac{99\,a^8\,b^4\,x^{185}}{37}+\frac{132\,a^7\,b^5\,x^{222}}{37}+\frac{132\,a^6\,b^6\,x^{259}}{37}+\frac{99\,a^5\,b^7\,x^{296}}{37}+\frac{55\,a^4\,b^8\,x^{333}}{37}+\frac{22\,a^3\,b^9\,x^{370}}{37}+\frac{6\,a^2\,b^{10}\,x^{407}}{37}+\frac{a\,b^{11}\,x^{444}}{37}+\frac{b^{12}\,x^{481}}{481}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a*x^2 + b*x^39)^12,x)
```

```
[Out] (a^12*x^37)/37 + (b^12*x^481)/481 + (6*a^11*b*x^74)/37 + (a*b^11*x^444)/37
+ (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37 +
(132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37 + (
55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37
```

sympy [B] time = 0.14, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37} + \frac{6a^{11}bx^{74}}{37} + \frac{22a^{10}b^2x^{111}}{37} + \frac{55a^9b^3x^{148}}{37} + \frac{99a^8b^4x^{185}}{37} + \frac{132a^7b^5x^{222}}{37} + \frac{132a^6b^6x^{259}}{37} + \frac{99a^5b^7x^{296}}{37} + \frac{55a^4b^8x^{333}}{37} + \frac{22a^3b^9x^{370}}{37} + \frac{6a^2b^{10}x^{407}}{37} + \frac{ab^{11}x^{444}}{37} + \frac{b^{12}x^{481}}{481}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b*x**39+a*x**2)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

3.255 $\int x^{24} \left(a + bx^{25}\right)^{12} \, dx$

Optimal. Leaf size=16

$$\frac{\left(a + bx^{25}\right)^{13}}{325b}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{\left(a + bx^{25}\right)^{13}}{325b}$$

Antiderivative was successfully verified.

```
[In] Int[x^24*(a + b*x^25)^12,x]
```

```
[Out] (a + b*x^25)^13/(325*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{24} \left(a + bx^{25}\right)^{12} \, dx = \frac{\left(a + bx^{25}\right)^{13}}{325b}$$

Mathematica [B] time = 0.01, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25} + \frac{6}{25}a^{11}bx^{50} + \frac{22}{25}a^{10}b^2x^{75} + \frac{11}{5}a^9b^3x^{100} + \frac{99}{25}a^8b^4x^{125} + \frac{132}{25}a^7b^5x^{150} + \frac{132}{25}a^6b^6x^{175} + \frac{99}{25}a^5b^7x^{200} + \frac{11}{5}a^4b^8x^{225} + \frac{22}{25}a^3b^9x^{250} + \frac{6}{25}a^2b^{10}x^{275} + \frac{1}{25}ab^{11}x^{300} + \frac{b^{12}x^{325}}{325}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^24*(a + b*x^25)^12,x]
```

```
[Out] (a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x
^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^1
75)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/
25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{24} \left(a + bx^{25}\right)^{12} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^24*(a + b*x^25)^12,x]
```

```
[Out] IntegrateAlgebraic[x^24*(a + b*x^25)^12, x]
```

fricas [B] time = 0.35, size = 134, normalized size = 8.38

$$\frac{1}{325}x^{325}b^{12} + \frac{1}{25}x^{300}b^{11}a + \frac{6}{25}x^{275}b^{10}a^2 + \frac{22}{25}x^{250}b^9a^3 + \frac{11}{5}x^{225}b^8a^4 + \frac{99}{25}x^{200}b^7a^5 + \frac{132}{25}x^{175}b^6a^6 + \frac{132}{25}x^{150}b^5a^7 + \frac{99}{25}x^{125}b^4a^8 + \frac{11}{5}x^{100}b^3a^9 + \frac{22}{25}x^{75}b^2a^{10} + \frac{6}{25}x^{50}ba^{11} + \frac{1}{25}x^{25}a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^25+a)^12,x, algorithm="fricas")
```

```
[Out] 1/325*x^325*b^12 + 1/25*x^300*b^11*a + 6/25*x^275*b^10*a^2 + 22/25*x^250*b^
9*a^3 + 11/5*x^225*b^8*a^4 + 99/25*x^200*b^7*a^5 + 132/25*x^175*b^6*a^6 + 1
32/25*x^150*b^5*a^7 + 99/25*x^125*b^4*a^8 + 11/5*x^100*b^3*a^9 + 22/25*x^75
*b^2*a^10 + 6/25*x^50*b*a^11 + 1/25*x^25*a^12
```

giac [A] time = 0.15, size = 14, normalized size = 0.88

$$\frac{\left(bx^{25}+a\right)^{13}}{325\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^25+a)^12,x, algorithm="giac")
```

```
[Out] 1/325*(b*x^25 + a)^13/b
```

maple [B] time = 0.04, size = 135, normalized size = 8.44

$$\frac{1}{325}b^{12}x^{325}+\frac{1}{25}a\,b^{11}x^{300}+\frac{6}{25}a^{2}b^{10}x^{275}+\frac{22}{25}a^{3}b^{9}x^{250}+\frac{11}{5}a^{4}b^{8}x^{225}+\frac{99}{25}a^{5}b^{7}x^{200}+\frac{132}{25}a^{6}b^{6}x^{175}+\frac{132}{25}a^{7}b^{5}x^{150}+\frac{99}{25}a^{8}b^{4}x^{125}+\frac{11}{5}a^{9}b^{3}x^{100}+\frac{22}{25}a^{10}b^{2}x^{75}+\frac{6}{25}a^{11}b\,x^{50}+\frac{1}{25}a^{12}x^{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24*(b*x^25+a)^12,x)
```

```
[Out] 1/325*b^12*x^325+1/25*a*b^11*x^300+6/25*a^2*b^10*x^275+22/25*a^3*b^9*x^250+
11/5*a^4*b^8*x^225+99/25*a^5*b^7*x^200+132/25*a^6*b^6*x^175+132/25*a^7*b^5*
x^150+99/25*a^8*b^4*x^125+11/5*a^9*b^3*x^100+22/25*a^10*b^2*x^75+6/25*a^11*
b*x^50+1/25*a^12*x^25
```

maxima [A] time = 1.35, size = 14, normalized size = 0.88

$$\frac{\left(bx^{25}+a\right)^{13}}{325\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^25+a)^12,x, algorithm="maxima")
```

```
[Out] 1/325*(b*x^25 + a)^13/b
```

mupad [B] time = 5.20, size = 14, normalized size = 0.88

$$\frac{\left(b\,x^{25}+a\right)^{13}}{325\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24*(a + b*x^25)^12,x)
```

```
[Out] (a + b*x^25)^13/(325*b)
```

sympy [B] time = 0.11, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25}+\frac{6a^{11}bx^{50}}{25}+\frac{22a^{10}b^{2}x^{75}}{25}+\frac{11a^{9}b^{3}x^{100}}{5}+\frac{99a^{8}b^{4}x^{125}}{25}+\frac{132a^{7}b^{5}x^{150}}{25}+\frac{132a^{6}b^{6}x^{175}}{25}+\frac{99a^{5}b^{7}x^{200}}{25}+\frac{11a^{4}b^{8}x^{225}}{5}+\frac{22a^{3}b^{9}x^{250}}{25}+\frac{6a^{2}b^{10}x^{275}}{25}+\frac{ab^{11}x^{300}}{25}+\frac{b^{12}x^{325}}{325}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**24*(b*x**25+a)**12,x)
```

```
[Out] a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**3
*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b**
6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b**9
*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/325
```

3.256 $\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 261}

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

Antiderivative was successfully verified.

```
[In] Int[x^24*(a*x + b*x^38)^12,x]
```

```
[Out] (a + b*x^37)^13/(481*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx &= \int x^{36}\left(a+bx^{37}\right)^{12}\,dx\\ &= \frac{\left(a+bx^{37}\right)^{13}}{481b}\end{aligned}$$

Mathematica [B] time = 0.01, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{333}+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^2b^{10}x^{407}+\frac{1}{37}ab^{11}x^{444}+\frac{b^{12}x^{481}}{481}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^24*(a*x + b*x^38)^12,x]
```

```
[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^24*(a*x + b*x^38)^12,x]
```

```
[Out] IntegrateAlgebraic[x^24*(a*x + b*x^38)^12, x]
```

fricas [B] time = 0.35, size = 134, normalized size = 8.38

$$\frac{1}{481}x^{481}b^{12}+\frac{1}{37}x^{444}b^{11}a+\frac{6}{37}x^{407}b^{10}a^{2}+\frac{22}{37}x^{370}b^{9}a^{3}+\frac{55}{37}x^{333}b^{8}a^{4}+\frac{99}{37}x^{296}b^{7}a^{5}+\frac{132}{37}x^{259}b^{6}a^{6}+\frac{132}{37}x^{222}b^{5}a^{7}+\frac{99}{37}x^{185}b^{4}a^{8}+\frac{55}{37}x^{148}b^{3}a^{9}+\frac{22}{37}x^{111}b^{2}a^{10}+\frac{6}{37}x^{74}ba^{11}+\frac{1}{37}x^{37}a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/481*x^481*b^12 + 1/37*x^444*b^11*a + 6/37*x^407*b^10*a^2 + 22/37*x^370*b^
9*a^3 + 55/37*x^333*b^8*a^4 + 99/37*x^296*b^7*a^5 + 132/37*x^259*b^6*a^6 +
132/37*x^222*b^5*a^7 + 99/37*x^185*b^4*a^8 + 55/37*x^148*b^3*a^9 + 22/37*x^
111*b^2*a^10 + 6/37*x^74*b*a^11 + 1/37*x^37*a^12
```

giac [B] time = 0.21, size = 134, normalized size = 8.38

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}ab^{11}x^{444}+\frac{6}{37}a^{2}b^{10}x^{407}+\frac{22}{37}a^{3}b^{9}x^{370}+\frac{55}{37}a^{4}b^{8}x^{333}+\frac{99}{37}a^{5}b^{7}x^{296}+\frac{132}{37}a^{6}b^{6}x^{259}+\frac{132}{37}a^{7}b^{5}x^{222}+\frac{99}{37}a^{8}b^{4}x^{185}+\frac{55}{37}a^{9}b^{3}x^{148}+\frac{22}{37}a^{10}b^{2}x^{111}+\frac{6}{37}a^{11}bx^{74}+\frac{1}{37}a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

maple [B] time = 0.04, size = 135, normalized size = 8.44

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}a\,b^{11}x^{444}+\frac{6}{37}a^{2}b^{10}x^{407}+\frac{22}{37}a^{3}b^{9}x^{370}+\frac{55}{37}a^{4}b^{8}x^{333}+\frac{99}{37}a^{5}b^{7}x^{296}+\frac{132}{37}a^{6}b^{6}x^{259}+\frac{132}{37}a^{7}b^{5}x^{222}+\frac{99}{37}a^{8}b^{4}x^{185}+\frac{55}{37}a^{9}b^{3}x^{148}+\frac{22}{37}a^{10}b^{2}x^{111}+\frac{6}{37}a^{11}b\,x^{74}+\frac{1}{37}a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24*(b*x^38+a*x)^12,x)
```

```
[Out] 1/481*b^12*x^481+1/37*a*b^11*x^444+6/37*a^2*b^10*x^407+22/37*a^3*b^9*x^370+
55/37*a^4*b^8*x^333+99/37*a^5*b^7*x^296+132/37*a^6*b^6*x^259+132/37*a^7*b^5
*x^222+99/37*a^8*b^4*x^185+55/37*a^9*b^3*x^148+22/37*a^10*b^2*x^111+6/37*a^
11*b*x^74+1/37*a^12*x^37
```

maxima [B] time = 1.31, size = 134, normalized size = 8.38

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}ab^{11}x^{444}+\frac{6}{37}a^{2}b^{10}x^{407}+\frac{22}{37}a^{3}b^{9}x^{370}+\frac{55}{37}a^{4}b^{8}x^{333}+\frac{99}{37}a^{5}b^{7}x^{296}+\frac{132}{37}a^{6}b^{6}x^{259}+\frac{132}{37}a^{7}b^{5}x^{222}+\frac{99}{37}a^{8}b^{4}x^{185}+\frac{55}{37}a^{9}b^{3}x^{148}+\frac{22}{37}a^{10}b^{2}x^{111}+\frac{6}{37}a^{11}bx^{74}+\frac{1}{37}a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

mupad [B] time = 0.00, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{37}}{37}+\frac{6\,a^{11}\,b\,x^{74}}{37}+\frac{22\,a^{10}\,b^2\,x^{111}}{37}+\frac{55\,a^9\,b^3\,x^{148}}{37}+\frac{99\,a^8\,b^4\,x^{185}}{37}+\frac{132\,a^7\,b^5\,x^{222}}{37}+\frac{132\,a^6\,b^6\,x^{259}}{37}+\frac{99\,a^5\,b^7\,x^{296}}{37}+\frac{55\,a^4\,b^8\,x^{333}}{37}+\frac{22\,a^3\,b^9\,x^{370}}{37}+\frac{6\,a^2\,b^{10}\,x^{407}}{37}+\frac{a\,b^{11}\,x^{444}}{37}+\frac{b^{12}\,x^{481}}{481}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24*(a*x + b*x^38)^12,x)
```

```
[Out] (a^12*x^37)/37 + (b^12*x^481)/481 + (6*a^11*b*x^74)/37 + (a*b^11*x^444)/37
+ (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37 +
(132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37 + (
55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37
```

sympy [B] time = 0.13, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37} + \frac{6a^{11}bx^{74}}{37} + \frac{22a^{10}b^2x^{111}}{37} + \frac{55a^9b^3x^{148}}{37} + \frac{99a^8b^4x^{185}}{37} + \frac{132a^7b^5x^{222}}{37} + \frac{132a^6b^6x^{259}}{37} + \frac{99a^5b^7x^{296}}{37} + \frac{55a^4b^8x^{333}}{37} + \frac{22a^3b^9x^{370}}{37} + \frac{6a^2b^{10}x^{407}}{37} + \frac{ab^{11}x^{444}}{37} + \frac{b^{12}x^{481}}{481}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**24*(b*x**38+a*x)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

3.257 $\int x^{36} \left(a+b x^{37}\right)^{12} \, dx$

Optimal. Leaf size=16

$$\frac{\left(a+b x^{37}\right)^{13}}{481 b}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{\left(a+b x^{37}\right)^{13}}{481 b}$$

Antiderivative was successfully verified.

```
[In] Int[x^36*(a + b*x^37)^12,x]
```

```
[Out] (a + b*x^37)^13/(481*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{36} \left(a+b x^{37}\right)^{12} \, dx=\frac{\left(a+b x^{37}\right)^{13}}{481 b}$$

Mathematica [B] time = 0.01, size = 160, normalized size = 10.00

$$\frac{a^{12} x^{37}}{37}+\frac{6}{37} a^{11} b x^{74}+\frac{22}{37} a^{10} b^2 x^{111}+\frac{55}{37} a^9 b^3 x^{148}+\frac{99}{37} a^8 b^4 x^{185}+\frac{132}{37} a^7 b^5 x^{222}+\frac{132}{37} a^6 b^6 x^{259}+\frac{99}{37} a^5 b^7 x^{296}+\frac{55}{37} a^4 b^8 x^{333}+\frac{22}{37} a^3 b^9 x^{370}+\frac{6}{37} a^2 b^{10} x^{407}+\frac{1}{37} a b^{11} x^{444}+\frac{b^{12} x^{481}}{481}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^36*(a + b*x^37)^12,x]
```

```
[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{36} \left(a+b x^{37}\right)^{12} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^36*(a + b*x^37)^12,x]
```

```
[Out] IntegrateAlgebraic[x^36*(a + b*x^37)^12, x]
```

fricas [B] time = 0.36, size = 134, normalized size = 8.38

$$\frac{1}{481} x^{481} b^{12}+\frac{1}{37} x^{444} b^{11} a+\frac{6}{37} x^{407} b^{10} a^2+\frac{22}{37} x^{370} b^9 a^3+\frac{55}{37} x^{333} b^8 a^4+\frac{99}{37} x^{296} b^7 a^5+\frac{132}{37} x^{259} b^6 a^6+\frac{132}{37} x^{222} b^5 a^7+\frac{99}{37} x^{185} b^4 a^8+\frac{55}{37} x^{148} b^3 a^9+\frac{22}{37} x^{111} b^2 a^{10}+\frac{6}{37} x^{74} b a^{11}+\frac{1}{37} x^{37} a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^36*(b*x^37+a)^12,x, algorithm="fricas")
```

```
[Out] 1/481*x^481*b^12 + 1/37*x^444*b^11*a + 6/37*x^407*b^10*a^2 + 22/37*x^370*b^
9*a^3 + 55/37*x^333*b^8*a^4 + 99/37*x^296*b^7*a^5 + 132/37*x^259*b^6*a^6 +
132/37*x^222*b^5*a^7 + 99/37*x^185*b^4*a^8 + 55/37*x^148*b^3*a^9 + 22/37*x^
111*b^2*a^10 + 6/37*x^74*b*a^11 + 1/37*x^37*a^12
```

giac [A] time = 0.15, size = 14, normalized size = 0.88

$$\frac{\left(bx^{37}+a\right)^{13}}{481\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^36*(b*x^37+a)^12,x, algorithm="giac")
```

```
[Out] 1/481*(b*x^37 + a)^13/b
```

maple [B] time = 0.04, size = 135, normalized size = 8.44

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}a\,b^{11}x^{444}+\frac{6}{37}a^{2}b^{10}x^{407}+\frac{22}{37}a^{3}b^{9}x^{370}+\frac{55}{37}a^{4}b^{8}x^{333}+\frac{99}{37}a^{5}b^{7}x^{296}+\frac{132}{37}a^{6}b^{6}x^{259}+\frac{132}{37}a^{7}b^{5}x^{222}+\frac{99}{37}a^{8}b^{4}x^{185}+\frac{55}{37}a^{9}b^{3}x^{148}+\frac{22}{37}a^{10}b^{2}x^{111}+\frac{6}{37}a^{11}b\,x^{74}+\frac{1}{37}a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^36*(b*x^37+a)^12,x)
```

```
[Out] 1/481*b^12*x^481+1/37*a*b^11*x^444+6/37*a^2*b^10*x^407+22/37*a^3*b^9*x^370+
55/37*a^4*b^8*x^333+99/37*a^5*b^7*x^296+132/37*a^6*b^6*x^259+132/37*a^7*b^5
*x^222+99/37*a^8*b^4*x^185+55/37*a^9*b^3*x^148+22/37*a^10*b^2*x^111+6/37*a^
11*b*x^74+1/37*a^12*x^37
```

maxima [A] time = 1.30, size = 14, normalized size = 0.88

$$\frac{\left(bx^{37}+a\right)^{13}}{481\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^36*(b*x^37+a)^12,x, algorithm="maxima")
```

```
[Out] 1/481*(b*x^37 + a)^13/b
```

mupad [B] time = 5.18, size = 14, normalized size = 0.88

$$\frac{\left(b\,x^{37}+a\right)^{13}}{481\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^36*(a + b*x^37)^12,x)
```

```
[Out] (a + b*x^37)^13/(481*b)
```

sympy [B] time = 0.12, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37}+\frac{6a^{11}bx^{74}}{37}+\frac{22a^{10}b^{2}x^{111}}{37}+\frac{55a^{9}b^{3}x^{148}}{37}+\frac{99a^{8}b^{4}x^{185}}{37}+\frac{132a^{7}b^{5}x^{222}}{37}+\frac{132a^{6}b^{6}x^{259}}{37}+\frac{99a^{5}b^{7}x^{296}}{37}+\frac{55a^{4}b^{8}x^{333}}{37}+\frac{22a^{3}b^{9}x^{370}}{37}+\frac{6a^{2}b^{10}x^{407}}{37}+\frac{ab^{11}x^{444}}{37}+\frac{b^{12}x^{481}}{481}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**36*(b*x**37+a)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

3.258 $\int \frac{1}{ax+bx^n}\,dx$

Optimal. Leaf size=23

$$\frac{\log\left(ax^{1-n}+b\right)}{a(1-n)}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 260}

$$\frac{\log\left(ax^{1-n}+b\right)}{a(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^n)^(-1),x]
```

```
[Out] Log[b + a*x^(1 - n)]/(a*(1 - n))
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax+bx^n}\,dx &= \int \frac{x^{-n}}{b+ax^{1-n}}\,dx\\ &= \frac{\log\left(b+ax^{1-n}\right)}{a(1-n)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\log\left(ax^{1-n}+b\right)}{a(1-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^n)^(-1),x]
```

```
[Out] Log[b + a*x^(1 - n)]/(a*(1 - n))
```

IntegrateAlgebraic [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{ax+bx^n}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x + b*x^n)^(-1),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a*x + b*x^n)^(-1), x]
```

fricas [A] time = 0.41, size = 27, normalized size = 1.17

$$\frac{n\log(x) - \log(ax + bx^n)}{an - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^n),x, algorithm="fricas")
[Out] (n*log(x) - log(a*x + b*x^n))/(a*n - a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ax + bx^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^n),x, algorithm="giac")
[Out] integrate(1/(a*x + b*x^n), x)
```

maple [A] time = 0.05, size = 36, normalized size = 1.57

$$\frac{n\ln(x)}{(n-1)\,a} - \frac{\ln\left(ax + b\,e^{n\ln(x)}\right)}{(n-1)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x^n),x)
[Out] n/a/(n-1)*ln(x)-1/a/(n-1)*ln(a*x+b*exp(n*ln(x)))
```

maxima [A] time = 1.42, size = 37, normalized size = 1.61

$$\frac{n\log(x)}{a(n-1)} - \frac{\log\left(\frac{ax+bx^n}{b}\right)}{a(n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^n),x, algorithm="maxima")
[Out] n*log(x)/(a*(n - 1)) - log((a*x + b*x^n)/b)/(a*(n - 1))
```

mupad [B] time = 5.26, size = 26, normalized size = 1.13

$$-\frac{\ln(b\,x^n + a\,x) - n\,\ln(x)}{a\,(n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n + a*x),x)
[Out] -(log(b*x^n + a*x) - n*log(x))/(a*(n - 1))
```

sympy [A] time = 0.68, size = 53, normalized size = 2.30

$$\begin{cases} \tilde{\infty}\log(x) & \text{for } a = 0 \wedge b = 0 \wedge n = 1 \\ -\frac{x}{b(nx^n - x^n)} & \text{for } a = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 1 \\ \frac{\log(x)}{a} & \text{for } b = 0 \\ \frac{n\log(x)}{an-a} - \frac{\log\left(\frac{ax}{b} + x^n\right)}{an-a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x**n),x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 1)), (-x/(b*(n*x**n - x*
*n)), Eq(a, 0)), (log(x)/(a + b), Eq(n, 1)), (log(x)/a, Eq(b, 0)), (n*log(x
)/(a*n - a) - log(a*x/b + x**n)/(a*n - a), True))
```

3.259 $\int \frac{1}{ax+bx^{1+n}}\,dx$

Optimal. Leaf size=23

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^n\right)}{an}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1593, 266, 36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^n\right)}{an}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^(1 + n))^(-1),x]
```

```
[Out] Log[x]/a - Log[a + b*x^n]/(a*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{ax+bx^{1+n}}\,dx &= \int \frac{1}{x\left(a+bx^n\right)}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^n\right)}{an}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx,x,x^n\right)}{an}\\
&= \frac{\log(x)}{a}-\frac{\log\left(a+bx^n\right)}{an}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.96

$$\frac{n\log(x)-\log\left(a+bx^n\right)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^(1 + n))^(-1),x]
[Out] (n*Log[x] - Log[a + b*x^n])/(a*n)
```

IntegrateAlgebraic [A] time = 0.03, size = 34, normalized size = 1.48

$$\frac{\log\left(x^n\right)}{an}-\frac{\log\left(a^2n+abnx^n\right)}{an}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x + b*x^(1 + n))^(-1),x]
[Out] Log[x^n]/(a*n) - Log[a^2*n + a*b*n*x^n]/(a*n)
```

fricas [A] time = 0.42, size = 28, normalized size = 1.22

$$\frac{(n+1)\log(x)-\log\left(ax+bx^{n+1}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1+n)),x, algorithm="fricas")
[Out] ((n + 1)*log(x) - log(a*x + b*x^(n + 1)))/(a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{ax+bx^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1+n)),x, algorithm="giac")
[Out] integrate(1/(a*x + b*x^(n + 1)), x)
```

maple [A] time = 0.05, size = 39, normalized size = 1.70

$$\frac{\ln(x)}{a}+\frac{\ln(x)}{an}-\frac{\ln\left(ax+b\,\mathrm{e}^{(n+1)\ln(x)}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x^(n+1)),x)
[Out] 1/a*ln(x)+1/a/n*ln(x)-1/a/n*ln(a*x+b*exp((n+1)*ln(x)))
```

maxima [A] time = 1.33, size = 27, normalized size = 1.17

$$\frac{\log(x)}{a}-\frac{\log\left(\frac{bx^n+a}{b}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1+n)),x, algorithm="maxima")
```

```
[Out] log(x)/a - log((b*x^n + a)/b)/(a*n)
```

mupad [B] time = 5.23, size = 31, normalized size = 1.35

$$\frac{\ln(x)\ (n+1)}{a\,n}-\frac{\ln\left(x\ (a+b\,x^n)\right)}{a\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^(n + 1)),x)
```

```
[Out] (log(x)*(n + 1))/(a*n) - log(x*(a + b*x^n))/(a*n)
```

sympy [A] time = 1.81, size = 41, normalized size = 1.78

$$\begin{cases} \tilde{\infty}\log{(x)} & \text{for}\: a = 0 \wedge b = 0 \wedge n = 0 \\ \frac{\log{(x)}}{a+b} & \text{for}\: n = 0 \\ -\frac{x^{-n}}{bn} & \text{for}\: a = 0 \\ \frac{\log{(x)}}{a} & \text{for}\: b = 0 \\ \frac{\log{(x)}}{a} - \frac{\log{\left(\frac{a}{b}+x^{n} \right)}}{an} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x**(1+n)),x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/(a + b), Eq
(n, 0)), (-x**(-n)/(b*n), Eq(a, 0)), (log(x)/a, Eq(b, 0)), (log(x)/a - log(
a/b + x**n)/(a*n), True))
```

3.260 $\int \frac{1}{ax+bx^{1-n}} \, dx$

Optimal. Leaf size=15

$$\frac{\log(ax^n+b)}{an}$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = $\{1593, 260\}$

$$\frac{\log(ax^n+b)}{an}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^(1 - n))^(-1),x]
```

```
[Out] Log[b + a*x^n]/(a*n)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{ax+bx^{1-n}} \, dx &= \int \frac{x^{-1+n}}{b+ax^n} \, dx \\ &= \frac{\log(b+ax^n)}{an} \end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log(ax^n+b)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^(1 - n))^(-1),x]
```

```
[Out] Log[b + a*x^n]/(a*n)
```

IntegrateAlgebraic [A] time = 0.03, size = 18, normalized size = 1.20

$$\frac{\log(anx^n+bn)}{an}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x + b*x^(1 - n))^(-1),x]
```

```
[Out] Log[b*n + a*n*x^n]/(a*n)
```

fricas [**A**] time = 0.42, size = 28, normalized size = 1.87

$$\frac{(n-1)\log(x) + \log\left(ax + bx^{-n+1}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1-n)),x, algorithm="fricas")
[Out] ((n - 1)*log(x) + log(a*x + b*x^(-n + 1)))/(a*n)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ax + bx^{-n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1-n)),x, algorithm="giac")
[Out] integrate(1/(a*x + b*x^(-n + 1)), x)
```

maple [**B**] time = 0.06, size = 41, normalized size = 2.73

$$\frac{\ln(x)}{a} - \frac{\ln(x)}{an} + \frac{\ln\left(ax + b\,\mathrm{e}^{(-n+1)\ln(x)}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x^(-n+1)),x)
[Out] 1/a*ln(x)-1/a/n*ln(x)+1/a/n*ln(a*x+b*exp((-n+1)*ln(x)))
```

maxima [**A**] time = 1.37, size = 19, normalized size = 1.27

$$\frac{\log\left(\frac{ax^n+b}{a}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1-n)),x, algorithm="maxima")
[Out] log((a*x^n + b)/a)/(a*n)
```

mupad [**B**] time = 5.22, size = 34, normalized size = 2.27

$$\frac{\ln\left(a\,x + b\,x^{1-n}\right)}{a\,n} + \frac{\ln(x)\,(n-1)}{a\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^(1 - n)),x)
[Out] log(a*x + b*x^(1 - n))/(a*n) + (log(x)*(n - 1))/(a*n)
```

sympy [**A**] time = 2.14, size = 39, normalized size = 2.60

$$\begin{cases} \tilde{\infty}\log(x) & \text{for}\ a = 0 \wedge b = 0 \wedge n = 0 \\ \frac{\log(x)}{a+b} & \text{for}\ n = 0 \\ \frac{x^n}{bn} & \text{for}\ a = 0 \\ \frac{\log(x)}{a} & \text{for}\ b = 0 \\ \frac{\log(x)}{a} + \frac{\log\left(\frac{a}{b} + x^{-n}\right)}{an} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x**(1-n)),x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/(a + b), Eq
(n, 0)), (x**n/(b*n), Eq(a, 0)), (log(x)/a, Eq(b, 0)), (log(x)/a + log(a/b
+ x**(-n))/(a*n), True))
```

3.261 $\int \frac{1}{2x+3x^{1+n}}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{2}-\frac{\log(3x^n+2)}{2n}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1593, 266, 36, 29, 31}

$$\frac{\log(x)}{2}-\frac{\log(3x^n+2)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + 3*x^(1 + n))^(-1),x]
```

```
[Out] Log[x]/2 - Log[2 + 3*x^n]/(2*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{2x+3x^{1+n}}\,dx &= \int \frac{1}{x\,(2+3x^n)}\,dx\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x(2+3x)}\,dx,x,x^n\right)}{n}\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^n\right)}{2n}-\frac{3\operatorname{Subst}\left(\int \frac{1}{2+3x}\,dx,x,x^n\right)}{2n}\\
&= \frac{\log(x)}{2}-\frac{\log(2+3x^n)}{2n}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{n\log(x)-\log(3x^n+2)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + 3*x^(1 + n))^(-1),x]
[Out] (n*Log[x] - Log[2 + 3*x^n])/(2*n)
```

IntegrateAlgebraic [A] time = 0.03, size = 30, normalized size = 1.36

$$\frac{\log(x^n)}{2n}-\frac{\log(3nx^n+2n)}{2n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*x + 3*x^(1 + n))^(-1),x]
[Out] Log[x^n]/(2*n) - Log[2*n + 3*n*x^n]/(2*n)
```

fricas [A] time = 0.40, size = 26, normalized size = 1.18

$$\frac{(n+1)\log(x)-\log\left(3\,x^{n+1}+2\,x\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x^(1+n)),x, algorithm="fricas")
[Out] 1/2*((n + 1)*log(x) - log(3*x^(n + 1) + 2*x))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{3\,x^{n+1}+2\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x^(1+n)),x, algorithm="giac")
[Out] integrate(1/(3*x^(n + 1) + 2*x), x)
```

maple [A] time = 0.05, size = 32, normalized size = 1.45

$$\frac{\ln(x)}{2}+\frac{\ln(x)}{2n}-\frac{\ln\left(2x+3\,\mathrm{e}^{(n+1)\ln(x)}\right)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x+3*x^(n+1)),x)
[Out] 1/2*ln(x)+1/2/n*ln(x)-1/2/n*ln(2*x+3*exp((n+1)*ln(x)))
```

maxima [A] time = 1.32, size = 16, normalized size = 0.73

$$-\frac{\log\left(x^n+\frac{2}{3}\right)}{2\,n}+\frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x^(1+n)),x, algorithm="maxima")
```

```
[Out] -1/2*log(x^n + 2/3)/n + 1/2*log(x)
```

mupad [B] time = 5.23, size = 26, normalized size = 1.18

$$\frac{\ln(x)\ (n+1)}{2\,n}-\frac{\ln\left(\frac{2\,x}{3}+x^{n+1}\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x + 3*x^(n + 1)),x)
[Out] (log(x)*(n + 1))/(2*n) - log((2*x)/3 + x^(n + 1))/(2*n)
```

sympy [A] time = 1.51, size = 20, normalized size = 0.91

$$\begin{cases} \frac{\log(x)}{2}-\frac{\log\left(x^n+\frac{2}{3}\right)}{2n} & \text{for } n \neq 0 \\ \frac{\log(x)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x**(1+n)),x)
[Out] Piecewise((log(x)/2 - log(x**n + 2/3)/(2*n), Ne(n, 0)), (log(x)/5, True))
```

3.262 $\int \frac{1}{2x+3x^{1-n}}\,dx$

Optimal. Leaf size=15

$$\frac{\log(2x^n+3)}{2n}$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = $\{1593, 260\}$

$$\frac{\log(2x^n+3)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + 3*x^(1 - n))^(-1),x]
```

```
[Out] Log[3 + 2*x^n]/(2*n)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{2x+3x^{1-n}}\,dx &= \int \frac{x^{-1+n}}{3+2x^n}\,dx \\ &= \frac{\log(3+2x^n)}{2n} \end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log(2x^n+3)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + 3*x^(1 - n))^(-1),x]
```

```
[Out] Log[3 + 2*x^n]/(2*n)
```

IntegrateAlgebraic [A] time = 0.02, size = 18, normalized size = 1.20

$$\frac{\log(2nx^n+3n)}{2n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*x + 3*x^(1 - n))^(-1),x]
```

```
[Out] Log[3*n + 2*n*x^n]/(2*n)
```

fricas [A] time = 0.40, size = 26, normalized size = 1.73

$$\frac{(n-1)\log(x) + \log\left(3x^{-n+1} + 2x\right)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x^(1-n)),x, algorithm="fricas")
[Out] 1/2*((n - 1)*log(x) + log(3*x^(-n + 1) + 2*x))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{3x^{-n+1} + 2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x^(1-n)),x, algorithm="giac")
[Out] integrate(1/(3*x^(-n + 1) + 2*x), x)
```

maple [B] time = 0.05, size = 34, normalized size = 2.27

$$\frac{\ln(x)}{2} - \frac{\ln(x)}{2n} + \frac{\ln\left(2x + 3\,e^{(-n+1)\ln(x)}\right)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x+3*x^(-n+1)),x)
[Out] 1/2*ln(x)-1/2/n*ln(x)+1/2/n*ln(2*x+3*exp((-n+1)*ln(x)))
```

maxima [A] time = 1.32, size = 11, normalized size = 0.73

$$\frac{\log\left(x^n + \frac{3}{2}\right)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x^(1-n)),x, algorithm="maxima")
[Out] 1/2*log(x^n + 3/2)/n
```

mupad [B] time = 5.20, size = 28, normalized size = 1.87

$$\frac{\ln\left(\frac{2x}{3} + x^{1-n}\right)}{2n} + \frac{\ln(x)\ (n-1)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x + 3*x^(1 - n)),x)
[Out] log((2*x)/3 + x^(1 - n))/(2*n) + (log(x)*(n - 1))/(2*n)
```

sympy [A] time = 1.70, size = 22, normalized size = 1.47

$$\begin{cases} \frac{\log(x)}{2} + \frac{\log\left(\frac{2}{3} + x^{-n}\right)}{2n} & \text{for}\ n \neq 0 \\ \frac{\log(x)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x**(1-n)),x)
```

```
[Out] Piecewise((log(x)/2 + log(2/3 + x**(-n))/(2*n), Ne(n, 0)), (log(x)/5, True)
)
```

3.263 $\int \frac{1}{-\sqrt{x}+x}\,dx$

Optimal. Leaf size=12

$$2\log\left(1-\sqrt{x}\right)$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 260}

$$2\log\left(1-\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[x] + x)^(-1),x]
```

```
[Out] 2*Log[1 - Sqrt[x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-\sqrt{x}+x}\,dx &= \int \frac{1}{\left(-1+\sqrt{x}\right)\sqrt{x}}\,dx\\ &= 2\log\left(1-\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$2\log\left(1-\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[x] + x)^(-1),x]
```

```
[Out] 2*Log[1 - Sqrt[x]]
```

IntegrateAlgebraic [A] time = 0.01, size = 10, normalized size = 0.83

$$2\log\left(\sqrt{x}-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-Sqrt[x] + x)^(-1),x]
```

```
[Out] 2*Log[-1 + Sqrt[x]]
```

fricas [A] time = 0.39, size = 8, normalized size = 0.67

$$2\,\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-x^(1/2)),x, algorithm="fricas")
[Out] 2*log(sqrt(x) - 1)
```

giac [A] time = 0.15, size = 9, normalized size = 0.75

$$2 \log \left(\left| \sqrt{x} - 1 \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-x^(1/2)),x, algorithm="giac")
[Out] 2*log(abs(sqrt(x) - 1))
```

maple [A] time = 0.05, size = 12, normalized size = 1.00

$$-2 \operatorname{arctanh}\left(\sqrt{x}\right) + \ln (x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-x^(1/2)),x)
[Out] ln(x-1)-2*arctanh(x^(1/2))
```

maxima [A] time = 1.33, size = 8, normalized size = 0.67

$$2 \log \left(\sqrt{x} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-x^(1/2)),x, algorithm="maxima")
[Out] 2*log(sqrt(x) - 1)
```

mupad [B] time = 0.10, size = 8, normalized size = 0.67

$$2 \ln \left(\sqrt{x} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - x^(1/2)),x)
[Out] 2*log(x^(1/2) - 1)
```

sympy [A] time = 0.18, size = 8, normalized size = 0.67

$$2 \log \left(\sqrt{x} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-x**(1/2)),x)
[Out] 2*log(sqrt(x) - 1)
```

3.264 $\int \frac{1}{-x^{3/5}+x}\,dx$

Optimal. Leaf size=14

$$\frac{5}{2}\log\left(1-x^{2/5}\right)$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 260}

$$\frac{5}{2}\log\left(1-x^{2/5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x^(3/5) + x)^(-1),x]
[Out] (5*Log[1 - x^(2/5)])/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-x^{3/5}+x}\,dx &= \int \frac{1}{\left(-1+x^{2/5}\right)x^{3/5}}\,dx\\ &= \frac{5}{2}\log\left(1-x^{2/5}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{5}{2}\log\left(1-x^{2/5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^(3/5) + x)^(-1),x]
[Out] (5*Log[1 - x^(2/5)])/2
```

IntegrateAlgebraic [A] time = 0.01, size = 25, normalized size = 1.79

$$\frac{5}{2}\log\left(\sqrt[5]{x}-1\right)+\frac{5}{2}\log\left(\sqrt[5]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x^(3/5) + x)^(-1),x]
[Out] (5*Log[-1 + x^(1/5)])/2 + (5*Log[1 + x^(1/5)])/2
```

fricas [A] time = 0.40, size = 8, normalized size = 0.57

$$\frac{5}{2} \log\left(x^{\frac{2}{5}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(3/5)+x),x, algorithm="fricas")
```

```
[Out] 5/2*log(x^(2/5) - 1)
```

giac [A] time = 0.20, size = 18, normalized size = 1.29

$$\frac{5}{2} \log\left(x^{\frac{1}{5}} + 1\right) + \frac{5}{2} \log\left(\left|x^{\frac{1}{5}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(3/5)+x),x, algorithm="giac")
```

```
[Out] 5/2*log(x^(1/5) + 1) + 5/2*log(abs(x^(1/5) - 1))
```

maple [B] time = 0.44, size = 116, normalized size = 8.29

$$2 \ln\left(x^{\frac{1}{5}} + 1\right) + \frac{\ln(x-1)}{2} + \frac{\ln(x+1)}{2} + 2 \ln\left(x^{\frac{1}{5}} - 1\right) - \frac{\ln\left(2x^{\frac{2}{5}} - \sqrt{5}\, x^{\frac{1}{5}} - x^{\frac{1}{5}} + 2\right)}{2} - \frac{\ln\left(2x^{\frac{2}{5}} - \sqrt{5}\, x^{\frac{1}{5}} + x^{\frac{1}{5}} + 2\right)}{2} - \frac{\ln\left(2x^{\frac{2}{5}} + \sqrt{5}\, x^{\frac{1}{5}} - x^{\frac{1}{5}} + 2\right)}{2} - \frac{\ln\left(2x^{\frac{2}{5}} + \sqrt{5}\, x^{\frac{1}{5}} + x^{\frac{1}{5}} + 2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^(3/5)+x),x)
```

```
[Out] 1/2*ln(x+1)+1/2*ln(x-1)+2*ln(x^(1/5)-1)-1/2*ln(-5^(1/2)*x^(1/5)+2*x^(2/5)+x
^(1/5)+2)-1/2*ln(5^(1/2)*x^(1/5)+2*x^(2/5)+x^(1/5)+2)-1/2*ln(2*x^(2/5)-5^(1
/2)*x^(1/5)-x^(1/5)+2)-1/2*ln(2*x^(2/5)+5^(1/2)*x^(1/5)-x^(1/5)+2)+2*ln(x^(
1/5)+1)
```

maxima [A] time = 1.30, size = 17, normalized size = 1.21

$$\frac{5}{2} \log\left(x^{\frac{1}{5}} + 1\right) + \frac{5}{2} \log\left(x^{\frac{1}{5}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(3/5)+x),x, algorithm="maxima")
```

```
[Out] 5/2*log(x^(1/5) + 1) + 5/2*log(x^(1/5) - 1)
```

mupad [B] time = 5.30, size = 8, normalized size = 0.57

$$\frac{5 \ln\left(x^{2/5} - 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - x^(3/5)),x)
```

```
[Out] (5*log(x^(2/5) - 1))/2
```

sympy [B] time = 0.37, size = 22, normalized size = 1.57

$$\frac{5 \log\left(\sqrt[5]{x} - 1\right)}{2} + \frac{5 \log\left(\sqrt[5]{x} + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**(3/5)+x),x)
```

```
[Out] 5*log(x**(1/5) - 1)/2 + 5*log(x**(1/5) + 1)/2
```

3.265 $\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx$

Optimal. Leaf size=12

$$\frac{3}{4}\log\left(x^{4/3}+1\right)$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1593, 260}

$$\frac{3}{4}\log\left(x^{4/3}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1/3) + x)^(-1),x]
[Out] (3*Log[1 + x^(4/3)])/4
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx &= \int \frac{\sqrt[3]{x}}{1+x^{4/3}}\,dx\\ &= \frac{3}{4}\log\left(1+x^{4/3}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{3}{4}\log\left(x^{4/3}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1/3) + x)^(-1),x]
[Out] (3*Log[1 + x^(4/3)])/4
```

IntegrateAlgebraic [A] time = 0.01, size = 12, normalized size = 1.00

$$\frac{3}{4}\log\left(x^{4/3}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(-1/3) + x)^(-1),x]
[Out] (3*Log[1 + x^(4/3)])/4
```

fricas [A] time = 0.39, size = 8, normalized size = 0.67

$$\frac{3}{4}\,\log\left(x^{\frac{4}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="fricas")
```

```
[Out] 3/4*log(x^(4/3) + 1)
```

giac [B] time = 0.15, size = 32, normalized size = 2.67

$$\frac{3}{4}\,\log\left(\sqrt{2}\,x^{\frac{1}{3}} + x^{\frac{2}{3}} + 1\right) + \frac{3}{4}\,\log\left(-\sqrt{2}\,x^{\frac{1}{3}} + x^{\frac{2}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="giac")
```

```
[Out] 3/4*log(sqrt(2)*x^(1/3) + x^(2/3) + 1) + 3/4*log(-sqrt(2)*x^(1/3) + x^(2/3)
 + 1)
```

maple [A] time = 0.04, size = 9, normalized size = 0.75

$$\frac{3\ln\left(x^{\frac{4}{3}} + 1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^(1/3)+x),x)
```

```
[Out] 3/4*ln(1+x^(4/3))
```

maxima [A] time = 2.89, size = 8, normalized size = 0.67

$$\frac{3}{4}\,\log\left(x^{\frac{4}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="maxima")
```

```
[Out] 3/4*log(x^(4/3) + 1)
```

mupad [B] time = 0.09, size = 8, normalized size = 0.67

$$\frac{3\,\ln\left(x^{4/3} + 1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + 1/x^(1/3)),x)
```

```
[Out] (3*log(x^(4/3) + 1))/4
```

sympy [A] time = 0.26, size = 10, normalized size = 0.83

$$\frac{3\log\left(x^{\frac{4}{3}} + 1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x**(1/3)+x),x)
```

```
[Out] 3*log(x**(4/3) + 1)/4
```

3.266 $\int \frac{1}{x+x^{\sqrt{2}}}\,dx$

Optimal. Leaf size=24

$$\log(x)-\left(1+\sqrt{2}\right)\log\left(x^{\sqrt{2}-1}+1\right)$$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1593, 266, 36, 29, 31}

$$\log(x)-\left(1+\sqrt{2}\right)\log\left(x^{\sqrt{2}-1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^Sqrt[2])^(-1),x]
```

```
[Out] Log[x] - (1 + Sqrt[2])*Log[1 + x^(-1 + Sqrt[2])]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x+x^{\sqrt{2}}}\,dx &= \int \frac{1}{x\left(1+x^{-1+\sqrt{2}}\right)}\,dx\\
&= \left(1+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx,x,x^{-1+\sqrt{2}}\right)\\
&= \left(-1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^{-1+\sqrt{2}}\right)+\left(1+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^{-1+\sqrt{2}}\right)\\
&= \log(x)-\left(1+\sqrt{2}\right)\log\left(1+x^{-1+\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 1.00

$$\log(x) - \left(1 + \sqrt{2}\right)\log\left(x^{\sqrt{2}-1} + 1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^Sqrt[2])^(-1),x]
[Out] Log[x] - (1 + Sqrt[2])*Log[1 + x^(-1 + Sqrt[2])]
```

IntegrateAlgebraic [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x + x^{\sqrt{2}}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x + x^Sqrt[2])^(-1),x]
[Out] Defer[IntegrateAlgebraic][(x + x^Sqrt[2])^(-1), x]
```

fricas [A] time = 0.42, size = 24, normalized size = 1.00

$$-\left(\sqrt{2} + 1\right)\log\left(x + x^{\left(\sqrt{2}\right)}\right) + \left(\sqrt{2} + 2\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(2^(1/2))),x, algorithm="fricas")
[Out] -(sqrt(2) + 1)*log(x + x^sqrt(2)) + (sqrt(2) + 2)*log(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x + x^{\left(\sqrt{2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(2^(1/2))),x, algorithm="giac")
[Out] integrate(1/(x + x^sqrt(2)), x)
```

maple [A] time = 0.16, size = 39, normalized size = 1.62

$$\sqrt{2}\,\ln(x) + 2\ln(x) - \sqrt{2}\,\ln\left(x + e^{\sqrt{2}\,\ln(x)}\right) - \ln\left(x + e^{\sqrt{2}\,\ln(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+x^(2^(1/2))),x)
[Out] 2^(1/2)*ln(x)+2*ln(x)-ln(x+exp(2^(1/2)*ln(x)))*2^(1/2)-ln(x+exp(2^(1/2)*ln(
x)))
```

maxima [A] time = 2.93, size = 31, normalized size = 1.29

$$\frac{\sqrt{2}\log(x)}{\sqrt{2} - 1} - \frac{\log\left(x + x^{\left(\sqrt{2}\right)}\right)}{\sqrt{2} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(2^(1/2))),x, algorithm="maxima")
```

```
[Out] sqrt(2)*log(x)/(sqrt(2) - 1) - log(x + x^sqrt(2))/(sqrt(2) - 1)
```

mupad [B] time = 5.25, size = 26, normalized size = 1.08

$$\ln(x)\left(\sqrt{2}+2\right)-\frac{\ln\left(x+x^{\sqrt{2}}\right)}{\sqrt{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + x^(2^(1/2))),x)
[Out] log(x)*(2^(1/2) + 2) - log(x + x^(2^(1/2)))/(2^(1/2) - 1)
```

sympy [A] time = 0.44, size = 32, normalized size = 1.33

$$-\frac{2\log(x)}{-2+\sqrt{2}}+\frac{\sqrt{2}\log\left(x+x^{\sqrt{2}}\right)}{-2+\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x**(2**(1/2))),x)
[Out] -2*log(x)/(-2 + sqrt(2)) + sqrt(2)*log(x + x**(sqrt(2)))/(-2 + sqrt(2))
```

3.267 $\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx$

Optimal. Leaf size=75

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}$$

Rubi [A] time = 0.11, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2028, 2029, 206}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (-2*Sqrt[a*x^j + b*x^n])/((j - n)*x^(j/2)) + (2*Sqrt[a]*ArcTanh[(Sqrt[a]*x^
(j/2))/Sqrt[a*x^j + b*x^n]])/(j - n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx &= -\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}+a\int\frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx\\ &= -\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}+\frac{(2a)\,\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}\\ &= -\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}\end{aligned}$$

Mathematica [A] time = 0.23, size = 104, normalized size = 1.39

$$-\frac{2x^{-j/2}\left(-\sqrt{a}\sqrt{b}\,x^{\frac{j+n}{2}}\sqrt{\frac{ax^{j-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{j-n}{2}}}{\sqrt{b}}\right)+ax^j+bx^n\right)}{(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (-2*(a*x^j + b*x^n - Sqrt[a]*Sqrt[b]*x^((j + n)/2)*Sqrt[1 + (a*x^(j - n))/b
]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[b]]))/((j - n)*x^(j/2)*Sqrt[a*x^j +
b*x^n])
```

IntegrateAlgebraic [F] time = 0.05, size = 0, normalized size = 0.00

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 - j/2)*Sqrt[a*x^j + b*x^n], x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^j+bx^n}\,x^{-\frac{1}{2}j-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^j + b*x^n)*x^(-1/2*j - 1), x)
```

maple [F] time = 0.87, size = 0, normalized size = 0.00

$$\int \sqrt{a\,x^j+b\,x^n}\;x^{-\frac{j}{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
```

```
[Out] int(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^j+bx^n}\,x^{-\frac{1}{2}j-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^j + b*x^n)*x^(-1/2*j - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x^j+b\,x^n}}{x^{\frac{j}{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^j + b*x^n)^(1/2)/x^(j/2 + 1),x)
[Out] int((a*x^j + b*x^n)^(1/2)/x^(j/2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-\frac{j}{2}-1}\sqrt{ax^j+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/2*j)*(a*x**j+b*x**n)**(1/2),x)
[Out] Integral(x**(-j/2 - 1)*sqrt(a*x**j + b*x**n), x)
```

3.268 $\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx$

Optimal. Leaf size=99

$$\frac{2\sqrt{a}\,x^{j/2}(cx)^{-j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}-\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)}$$

Rubi [A] time = 0.16, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2031, 2028, 2029, 206}

$$\frac{2\sqrt{a}\,x^{j/2}(cx)^{-j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}-\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (-2*Sqrt[a*x^j + b*x^n])/(c*(j - n)*(c*x)^(j/2)) + (2*Sqrt[a]*x^(j/2)*ArcTa
nh[(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(c*(j - n)*(c*x)^(j/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx = \frac{\left(x^{j/2}(cx)^{-j/2}\right)\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx}{c}$$
$$= -\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)} + \frac{\left(ax^{j/2}(cx)^{-j/2}\right)\int \frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx}{c}$$
$$= -\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)} + \frac{\left(2ax^{j/2}(cx)^{-j/2}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}$$
$$= -\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)} + \frac{2\sqrt{a}\,x^{j/2}(cx)^{-j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}$$

Mathematica [A] time = 0.07, size = 109, normalized size = 1.10

$$-\frac{2(cx)^{-j/2}\left(-\sqrt{a}\sqrt{b}\,x^{\frac{j+n}{2}}\sqrt{\frac{ax^{j-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{j-n}{2}}}{\sqrt{b}}\right)+ax^j+bx^n\right)}{c(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
[Out] (-2*(a*x^j + b*x^n - Sqrt[a]*Sqrt[b]*x^((j + n)/2)*Sqrt[1 + (a*x^(j - n))/b
]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[b]]))/(c*(j - n)*(c*x)^(j/2)*Sqrt[a*
x^j + b*x^n])
```

IntegrateAlgebraic [F] time = 0.12, size = 0, normalized size = 0.00

$$\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c*x)^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
[Out] Defer[IntegrateAlgebraic][(c*x)^(-1 - j/2)*Sqrt[a*x^j + b*x^n], x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^j+bx^n}\,(cx)^{-\frac{1}{2}j-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x^j + b*x^n)*(c*x)^(-1/2*j - 1), x)
```

maple [F] time = 0.95, size = 0, normalized size = 0.00

$$\int \sqrt{a\,x^{j}+b\,x^{n}}\;(c x)^{-\frac{j}{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1/2*j-1)*(a*x^j+b*x^n)^(1/2),x)
[Out] int((c*x)^(-1/2*j-1)*(a*x^j+b*x^n)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^{j}+bx^{n}}\,(cx)^{-\frac{1}{2}j-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^j + b*x^n)*(c*x)^(-1/2*j - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x^{j}+b\,x^{n}}}{{\left(c\,x\right)}^{\frac{j}{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^j + b*x^n)^(1/2)/(c*x)^(j/2 + 1),x)
[Out] int((a*x^j + b*x^n)^(1/2)/(c*x)^(j/2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx)^{-\frac{j}{2}-1}\sqrt{ax^{j}+bx^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1-1/2*j)*(a*x**j+b*x**n)**(1/2),x)
[Out] Integral((c*x)**(-j/2 - 1)*sqrt(a*x**j + b*x**n), x)
```

3.269 $\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx$

Optimal. Leaf size=91

$$\frac{2\sqrt{a}\,\sqrt{cx}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}$$

Rubi [A] time = 0.16, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2028, 2031, 2029, 206}

$$\frac{2\sqrt{a}\,\sqrt{cx}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^3 + b*x^n]/(c*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^n])/(c*(3 - n)*(c*x)^(3/2)) + (2*Sqrt[a]*Sqrt[c*x]*Arc
Tanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(c^3*(3 - n)*Sqrt[x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx = -\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}} + \frac{a\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx}{c^3}$$
$$= -\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}} + \frac{\left(a\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{ax^3+bx^n}}\,dx}{c^3\sqrt{x}}$$
$$= -\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}} + \frac{\left(2a\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}$$
$$= -\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}} + \frac{2\sqrt{a}\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}$$

Mathematica [A] time = 0.27, size = 103, normalized size = 1.13

$$\frac{2x\left(-\sqrt{a}\sqrt{b}\,x^{\frac{n+3}{2}}\sqrt{\frac{ax^{3-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)+ax^3+bx^n\right)}{(n-3)(cx)^{5/2}\sqrt{ax^3+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^3 + b*x^n]/(c*x)^(5/2),x]
```

```
[Out] (2*x*(a*x^3 + b*x^n - Sqrt[a]*Sqrt[b]*x^((3 + n)/2)*Sqrt[1 + (a*x^(3 - n))/
b]*ArcSinh[(Sqrt[a]*x^(3/2 - n/2))/Sqrt[b]]))/((-3 + n)*(c*x)^(5/2)*Sqrt[a*
x^3 + b*x^n])
```

IntegrateAlgebraic [F] time = 3.30, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[a*x^3 + b*x^n]/(c*x)^(5/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][Sqrt[a*x^3 + b*x^n]/(c*x)^(5/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^3 + b*x^n)/(c*x)^(5/2), x)
```

maple [F] time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\,x^{3}+b\,x^{n}}}{\left(cx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x)
[Out] int((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{3}+bx^{n}}}{\left(cx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^3 + b*x^n)/(c*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^{n}+a\,x^{3}}}{{\left(c\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x^3)^(1/2)/(c*x)^(5/2),x)
[Out] int((b*x^n + a*x^3)^(1/2)/(c*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{3}+bx^{n}}}{\left(cx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b*x**n)**(1/2)/(c*x)**(5/2),x)
[Out] Integral(sqrt(a*x**3 + b*x**n)/(c*x)**(5/2), x)
```

3.270 $\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx$

Optimal. Leaf size=71

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{c^2(2-n)}-\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x}$$

Rubi [A] time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 2028, 2008, 206}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{c^2(2-n)}-\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^n]/(c^2*x^2),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^n])/(c^2*(2 - n)*x) + (2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/S
qrt[a*x^2 + b*x^n]])/(c^2*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx = \frac{\int \frac{\sqrt{ax^2+bx^n}}{x^2}\,dx}{c^2}$$
$$= -\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x} + \frac{a\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx}{c^2}$$
$$= -\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x} + \frac{(2a)\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^n}}\right)}{c^2(2-n)}$$
$$= -\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x} + \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{c^2(2-n)}$$

Mathematica [A] time = 0.16, size = 99, normalized size = 1.39

$$\frac{2\left(-\sqrt{a}\,\sqrt{b}\,x^{\frac{n}{2}+1}\sqrt{\frac{ax^{2-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{1-\frac{n}{2}}}{\sqrt{b}}\right)+ax^2+bx^n\right)}{c^2(n-2)x\sqrt{ax^2+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^n]/(c^2*x^2),x]
```

```
[Out] (2*(a*x^2 + b*x^n - Sqrt[a]*Sqrt[b]*x^(1 + n/2)*Sqrt[1 + (a*x^(2 - n))/b]*A
rcSinh[(Sqrt[a]*x^(1 - n/2))/Sqrt[b]]))/(c^2*(-2 + n)*x*Sqrt[a*x^2 + b*x^n]
)
```

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[a*x^2 + b*x^n]/(c^2*x^2),x]
```

```
[Out] Defer[IntegrateAlgebraic][Sqrt[a*x^2 + b*x^n]/(c^2*x^2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x^n)^(1/2)/c^2/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x^n)^(1/2)/c^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^2 + b*x^n)/(c^2*x^2), x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\,x^{2}+b\,x^{n}}}{c^{2}x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b*x^n)^(1/2)/c^2/x^2,x)
[Out] int((a*x^2+b*x^n)^(1/2)/c^2/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{ax^{2}+bx^{n}}}{x^{2}}\, dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x^n)^(1/2)/c^2/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(a*x^2 + b*x^n)/x^2, x)/c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^{n}+a\,x^{2}}}{c^{2}\,x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x^2)^(1/2)/(c^2*x^2),x)
[Out] int((b*x^n + a*x^2)^(1/2)/(c^2*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{ax^{2}+bx^{n}}}{x^{2}}\, dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b*x**n)**(1/2)/c**2/x**2,x)
[Out] Integral(sqrt(a*x**2 + b*x**n)/x**2, x)/c**2
```

3.271 $\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}} dx$

Optimal. Leaf size=87

$$\frac{2\sqrt{a}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}} - \frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}$$

Rubi [A] time = 0.14, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2028, 2031, 2029, 206}

$$\frac{2\sqrt{a}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}} - \frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*x^n]/(c*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[a*x + b*x^n])/(c*(1 - n)*Sqrt[c*x]) + (2*Sqrt[a]*Sqrt[x]*ArcTanh[(
Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(c*(1 - n)*Sqrt[c*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx = -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}} + \frac{a\int \frac{1}{\sqrt{cx}\,\sqrt{ax+bx^n}}\,dx}{c}$$
$$= -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}} + \frac{\left(a\sqrt{x}\right)\int \frac{1}{\sqrt{x}\,\sqrt{ax+bx^n}}\,dx}{c\sqrt{cx}}$$
$$= -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}} + \frac{\left(2a\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}}$$
$$= -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}} + \frac{2\sqrt{a}\,\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}}$$

Mathematica [A] time = 0.29, size = 100, normalized size = 1.15

$$\frac{x\left(-2\sqrt{a}\,\sqrt{b}\,x^{\frac{n+1}{2}}\sqrt{\frac{ax^{1-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)+2ax+2bx^n\right)}{(n-1)(cx)^{3/2}\sqrt{ax+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^n]/(c*x)^(3/2),x]
```

```
[Out] (x*(2*a*x + 2*b*x^n - 2*Sqrt[a]*Sqrt[b]*x^((1 + n)/2)*Sqrt[1 + (a*x^(1 - n)
)/b]*ArcSinh[(Sqrt[a]*x^(1/2 - n/2))/Sqrt[b]]))/((-1 + n)*(c*x)^(3/2)*Sqrt[
a*x + b*x^n])
```

IntegrateAlgebraic [F] time = 1.03, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[a*x + b*x^n]/(c*x)^(3/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][Sqrt[a*x + b*x^n]/(c*x)^(3/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+bx^n}}{(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + b*x^n)/(c*x)^(3/2), x)
```

maple [F] time = 0.97, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+b\,x^{n}}}{(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x)
[Out] int((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+bx^{n}}}{(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x + b*x^n)/(c*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^{n}+a\,x}}{{(c\,x)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x)^(1/2)/(c*x)^(3/2),x)
[Out] int((b*x^n + a*x)^(1/2)/(c*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+bx^{n}}}{(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x**n)**(1/2)/(c*x)**(3/2),x)
[Out] Integral(sqrt(a*x + b*x**n)/(c*x)**(3/2), x)
```

3.272 $\int \frac{\sqrt{a+bx^n}}{cx}\,dx$

Optimal. Leaf size=51

$$\frac{2\sqrt{a+bx^n}}{cn}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {12, 266, 50, 63, 208}

$$\frac{2\sqrt{a+bx^n}}{cn}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^n]/(c*x),x]
```

```
[Out] (2*Sqrt[a + b*x^n])/(c*n) - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(c
*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^n}}{cx}\,dx = \frac{\int \frac{\sqrt{a+bx^n}}{x}\,dx}{c}$$
$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^n\right)}{cn}$$
$$= \frac{2\sqrt{a+bx^n}}{cn} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{cn}$$
$$= \frac{2\sqrt{a+bx^n}}{cn} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{bcn}$$
$$= \frac{2\sqrt{a+bx^n}}{cn} - \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.90

$$\frac{2\sqrt{a+bx^n} - 2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^n]/(c*x),x]
[Out] (2*Sqrt[a + b*x^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(c*n)
```

IntegrateAlgebraic [A] time = 0.05, size = 51, normalized size = 1.00

$$\frac{2\sqrt{a+bx^n}}{cn} - \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^n]/(c*x),x]
[Out] (2*Sqrt[a + b*x^n])/(c*n) - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(c
*n)
```

fricas [A] time = 0.43, size = 97, normalized size = 1.90

$$\left[\frac{\sqrt{a}\log\left(\frac{bx^n - 2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right) + 2\sqrt{bx^n+a}}{cn}, \frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right) + \sqrt{bx^n+a}\right)}{cn}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2)/c/x,x, algorithm="fricas")
[Out] [(sqrt(a)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*sqrt(b*x^n
 + a))/(c*n), 2*(sqrt(-a)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a) + sqrt(b*x^n +
 a))/(c*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n+a}}{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2)/c/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)/(c*x), x)
```

maple [A] time = 0.04, size = 39, normalized size = 0.76

$$\frac{-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{n}+a}}{\sqrt{a}}\right)+2\sqrt{b\,x^{n}+a}}{c n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^(1/2)/c/x,x)
```

```
[Out] 1/c/n*(2*(b*x^n+a)^(1/2)-2*a^(1/2)*arctanh((b*x^n+a)^(1/2)/a^(1/2)))
```

maxima [A] time = 2.96, size = 58, normalized size = 1.14

$$\frac{\frac{\sqrt{a}\log\left(\frac{\sqrt{bx^{n}+a}-\sqrt{a}}{\sqrt{bx^{n}+a}+\sqrt{a}}\right)}{n}+\frac{2\,\sqrt{bx^{n}+a}}{n}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2)/c/x,x, algorithm="maxima")
```

```
[Out] (sqrt(a)*log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/n + 2
*sqrt(b*x^n + a)/n)/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+b\,x^{n}}}{c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(1/2)/(c*x),x)
```

```
[Out] int((a + b*x^n)^(1/2)/(c*x), x)
```

sympy [A] time = 1.78, size = 78, normalized size = 1.53

$$\frac{-\frac{2\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{n}+\frac{2ax^{-\frac{n}{2}}}{\sqrt{b}n\sqrt{\frac{ax^{-n}}{b}+1}}+\frac{2\sqrt{b}x^{\frac{n}{2}}}{n\sqrt{\frac{ax^{-n}}{b}+1}}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(1/2)/c/x,x)
```

```
[Out] (-2*sqrt(a)*asinh(sqrt(a)*x**(-n/2)/sqrt(b))/n + 2*a*x**(-n/2)/(sqrt(b)*n*s
qrt(a*x**(-n)/b + 1)) + 2*sqrt(b)*x**(n/2)/(n*sqrt(a*x**(-n)/b + 1)))/c
```

3.273 $\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx$

Optimal. Leaf size=84

$$\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(n+1)}-\frac{2\sqrt{a}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(n+1)\sqrt{cx}}$$

Rubi [A] time = 0.17, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2028, 2031, 2029, 206}

$$\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(n+1)}-\frac{2\sqrt{a}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(n+1)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x + b*x^n]/Sqrt[c*x],x]
```

```
[Out] (2*Sqrt[c*x]*Sqrt[a/x + b*x^n])/(c*(1 + n)) - (2*Sqrt[a]*Sqrt[x]*ArcTanh[Sq
rt[a]/(Sqrt[x]*Sqrt[a/x + b*x^n])])/((1 + n)*Sqrt[c*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx &= \frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(1+n)}+(ac)\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx\\ &= \frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(1+n)}+\frac{\left(a\sqrt{x}\right)\int \frac{1}{x^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx}{\sqrt{cx}}\\ &= \frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(1+n)}-\frac{\left(2a\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{1}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}}\\ &= \frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(1+n)}-\frac{2\sqrt{a}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 85, normalized size = 1.01

$$\frac{x\sqrt{\frac{a}{x}+bx^n}\left(2\sqrt{a+bx^{n+1}}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+1}}}{\sqrt{a}}\right)\right)}{(n+1)\sqrt{cx}\sqrt{a+bx^{n+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x + b*x^n]/Sqrt[c*x],x]
```

```
[Out] (x*Sqrt[a/x + b*x^n]*(2*Sqrt[a + b*x^(1 + n)] - 2*Sqrt[a]*ArcTanh[Sqrt[a +
b*x^(1 + n)]/Sqrt[a]]))/((1 + n)*Sqrt[c*x]*Sqrt[a + b*x^(1 + n)])
```

IntegrateAlgebraic [A] time = 1.27, size = 104, normalized size = 1.24

$$\frac{\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}\left(\frac{2\sqrt{a+bx^{n+1}}}{\sqrt{c}(n+1)}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+1}}}{\sqrt{a}}\right)}{\sqrt{c}(n+1)}\right)}{\sqrt{c}\sqrt{a+bx^{n+1}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a/x + b*x^n]/Sqrt[c*x],x]
```

```
[Out] (Sqrt[c*x]*Sqrt[a/x + b*x^n]*((2*Sqrt[a + b*x^(1 + n)])/(Sqrt[c]*(1 + n)) -
 (2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^(1 + n)]/Sqrt[a]])/(Sqrt[c]*(1 + n))))/(Sq
rt[c]*Sqrt[a + b*x^(1 + n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n+\frac{a}{x}}}{\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a/x)/sqrt(c*x), x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,x^{n}+\frac{a}{x}}}{\sqrt{cx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x)
```

```
[Out] int((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^{n}+\frac{a}{x}}}{\sqrt{cx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a/x)/sqrt(c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^{n}+\frac{a}{x}}}{\sqrt{c\,x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a/x)^(1/2)/(c*x)^(1/2),x)
```

```
[Out] int((b*x^n + a/x)^(1/2)/(c*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x}+bx^{n}}}{\sqrt{cx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x**n)**(1/2)/(c*x)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x + b*x**n)/sqrt(c*x), x)
```

3.274 $\int \sqrt{\frac{a}{x^2}+bx^n}\,dx$

Optimal. Leaf size=61

$$\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{n+2}$$

Rubi [A] time = 0.08, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2007, 2029, 206}

$$\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{n+2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^2 + b*x^n],x]
```

```
[Out] (2*x*Sqrt[a/x^2 + b*x^n])/(2 + n) - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^
2 + b*x^n])])/(2 + n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2007

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{a}{x^2}+bx^n}\,dx &= \frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}+a\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx\\ &= \frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}-\frac{(2a)\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{1}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}\\ &= \frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}\end{aligned}$$

Mathematica [A] time = 0.06, size = 78, normalized size = 1.28

$$\frac{x\sqrt{\frac{a}{x^2}+bx^n}\left(2\sqrt{a+bx^{n+2}}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+2}}}{\sqrt{a}}\right)\right)}{(n+2)\sqrt{a+bx^{n+2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^2 + b*x^n],x]
```

```
[Out] (x*Sqrt[a/x^2 + b*x^n]*(2*Sqrt[a + b*x^(2 + n)] - 2*Sqrt[a]*ArcTanh[Sqrt[a
+ b*x^(2 + n)]/Sqrt[a]]))/((2 + n)*Sqrt[a + b*x^(2 + n)])
```

IntegrateAlgebraic [A] time = 0.10, size = 83, normalized size = 1.36

$$\frac{x\sqrt{\frac{a}{x^2}+bx^n}\left(\frac{2\sqrt{a+bx^{n+2}}}{n+2}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+2}}}{\sqrt{a}}\right)}{n+2}\right)}{\sqrt{a+bx^{n+2}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a/x^2 + b*x^n],x]
```

```
[Out] (x*Sqrt[a/x^2 + b*x^n]*((2*Sqrt[a + b*x^(2 + n)])/(2 + n) - (2*Sqrt[a]*ArcT
anh[Sqrt[a + b*x^(2 + n)]/Sqrt[a]])/(2 + n)))/Sqrt[a + b*x^(2 + n)]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{bx^n+\frac{a}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a/x^2), x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int\sqrt{b\,x^n+\frac{a}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^2+b*x^n)^(1/2),x)
```

```
[Out] int((a/x^2+b*x^n)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{bx^n+\frac{a}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a/x^2), x)
```

mupad [B] time = 5.17, size = 97, normalized size = 1.59

$$\frac{x\sqrt{b\,x^n+\frac{a}{x^2}}}{\frac{n}{2}+1}+\frac{\sqrt{a}\,x\,\mathrm{asin}\left(\frac{\sqrt{a}\,1\mathrm{i}}{\sqrt{b}\,x^{\frac{n}{2}+1}}\right)\sqrt{b\,x^n+\frac{a}{x^2}}\,1\mathrm{i}}{\sqrt{b}\,x^{\frac{n}{2}+1}\,\left(\frac{n}{2}+1\right)\sqrt{\frac{a}{b\,x^{n+2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a/x^2)^(1/2),x)
```

```
[Out] (x*(b*x^n + a/x^2)^(1/2))/(n/2 + 1) + (a^(1/2)*x*asin((a^(1/2)*1i)/(b^(1/2)
*x^(n/2 + 1)))*(b*x^n + a/x^2)^(1/2)*1i)/(b^(1/2)*x^(n/2 + 1)*(n/2 + 1)*(a/
(b*x^(n + 2)) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{a}{x^2}+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**2+b*x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**2 + b*x**n), x)
```

3.275 $\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx$

Optimal. Leaf size=85

$$\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(n+3)}-\frac{2\sqrt{a}\,c\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(n+3)\sqrt{cx}}$$

Rubi [A] time = 0.20, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2028, 2031, 2029, 206}

$$\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(n+3)}-\frac{2\sqrt{a}\,c\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(n+3)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]*Sqrt[a/x^3 + b*x^n],x]
```

```
[Out] (2*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n])/(c*(3 + n)) - (2*Sqrt[a]*c*Sqrt[x]*ArcT
anh[Sqrt[a]/(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/((3 + n)*Sqrt[c*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx &= \frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(3+n)} + \left(ac^3\right)\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx \\ &= \frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(3+n)} + \frac{\left(ac\sqrt{x}\right)\int \frac{1}{x^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx}{\sqrt{cx}} \\ &= \frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(3+n)} - \frac{\left(2ac\sqrt{x}\right)\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(3+n)\sqrt{cx}} \\ &= \frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(3+n)} - \frac{2\sqrt{a}\,c\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(3+n)\sqrt{cx}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 85, normalized size = 1.00

$$\frac{x\sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\left(2\sqrt{a+bx^{n+3}}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+3}}}{\sqrt{a}}\right)\right)}{(n+3)\sqrt{a+bx^{n+3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]*Sqrt[a/x^3 + b*x^n],x]
```

```
[Out] (x*Sqrt[c*x]*Sqrt[a/x^3 + b*x^n]*(2*Sqrt[a + b*x^(3 + n)] - 2*Sqrt[a]*ArcTa
nh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]]))/((3 + n)*Sqrt[a + b*x^(3 + n)])
```

IntegrateAlgebraic [A] time = 1.37, size = 104, normalized size = 1.22

$$\frac{(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}\left(\frac{2\sqrt{c}\sqrt{a+bx^{n+3}}}{n+3}-\frac{2\sqrt{a}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+3}}}{\sqrt{a}}\right)}{n+3}\right)}{c^{3/2}\sqrt{a+bx^{n+3}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x]*Sqrt[a/x^3 + b*x^n],x]
```

```
[Out] ((c*x)^(3/2)*Sqrt[a/x^3 + b*x^n]*((2*Sqrt[c]*Sqrt[a + b*x^(3 + n)])/(3 + n)
 - (2*Sqrt[a]*Sqrt[c]*ArcTanh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]])/(3 + n)))/(c^
(3/2)*Sqrt[a + b*x^(3 + n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n+\frac{a}{x^3}}\sqrt{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a/x^3)*sqrt(c*x), x)
```

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int \sqrt{cx}\, \sqrt{b\, x^{n}+\frac{a}{x^{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^{n}+\frac{a}{x^{3}}}\, \sqrt{cx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a/x^3)*sqrt(c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\, x}\, \sqrt{b\, x^{n}+\frac{a}{x^{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(b*x^n + a/x^3)^(1/2),x)
```

```
[Out] int((c*x)^(1/2)*(b*x^n + a/x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx}\, \sqrt{\frac{a}{x^{3}}+bx^{n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)*(a/x**3+b*x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(c*x)*sqrt(a/x**3 + b*x**n), x)
```

3.276 $\int (cx)^{-1-\frac{3j}{2}} \left(ax^j + bx^n\right)^{3/2} dx$

Optimal. Leaf size=141

$$\frac{2a^{3/2}x^{3j/2}(cx)^{-3j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)} - \frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)} - \frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)}$$

Rubi [A] time = 0.23, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2031, 2028, 2029, 206}

$$\frac{2a^{3/2}x^{3j/2}(cx)^{-3j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)} - \frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)} - \frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - (3*j)/2)*(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (-2*a*x^j*Sqrt[a*x^j + b*x^n])/(c*(j - n)*(c*x)^((3*j)/2)) - (2*(a*x^j + b*
x^n)^(3/2))/(3*c*(j - n)*(c*x)^((3*j)/2)) + (2*a^(3/2)*x^((3*j)/2)*ArcTanh[
(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(c*(j - n)*(c*x)^((3*j)/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int (cx)^{-1-\frac{3j}{2}}\left(ax^j+bx^n\right)^{3/2}dx = \frac{\left(x^{3j/2}(cx)^{-3j/2}\right)\int x^{-1-\frac{3j}{2}}\left(ax^j+bx^n\right)^{3/2}dx}{c}$$
$$= -\frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)} + \frac{\left(ax^{3j/2}(cx)^{-3j/2}\right)\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx}{c}$$
$$= -\frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)} - \frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)} + \frac{\left(a^2x^{3j/2}(cx)^{-3j/2}\right)\int \frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx}}}{c}$$
$$= -\frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)} - \frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)} + \frac{\left(2a^2x^{3j/2}(cx)^{-3j/2}\right)\text{Subst}\left(\right.}{c(j-}$$
$$= -\frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)} - \frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)} + \frac{2a^{3/2}x^{3j/2}(cx)^{-3j/2}\tanh^{-1}\left(\right.}{c(j-n)}$$

Mathematica [A] time = 0.26, size = 131, normalized size = 0.93

$$-\frac{2(cx)^{-3j/2}\left(-3a^{3/2}\sqrt{b}\,x^{\frac{1}{2}(3j+n)}\sqrt{\frac{ax^{j-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{j-n}{2}}}{\sqrt{b}}\right)+4a^2x^{2j}+5abx^{j+n}+b^2x^{2n}\right)}{3c(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - (3*j)/2)*(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (-2*(4*a^2*x^(2*j) + b^2*x^(2*n) + 5*a*b*x^(j + n) - 3*a^(3/2)*Sqrt[b]*x^((
3*j + n)/2)*Sqrt[1 + (a*x^(j - n))/b]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[
b]]))/(3*c*(j - n)*(c*x)^((3*j)/2)*Sqrt[a*x^j + b*x^n])
```

IntegrateAlgebraic [F] time = 0.14, size = 0, normalized size = 0.00

$$\int (cx)^{-1-\frac{3j}{2}}\left(ax^j+bx^n\right)^{3/2}dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c*x)^(-1 - (3*j)/2)*(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(c*x)^(-1 - (3*j)/2)*(a*x^j + b*x^n)^(3/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^j+bx^n\right)^{\frac{3}{2}}(cx)^{-\frac{3}{2}j-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate((a*x^j + b*x^n)^(3/2)*(c*x)^(-3/2*j - 1), x)
```

maple [F] time = 0.87, size = 0, normalized size = 0.00

$$\int \left(a\,x^{j}+b\,x^{n}\right)^{\frac{3}{2}} (cx)^{-\frac{3j}{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x)
[Out] int((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^{j}+bx^{n}\right)^{\frac{3}{2}} (cx)^{-\frac{3}{2}j-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate((a*x^j + b*x^n)^(3/2)*(c*x)^(-3/2*j - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x^{j}+b\,x^{n}\right)^{3/2}}{(c\,x)^{\frac{3j}{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^j + b*x^n)^(3/2)/(c*x)^((3*j)/2 + 1),x)
[Out] int((a*x^j + b*x^n)^(3/2)/(c*x)^((3*j)/2 + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1-3/2*j)*(a*x**j+b*x**n)**(3/2),x)
[Out] Timed out
```

3.277 $\int \frac{\left(ax^3+bx^n\right)^{3/2}}{(cx)^{11/2}}\,dx$

Optimal. Leaf size=128

$$\frac{2a^{3/2}\sqrt{cx}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^6(3-n)\sqrt{x}}-\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}}-\frac{2\left(ax^3+bx^n\right)^{3/2}}{3c(3-n)(cx)^{9/2}}$$

Rubi [A] time = 0.21, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2028, 2031, 2029, 206}

$$\frac{2a^{3/2}\sqrt{cx}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^6(3-n)\sqrt{x}}-\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}}-\frac{2\left(ax^3+bx^n\right)^{3/2}}{3c(3-n)(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2),x]
```

```
[Out] (-2*a*Sqrt[a*x^3 + b*x^n])/(c^4*(3 - n)*(c*x)^(3/2)) - (2*(a*x^3 + b*x^n)^(
3/2))/(3*c*(3 - n)*(c*x)^(9/2)) + (2*a^(3/2)*Sqrt[c*x]*ArcTanh[(Sqrt[a]*x^(
3/2))/Sqrt[a*x^3 + b*x^n]])/(c^6*(3 - n)*Sqrt[x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{\left(ax^3+bx^n\right)^{3/2}}{(cx)^{11/2}}\,dx = -\frac{2\left(ax^3+bx^n\right)^{3/2}}{3c(3-n)(cx)^{9/2}} + \frac{a\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx}{c^3}$$

$$= -\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}} - \frac{2\left(ax^3+bx^n\right)^{3/2}}{3c(3-n)(cx)^{9/2}} + \frac{a^2\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx}{c^6}$$

$$= -\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}} - \frac{2\left(ax^3+bx^n\right)^{3/2}}{3c(3-n)(cx)^{9/2}} + \frac{\left(a^2\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{ax^3+bx^n}}\,dx}{c^6\sqrt{x}}$$

$$= -\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}} - \frac{2\left(ax^3+bx^n\right)^{3/2}}{3c(3-n)(cx)^{9/2}} + \frac{\left(2a^2\sqrt{cx}\right)\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^6(3-n)\sqrt{x}}$$

$$= -\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}} - \frac{2\left(ax^3+bx^n\right)^{3/2}}{3c(3-n)(cx)^{9/2}} + \frac{2a^{3/2}\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^6(3-n)\sqrt{x}}$$

Mathematica [A] time = 0.33, size = 126, normalized size = 0.98

$$\frac{2\sqrt{cx}\left(-3a^{3/2}\sqrt{b}\,x^{\frac{n+9}{2}}\sqrt{\frac{ax^{3-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)+4a^2x^6+5abx^{n+3}+b^2x^{2n}\right)}{3c^6(n-3)x^5\sqrt{ax^3+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2),x]
```

```
[Out] (2*Sqrt[c*x]*(4*a^2*x^6 + b^2*x^(2*n) + 5*a*b*x^(3 + n) - 3*a^(3/2)*Sqrt[b]
*x^((9 + n)/2)*Sqrt[1 + (a*x^(3 - n))/b]*ArcSinh[(Sqrt[a]*x^(3/2 - n/2))/Sq
rt[b]]))/(3*c^6*(-3 + n)*x^5*Sqrt[a*x^3 + b*x^n])
```

IntegrateAlgebraic [F] time = 6.40, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^3+bx^n\right)^{3/2}}{(cx)^{11/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^3+bx^n\right)^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2), x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{3}+b\,x^{n}\right)^{\frac{3}{2}}}{\left(cx\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x)
```

```
[Out] int((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{3}+bx^{n}\right)^{\frac{3}{2}}}{\left(cx\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^n+a\,x^3\right)^{3/2}}{\left(c\,x\right)^{11/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x^3)^(3/2)/(c*x)^(11/2),x)
```

```
[Out] int((b*x^n + a*x^3)^(3/2)/(c*x)^(11/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b*x**n)**(3/2)/(c*x)**(11/2),x)
```

```
[Out] Timed out
```

3.278 $\int \frac{\left(ax^2+bx^n\right)^{3/2}}{c^4x^4} dx$

Optimal. Leaf size=104

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{c^4(2-n)}-\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x}-\frac{2\left(ax^2+bx^n\right)^{3/2}}{3c^4(2-n)x^3}$$

Rubi [A] time = 0.13, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 2028, 2008, 206}

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{c^4(2-n)}-\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x}-\frac{2\left(ax^2+bx^n\right)^{3/2}}{3c^4(2-n)x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^n)^(3/2)/(c^4*x^4),x]
```

```
[Out] (-2*a*Sqrt[a*x^2 + b*x^n])/(c^4*(2 - n)*x) - (2*(a*x^2 + b*x^n)^(3/2))/(3*c
^4*(2 - n)*x^3) + (2*a^(3/2)*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^n]])/(c^4
*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4}\,dx &= \frac{\int \frac{(ax^2+bx^n)^{3/2}}{x^4}\,dx}{c^4} \\
&= -\frac{2(ax^2+bx^n)^{3/2}}{3c^4(2-n)x^3} + \frac{a\int \frac{\sqrt{ax^2+bx^n}}{x^2}\,dx}{c^4} \\
&= -\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x} - \frac{2(ax^2+bx^n)^{3/2}}{3c^4(2-n)x^3} + \frac{a^2\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx}{c^4} \\
&= -\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x} - \frac{2(ax^2+bx^n)^{3/2}}{3c^4(2-n)x^3} + \frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^n}}\right)}{c^4(2-n)} \\
&= -\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x} - \frac{2(ax^2+bx^n)^{3/2}}{3c^4(2-n)x^3} + \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{c^4(2-n)}
\end{aligned}
$$

Mathematica [A] time = 0.17, size = 117, normalized size = 1.12

$$
\frac{2\left(-3a^{3/2}\sqrt{b}\,x^{\frac{n}{2}+3}\sqrt{\frac{ax^{2-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{1-\frac{n}{2}}}{\sqrt{b}}\right)+4a^2x^4+5abx^{n+2}+b^2x^{2n}\right)}{3c^4(n-2)x^3\sqrt{ax^2+bx^n}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^n)^(3/2)/(c^4*x^4),x]
```

```
[Out] (2*(4*a^2*x^4 + b^2*x^(2*n) + 5*a*b*x^(2 + n) - 3*a^(3/2)*Sqrt[b]*x^(3 + n/
2)*Sqrt[1 + (a*x^(2 - n))/b]*ArcSinh[(Sqrt[a]*x^(1 - n/2))/Sqrt[b]]))/(3*c^
4*(-2 + n)*x^3*Sqrt[a*x^2 + b*x^n])
```

IntegrateAlgebraic [F] time = 0.24, size = 0, normalized size = 0.00

$$
\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x^n)^(3/2)/(c^4*x^4),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a*x^2 + b*x^n)^(3/2)/(c^4*x^4), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x^n)^(3/2)/c^4/x^4,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{(ax^2+bx^n)^{\frac{3}{2}}}{c^4x^4}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x^n)^(3/2)/c^4/x^4,x, algorithm="giac")
[Out] integrate((a*x^2 + b*x^n)^(3/2)/(c^4*x^4), x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{2}+b\,x^{n}\right)^{\frac{3}{2}}}{c^{4}x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b*x^n)^(3/2)/c^4/x^4,x)
[Out] int((a*x^2+b*x^n)^(3/2)/c^4/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(ax^{2}+bx^{n}\right)^{\frac{3}{2}}}{x^{4}}\,dx}{c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x^n)^(3/2)/c^4/x^4,x, algorithm="maxima")
[Out] integrate((a*x^2 + b*x^n)^(3/2)/x^4, x)/c^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^n+a\,x^2\right)}^{3/2}}{c^4\,x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x^2)^(3/2)/(c^4*x^4),x)
[Out] int((b*x^n + a*x^2)^(3/2)/(c^4*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a\sqrt{ax^{2}+bx^{n}}}{x^{2}}\,dx+\int \frac{bx^{n}\sqrt{ax^{2}+bx^{n}}}{x^{4}}\,dx}{c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b*x**n)**(3/2)/c**4/x**4,x)
[Out] (Integral(a*sqrt(a*x**2 + b*x**n)/x**2, x) + Integral(b*x**n*sqrt(a*x**2 +
b*x**n)/x**4, x))/c**4
```

3.279 $\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx$

Optimal. Leaf size=122

$$\frac{2a^{3/2}\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}}-\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}}-\frac{2\left(ax+bx^n\right)^{3/2}}{3c(1-n)(cx)^{3/2}}$$

Rubi [A] time = 0.19, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2028, 2031, 2029, 206}

$$\frac{2a^{3/2}\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}}-\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}}-\frac{2\left(ax+bx^n\right)^{3/2}}{3c(1-n)(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^n)^(3/2)/(c*x)^(5/2),x]
```

```
[Out] (-2*a*Sqrt[a*x + b*x^n])/(c^2*(1 - n)*Sqrt[c*x]) - (2*(a*x + b*x^n)^(3/2))/
(3*c*(1 - n)*(c*x)^(3/2)) + (2*a^(3/2)*Sqrt[x]*ArcTanh[(Sqrt[a]*Sqrt[x])/Sq
rt[a*x + b*x^n]])/(c^2*(1 - n)*Sqrt[c*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx = -\frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}} + \frac{a\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx}{c}$$
$$= -\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}} - \frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}} + \frac{a^2\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx}{c^2}$$
$$= -\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}} - \frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}} + \frac{\left(a^2\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{ax+bx^n}}\,dx}{c^2\sqrt{cx}}$$
$$= -\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}} - \frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}} + \frac{\left(2a^2\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}}$$
$$= -\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}} - \frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}} + \frac{2a^{3/2}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}}$$

Mathematica [A] time = 0.28, size = 120, normalized size = 0.98

$$\frac{x\left(-6a^{3/2}\sqrt{b}\,x^{\frac{n+3}{2}}\sqrt{\frac{ax^{1-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)+8a^2x^2+10abx^{n+1}+2b^2x^{2n}\right)}{3(n-1)(cx)^{5/2}\sqrt{ax+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^n)^(3/2)/(c*x)^(5/2),x]
```

```
[Out] (x*(8*a^2*x^2 + 2*b^2*x^(2*n) + 10*a*b*x^(1 + n) - 6*a^(3/2)*Sqrt[b]*x^((3
+ n)/2)*Sqrt[1 + (a*x^(1 - n))/b]*ArcSinh[(Sqrt[a]*x^(1/2 - n/2))/Sqrt[b]])
)/(3*(-1 + n)*(c*x)^(5/2)*Sqrt[a*x + b*x^n])
```

IntegrateAlgebraic [F] time = 2.40, size = 0, normalized size = 0.00

$$\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x + b*x^n)^(3/2)/(c*x)^(5/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a*x + b*x^n)^(3/2)/(c*x)^(5/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+bx^n)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x, algorithm="giac")
[Out] integrate((a*x + b*x^n)^(3/2)/(c*x)^(5/2), x)
```

maple [F] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{(ax + b\,x^{n})^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x)
[Out] int((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax + bx^{n})^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x, algorithm="maxima")
[Out] integrate((a*x + b*x^n)^(3/2)/(c*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^n + a\,x)^{3/2}}{(c\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x)^(3/2)/(c*x)^(5/2),x)
[Out] int((b*x^n + a*x)^(3/2)/(c*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax + bx^{n})^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x**n)**(3/2)/(c*x)**(5/2),x)
[Out] Integral((a*x + b*x**n)**(3/2)/(c*x)**(5/2), x)
```

3.280 $\int \frac{(a+bx^n)^{3/2}}{cx} \, dx$

Optimal. Leaf size=73

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}+\frac{2a\sqrt{a+bx^n}}{cn}+\frac{2\left(a+bx^n\right)^{3/2}}{3cn}$$

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {12, 266, 50, 63, 208}

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}+\frac{2a\sqrt{a+bx^n}}{cn}+\frac{2\left(a+bx^n\right)^{3/2}}{3cn}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(3/2)/(c*x),x]
```

```
[Out] (2*a*Sqrt[a + b*x^n])/(c*n) + (2*(a + b*x^n)^(3/2))/(3*c*n) - (2*a^(3/2)*Ar
cTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(c*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^n)^{3/2}}{cx}\,dx = \frac{\int \frac{(a+bx^n)^{3/2}}{x}\,dx}{c}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^n\right)}{cn}$$
$$= \frac{2(a+bx^n)^{3/2}}{3cn} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^n\right)}{cn}$$
$$= \frac{2a\sqrt{a+bx^n}}{cn} + \frac{2(a+bx^n)^{3/2}}{3cn} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{cn}$$
$$= \frac{2a\sqrt{a+bx^n}}{cn} + \frac{2(a+bx^n)^{3/2}}{3cn} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{bcn}$$
$$= \frac{2a\sqrt{a+bx^n}}{cn} + \frac{2(a+bx^n)^{3/2}}{3cn} - \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 0.79

$$\frac{2\sqrt{a+bx^n}\,(4a+bx^n) - 6a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{3cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(3/2)/(c*x),x]
```

```
[Out] (2*Sqrt[a + b*x^n]*(4*a + b*x^n) - 6*a^(3/2)*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a
]])/(3*c*n)
```

IntegrateAlgebraic [A] time = 0.06, size = 68, normalized size = 0.93

$$\frac{2\left((a+bx^n)^{3/2} + 3a\sqrt{a+bx^n}\right)}{3cn} - \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^n)^(3/2)/(c*x),x]
```

```
[Out] (2*(3*a*Sqrt[a + b*x^n] + (a + b*x^n)^(3/2)))/(3*c*n) - (2*a^(3/2)*ArcTanh[
Sqrt[a + b*x^n]/Sqrt[a]])/(c*n)
```

fricas [A] time = 0.42, size = 120, normalized size = 1.64

$$\left[\frac{3\,a^{\frac{3}{2}}\log\left(\frac{bx^n - 2\sqrt{bx^n+a}\sqrt{a} + 2\,a}{x^n}\right) + 2\,(bx^n + 4\,a)\sqrt{bx^n + a}}{3\,cn}, \frac{2\left(3\sqrt{-a}\,a\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right) + (bx^n + 4\,a)\sqrt{bx^n + a}\right)}{3\,cn}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(3/2)/c/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*a^(3/2)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*(b*x
^n + 4*a)*sqrt(b*x^n + a))/(c*n), 2/3*(3*sqrt(-a)*a*arctan(sqrt(b*x^n + a)*
sqrt(-a)/a) + (b*x^n + 4*a)*sqrt(b*x^n + a))/(c*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{3}{2}}}{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(3/2)/c/x,x, algorithm="giac")
[Out] integrate((b*x^n + a)^(3/2)/(c*x), x)
```

maple [A] time = 0.05, size = 51, normalized size = 0.70

$$\frac{-2a^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^n+a}}{\sqrt{a}}\right)+2\sqrt{b\,x^n+a}\;a+\frac{2(b\,x^n+a)^{\frac{3}{2}}}{3}}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^(3/2)/c/x,x)
[Out] 1/c/n*(2/3*(b*x^n+a)^(3/2)+2*(b*x^n+a)^(1/2)*a-2*a^(3/2)*arctanh((b*x^n+a)^
(1/2)/a^(1/2)))
```

maxima [A] time = 3.00, size = 73, normalized size = 1.00

$$\frac{\frac{3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{n}+\frac{2\left((bx^n+a)^{\frac{3}{2}}+3\,\sqrt{bx^n+a}\,a\right)}{n}}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(3/2)/c/x,x, algorithm="maxima")
[Out] 1/3*(3*a^(3/2)*log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))
/n + 2*((b*x^n + a)^(3/2) + 3*sqrt(b*x^n + a)*a)/n)/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x^n)^{3/2}}{c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(3/2)/(c*x),x)
[Out] int((a + b*x^n)^(3/2)/(c*x), x)
```

sympy [A] time = 3.08, size = 88, normalized size = 1.21

$$\frac{\frac{8a^{\frac{3}{2}}\sqrt{1+\frac{bx^n}{a}}}{3n}+\frac{a^{\frac{3}{2}}\log\left(\frac{bx^n}{a}\right)}{n}-\frac{2a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{bx^n}{a}}+1\right)}{n}+\frac{2\sqrt{a}\,bx^n\sqrt{1+\frac{bx^n}{a}}}{3n}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(3/2)/c/x,x)
[Out] (8*a**(3/2)*sqrt(1 + b*x**n/a)/(3*n) + a**(3/2)*log(b*x**n/a)/n - 2*a**(3/2
)*log(sqrt(1 + b*x**n/a) + 1)/n + 2*sqrt(a)*b*x**n*sqrt(1 + b*x**n/a)/(3*n)
)/c
```

3.281 $\int \sqrt{cx} \left(\frac{a}{x} + bx^n\right)^{3/2} dx$

Optimal. Leaf size=117

$$-\frac{2a^{3/2}c\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(n+1)\sqrt{cx}}+\frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{n+1}+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(n+1)}$$

Rubi [A] time = 0.23, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2028, 2031, 2029, 206}

$$-\frac{2a^{3/2}c\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(n+1)\sqrt{cx}}+\frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{n+1}+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]*(a/x + b*x^n)^(3/2),x]
```

```
[Out] (2*a*Sqrt[c*x]*Sqrt[a/x + b*x^n])/(1 + n) + (2*(c*x)^(3/2)*(a/x + b*x^n)^(3
/2))/(3*c*(1 + n)) - (2*a^(3/2)*c*Sqrt[x]*ArcTanh[Sqrt[a]/(Sqrt[x]*Sqrt[a/x
 + b*x^n])])/((1 + n)*Sqrt[c*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2} dx = \frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)} + (ac)\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}} dx$$

$$= \frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{1+n} + \frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)} + \left(a^2c^2\right)\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}} dx$$

$$= \frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{1+n} + \frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)} + \frac{\left(a^2c\sqrt{x}\right)\int \frac{1}{x^{3/2}\sqrt{\frac{a}{x}+bx^n}} dx}{\sqrt{cx}}$$

$$= \frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{1+n} + \frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)} - \frac{\left(2a^2c\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{1}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}}$$

$$= \frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{1+n} + \frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)} - \frac{2a^{3/2}c\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}}$$

Mathematica [A] time = 0.08, size = 97, normalized size = 0.83

$$\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}\left(\sqrt{a+bx^{n+1}}\left(4a+bx^{n+1}\right)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+1}}}{\sqrt{a}}\right)\right)}{3(n+1)\sqrt{a+bx^{n+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]*(a/x + b*x^n)^(3/2),x]
```

```
[Out] (2*Sqrt[c*x]*Sqrt[a/x + b*x^n]*(Sqrt[a + b*x^(1 + n)]*(4*a + b*x^(1 + n)) -
 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(1 + n)]/Sqrt[a]]))/(3*(1 + n)*Sqrt[a + b*x
^(1 + n)])
```

IntegrateAlgebraic [A] time = 1.50, size = 123, normalized size = 1.05

$$\frac{\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}\left(\frac{2\sqrt{c}\left(\left(a+bx^{n+1}\right)^{3/2}+3a\sqrt{a+bx^{n+1}}\right)}{3(n+1)}-\frac{2a^{3/2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+1}}}{\sqrt{a}}\right)}{n+1}\right)}{\sqrt{c}\sqrt{a+bx^{n+1}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x]*(a/x + b*x^n)^(3/2),x]
```

```
[Out] (Sqrt[c*x]*Sqrt[a/x + b*x^n]*((2*Sqrt[c]*(3*a*Sqrt[a + b*x^(1 + n)] + (a +
b*x^(1 + n))^(3/2)))/(3*(1 + n)) - (2*a^(3/2)*Sqrt[c]*ArcTanh[Sqrt[a + b*x^
(1 + n)]/Sqrt[a]])/(1 + n)))/(Sqrt[c]*Sqrt[a + b*x^(1 + n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^n + \frac{a}{x}\right)^{\frac{3}{2}} \sqrt{cx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^n + a/x)^(3/2)*sqrt(c*x), x)
```

maple [F] time = 0.67, size = 0, normalized size = 0.00

$$\int \sqrt{cx}\, \left(b\, x^n + \frac{a}{x}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(b*x^n+a/x)^(3/2),x)
[Out] int((c*x)^(1/2)*(b*x^n+a/x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^n + \frac{a}{x}\right)^{\frac{3}{2}} \sqrt{cx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^n + a/x)^(3/2)*sqrt(c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\, x}\, \left(b\, x^n + \frac{a}{x}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(b*x^n + a/x)^(3/2),x)
[Out] int((c*x)^(1/2)*(b*x^n + a/x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx}\, \left(\frac{a}{x} + bx^n\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)*(a/x+b*x**n)**(3/2),x)
[Out] Integral(sqrt(c*x)*(a/x + b*x**n)**(3/2), x)
```

3.282 $\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}dx$

Optimal. Leaf size=98

$$-\frac{2a^{3/2}c^2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{n+2}+\frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}+\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(n+2)}$$

Rubi [A] time = 0.17, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {12, 2028, 2007, 2029, 206}

$$-\frac{2a^{3/2}c^2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{n+2}+\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(n+2)}+\frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}$$

Antiderivative was successfully verified.

```
[In] Int[c^2*x^2*(a/x^2 + b*x^n)^(3/2),x]
```

```
[Out] (2*a*c^2*x*Sqrt[a/x^2 + b*x^n])/(2 + n) + (2*c^2*x^3*(a/x^2 + b*x^n)^(3/2))
/(3*(2 + n)) - (2*a^(3/2)*c^2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x^n])])/(2
+ n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2007

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}dx=c^2\int x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}dx$$
$$=\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(2+n)}+\left(ac^2\right)\int\sqrt{\frac{a}{x^2}+bx^n}\,dx$$
$$=\frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}+\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(2+n)}+\left(a^2c^2\right)\int\frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx$$
$$=\frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}+\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(2+n)}-\frac{\left(2a^2c^2\right)\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{1}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}$$
$$=\frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}+\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(2+n)}-\frac{2a^{3/2}c^2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}$$

Mathematica [A] time = 0.07, size = 94, normalized size = 0.96

$$\frac{2c^2x\sqrt{\frac{a}{x^2}+bx^n}\left(\sqrt{a+bx^{n+2}}\left(4a+bx^{n+2}\right)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+2}}}{\sqrt{a}}\right)\right)}{3(n+2)\sqrt{a+bx^{n+2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[c^2*x^2*(a/x^2 + b*x^n)^(3/2),x]
```

```
[Out] (2*c^2*x*Sqrt[a/x^2 + b*x^n]*(Sqrt[a + b*x^(2 + n)]*(4*a + b*x^(2 + n)) - 3
*a^(3/2)*ArcTanh[Sqrt[a + b*x^(2 + n)]/Sqrt[a]]))/(3*(2 + n)*Sqrt[a + b*x^(
2 + n)])
```

IntegrateAlgebraic [A] time = 0.14, size = 108, normalized size = 1.10

$$\frac{x\sqrt{\frac{a}{x^2}+bx^n}\left(\frac{2c^2\left(\left(a+bx^{n+2}\right)^{3/2}+3a\sqrt{a+bx^{n+2}}\right)}{3(n+2)}-\frac{2a^{3/2}c^2\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+2}}}{\sqrt{a}}\right)}{n+2}\right)}{\sqrt{a+bx^{n+2}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[c^2*x^2*(a/x^2 + b*x^n)^(3/2),x]
```

```
[Out] (x*Sqrt[a/x^2 + b*x^n]*((2*c^2*(3*a*Sqrt[a + b*x^(2 + n)] + (a + b*x^(2 + n
))^(3/2)))/(3*(2 + n)) - (2*a^(3/2)*c^2*ArcTanh[Sqrt[a + b*x^(2 + n)]/Sqrt[
a]])/(2 + n)))/Sqrt[a + b*x^(2 + n)]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^2*(a/x^2+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^n + \frac{a}{x^2}\right)^{\frac{3}{2}} c^2 x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^2*(a/x^2+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^n + a/x^2)^(3/2)*c^2*x^2, x)
```

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \left(b\,x^n + \frac{a}{x^2}\right)^{\frac{3}{2}} c^2 x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c^2*x^2*(b*x^n+a/x^2)^(3/2),x)
[Out] int(c^2*x^2*(b*x^n+a/x^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2 \int \left(bx^n + \frac{a}{x^2}\right)^{\frac{3}{2}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^2*(a/x^2+b*x^n)^(3/2),x, algorithm="maxima")
[Out] c^2*integrate((b*x^n + a/x^2)^(3/2)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int c^2\,x^2\,{\left(b\,x^n + \frac{a}{x^2}\right)}^{3/2} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c^2*x^2*(b*x^n + a/x^2)^(3/2),x)
[Out] int(c^2*x^2*(b*x^n + a/x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2 \left(\int a\sqrt{\frac{a}{x^2} + bx^n}\, dx + \int bx^2x^n\sqrt{\frac{a}{x^2} + bx^n}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c**2*x**2*(a/x**2+b*x**n)**(3/2),x)
[Out] c**2*(Integral(a*sqrt(a/x**2 + b*x**n), x) + Integral(b*x**2*x**n*sqrt(a/x*
*2 + b*x**n), x))
```

3.283 $\int (cx)^{7/2} \left(\frac{a}{x^3}+bx^n\right)^{3/2} dx$

Optimal. Leaf size=122

$$-\frac{2a^{3/2}c^4\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(n+3)\sqrt{cx}}+\frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{n+3}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(n+3)}$$

Rubi [A] time = 0.28, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2028, 2031, 2029, 206}

$$-\frac{2a^{3/2}c^4\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(n+3)\sqrt{cx}}+\frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{n+3}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)*(a/x^3 + b*x^n)^(3/2),x]
```

```
[Out] (2*a*c^2*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n])/(3 + n) + (2*(c*x)^(9/2)*(a/x^3 +
 b*x^n)^(3/2))/(3*c*(3 + n)) - (2*a^(3/2)*c^4*Sqrt[x]*ArcTanh[Sqrt[a]/(x^(3
/2)*Sqrt[a/x^3 + b*x^n])])/((3 + n)*Sqrt[c*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int (cx)^{7/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}dx = \frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}+\left(ac^3\right)\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx$$

$$= \frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{3+n}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}+\left(a^2c^6\right)\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}dx$$

$$= \frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{3+n}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}+\frac{\left(a^2c^4\sqrt{x}\right)\int \frac{1}{x^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}dx}{\sqrt{cx}}$$

$$= \frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{3+n}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}-\frac{\left(2a^2c^4\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}dx, x\right.}{(3+n)\sqrt{cx}}$$

$$= \frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{3+n}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}-\frac{2a^{3/2}c^4\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right.}{(3+n)\sqrt{cx}}$$

Mathematica [A] time = 0.09, size = 100, normalized size = 0.82

$$\frac{2c^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}\left(\sqrt{a+bx^{n+3}}\left(4a+bx^{n+3}\right)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+3}}}{\sqrt{a}}\right)\right)}{3(n+3)\sqrt{a+bx^{n+3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)*(a/x^3 + b*x^n)^(3/2),x]
```

```
[Out] (2*c^2*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n]*(Sqrt[a + b*x^(3 + n)]*(4*a + b*x^(3
 + n)) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]]))/(3*(3 + n)*Sqrt
[a + b*x^(3 + n)])
```

IntegrateAlgebraic [A] time = 1.69, size = 123, normalized size = 1.01

$$\frac{(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}\left(\frac{2c^{7/2}\left(\left(a+bx^{n+3}\right)^{3/2}+3a\sqrt{a+bx^{n+3}}\right)}{3(n+3)}-\frac{2a^{3/2}c^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+3}}}{\sqrt{a}}\right)}{n+3}\right)}{c^{9/2}\left(a+bx^{n+3}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x)^(7/2)*(a/x^3 + b*x^n)^(3/2),x]
```

```
[Out] ((c*x)^(9/2)*(a/x^3 + b*x^n)^(3/2)*((2*c^(7/2)*(3*a*Sqrt[a + b*x^(3 + n)] +
 (a + b*x^(3 + n))^(3/2)))/(3*(3 + n)) - (2*a^(3/2)*c^(7/2)*ArcTanh[Sqrt[a
+ b*x^(3 + n)]/Sqrt[a]])/(3 + n)))/(c^(9/2)*(a + b*x^(3 + n))^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^n + \frac{a}{x^3}\right)^{\frac{3}{2}} (cx)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^n + a/x^3)^(3/2)*(c*x)^(7/2), x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{7}{2}} \left(b\,x^n + \frac{a}{x^3}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)*(b*x^n+a/x^3)^(3/2),x)
[Out] int((c*x)^(7/2)*(b*x^n+a/x^3)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^n + \frac{a}{x^3}\right)^{\frac{3}{2}} (cx)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^n + a/x^3)^(3/2)*(c*x)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{7/2} \left(b\,x^n + \frac{a}{x^3}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)*(b*x^n + a/x^3)^(3/2),x)
[Out] int((c*x)^(7/2)*(b*x^n + a/x^3)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)*(a/x**3+b*x**n)**(3/2),x)
[Out] Timed out
```

3.284 $\int c^5 x^5 \left(\frac{a}{x^4}+b x^n\right)^{3/2} \, dx$

Optimal. Leaf size=100

$$-\frac{2a^{3/2}c^5\tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{n+4}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(n+4)}+\frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{n+4}$$

Rubi [A] time = 0.21, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 2028, 2029, 206}

$$-\frac{2a^{3/2}c^5\tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{n+4}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(n+4)}+\frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{n+4}$$

Antiderivative was successfully verified.

```
[In] Int[c^5*x^5*(a/x^4 + b*x^n)^(3/2),x]
```

```
[Out] (2*a*c^5*x^2*Sqrt[a/x^4 + b*x^n])/(4 + n) + (2*c^5*x^6*(a/x^4 + b*x^n)^(3/2
))/(3*(4 + n)) - (2*a^(3/2)*c^5*ArcTanh[Sqrt[a]/(x^2*Sqrt[a/x^4 + b*x^n])])
/(4 + n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}dx=c^5\int x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}dx$$
$$=\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(4+n)}+\left(ac^5\right)\int x\sqrt{\frac{a}{x^4}+bx^n}\,dx$$
$$=\frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{4+n}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(4+n)}+\left(a^2c^5\right)\int\frac{1}{x^3\sqrt{\frac{a}{x^4}+bx^n}}\,dx$$
$$=\frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{4+n}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(4+n)}-\frac{\left(2a^2c^5\right)\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{1}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{4+n}$$
$$=\frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{4+n}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(4+n)}-\frac{2a^{3/2}c^5\tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{4+n}$$

Mathematica [A] time = 0.08, size = 96, normalized size = 0.96

$$\frac{2c^5x^2\sqrt{\frac{a}{x^4}+bx^n}\left(\sqrt{a+bx^{n+4}}\left(4a+bx^{n+4}\right)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+4}}}{\sqrt{a}}\right)\right)}{3(n+4)\sqrt{a+bx^{n+4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[c^5*x^5*(a/x^4 + b*x^n)^(3/2),x]
```

```
[Out] (2*c^5*x^2*Sqrt[a/x^4 + b*x^n]*(Sqrt[a + b*x^(4 + n)]*(4*a + b*x^(4 + n)) -
 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(4 + n)]/Sqrt[a]]))/(3*(4 + n)*Sqrt[a + b*x
^(4 + n)])
```

IntegrateAlgebraic [A] time = 0.15, size = 110, normalized size = 1.10

$$\frac{x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}\left(\frac{2c^5\left(\left(a+bx^{n+4}\right)^{3/2}+3a\sqrt{a+bx^{n+4}}\right)}{3(n+4)}-\frac{2a^{3/2}c^5\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+4}}}{\sqrt{a}}\right)}{n+4}\right)}{\left(a+bx^{n+4}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[c^5*x^5*(a/x^4 + b*x^n)^(3/2),x]
```

```
[Out] (x^6*(a/x^4 + b*x^n)^(3/2)*((2*c^5*(3*a*Sqrt[a + b*x^(4 + n)] + (a + b*x^(4
 + n))^(3/2)))/(3*(4 + n)) - (2*a^(3/2)*c^5*ArcTanh[Sqrt[a + b*x^(4 + n)]/S
qrt[a]])/(4 + n)))/(a + b*x^(4 + n))^(3/2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^5*x^5*(a/x^4+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^n + \frac{a}{x^4}\right)^{\frac{3}{2}} c^5 x^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^5*x^5*(a/x^4+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^n + a/x^4)^(3/2)*c^5*x^5, x)
```

maple [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \left(b\,x^n + \frac{a}{x^4}\right)^{\frac{3}{2}} c^5 x^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c^5*x^5*(a/x^4+b*x^n)^(3/2),x)
[Out] int(c^5*x^5*(a/x^4+b*x^n)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$c^5 \int \left(bx^n + \frac{a}{x^4}\right)^{\frac{3}{2}} x^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^5*x^5*(a/x^4+b*x^n)^(3/2),x, algorithm="maxima")
[Out] c^5*integrate((b*x^n + a/x^4)^(3/2)*x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int c^5\,x^5\,{\left(b\,x^n + \frac{a}{x^4}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c^5*x^5*(b*x^n + a/x^4)^(3/2),x)
[Out] int(c^5*x^5*(b*x^n + a/x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^5 \left(\int ax\sqrt{\frac{a}{x^4} + bx^n}\, dx + \int bx^5 x^n \sqrt{\frac{a}{x^4} + bx^n}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c**5*x**5*(a/x**4+b*x**n)**(3/2),x)
[Out] c**5*(Integral(a*x*sqrt(a/x**4 + b*x**n), x) + Integral(b*x**5*x**n*sqrt(a/
x**4 + b*x**n), x))
```

3.285 $\int \sqrt{\frac{a+bx}{x^2}}\, dx$

Optimal. Leaf size=51

$$2x\sqrt{\frac{a}{x^2}+\frac{b}{x}}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+\frac{b}{x}}}\right)$$

Rubi [A] time = 0.08, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1979, 2007, 2013, 620, 206}

$$2x\sqrt{\frac{a}{x^2}+\frac{b}{x}}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+\frac{b}{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x)/x^2],x]
```

```
[Out] 2*Sqrt[a/x^2 + b/x]*x - 2*Sqrt[a]*ArcTanh[Sqrt[a]/(Sqrt[a/x^2 + b/x]*x)]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2007

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{a+bx}{x^2}}\,dx &= \int \sqrt{\frac{a}{x^2}+\frac{b}{x}}\,dx \\
&= 2\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,x + a\int \frac{1}{\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,x^2}\,dx \\
&= 2\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,x - a\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \frac{1}{x}\right) \\
&= 2\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,x - (2a)\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,x}\right) \\
&= 2\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,x - 2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 58, normalized size = 1.14

$$
\frac{2x\sqrt{\frac{a+bx}{x^2}}\left(\sqrt{a+bx}-\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{a+bx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x)/x^2],x]
```

```
[Out] (2*x*Sqrt[(a + b*x)/x^2]*(Sqrt[a + b*x] - Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqr
t[a]]))/Sqrt[a + b*x]
```

IntegrateAlgebraic [A] time = 3.65, size = 59, normalized size = 1.16

$$
\frac{x\sqrt{\frac{a+bx}{x^2}}\left(2\sqrt{a+bx}-2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{a+bx}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(a + b*x)/x^2],x]
```

```
[Out] (x*Sqrt[(a + b*x)/x^2]*(2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/S
qrt[a]]))/Sqrt[a + b*x]
```

fricas [A] time = 0.42, size = 93, normalized size = 1.82

$$
\left[2\,x\sqrt{\frac{bx+a}{x^2}}+\sqrt{a}\,\log\left(\frac{bx-2\,\sqrt{a}\,x\sqrt{\frac{bx+a}{x^2}}+2\,a}{x}\right), 2\,x\sqrt{\frac{bx+a}{x^2}}+2\,\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\,x\sqrt{\frac{bx+a}{x^2}}}{a}\right)\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [2*x*sqrt((b*x + a)/x^2) + sqrt(a)*log((b*x - 2*sqrt(a)*x*sqrt((b*x + a)/x^
2) + 2*a)/x), 2*x*sqrt((b*x + a)/x^2) + 2*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((
b*x + a)/x^2)/a)]
```

giac [A] time = 0.16, size = 67, normalized size = 1.31

$$
\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}}+2\,\sqrt{bx+a}\,\mathrm{sgn}(x)-\frac{2\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\,\sqrt{a}\right)\mathrm{sgn}(x)}{\sqrt{-a}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*sqrt(b*x + a)*sgn(x)
 - 2*(a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)
```

maple [A] time = 0.08, size = 47, normalized size = 0.92

$$\frac{2\sqrt{\frac{bx+a}{x^2}}\left(-\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\sqrt{bx+a}\right)x}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)/x^2)^(1/2),x)
```

```
[Out] 2*((b*x+a)/x^2)^(1/2)*x*(-a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+(b*x+a)^(1
/2))/(b*x+a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{bx+a}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x + a)/x^2), x)
```

mupad [B] time = 5.18, size = 67, normalized size = 1.31

$$2\,x\,\sqrt{\frac{a}{x^2}+\frac{b}{x}}+\frac{\sqrt{a}\,\sqrt{x}\,\mathrm{asin}\left(\frac{\sqrt{a}\,1\mathrm{i}}{\sqrt{b}\,\sqrt{x}}\right)\,\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,2\mathrm{i}}{\sqrt{b}\,\sqrt{\frac{a}{b\,x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)/x^2)^(1/2),x)
```

```
[Out] 2*x*(a/x^2 + b/x)^(1/2) + (a^(1/2)*x^(1/2)*asin((a^(1/2)*1i)/(b^(1/2)*x^(1/
2)))*(a/x^2 + b/x)^(1/2)*2i)/(b^(1/2)*(a/(b*x) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((a + b*x)/x**2), x)
```

3.286 $\int \sqrt{\frac{a+bx^2}{x^2}}\,dx$

Optimal. Leaf size=42

$$x\sqrt{\frac{a}{x^2}+b}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+b}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1972, 242, 277, 217, 206}

$$x\sqrt{\frac{a}{x^2}+b}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x^2)/x^2],x]
```

```
[Out] Sqrt[b + a/x^2]*x - Sqrt[a]*ArcTanh[Sqrt[a]/(Sqrt[b + a/x^2]*x)]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{a+bx^2}{x^2}}\,dx &= \int \sqrt{b+\frac{a}{x^2}}\,dx \\
&= -\operatorname{Subst}\left(\int \frac{\sqrt{b+ax^2}}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{b+\frac{a}{x^2}}\,x - a\operatorname{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{b+\frac{a}{x^2}}\,x - a\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{\sqrt{b+\frac{a}{x^2}}\,x}\right) \\
&= \sqrt{b+\frac{a}{x^2}}\,x - \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{b+\frac{a}{x^2}}\,x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 62, normalized size = 1.48

$$
x\sqrt{\frac{a}{x^2}+b} - \frac{\sqrt{a}\,x\sqrt{\frac{a}{x^2}+b}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x^2)/x^2],x]
```

```
[Out] Sqrt[b + a/x^2]*x - (Sqrt[a]*Sqrt[b + a/x^2]*x*ArcTanh[Sqrt[a + b*x^2]/Sqrt
[a]])/Sqrt[a + b*x^2]
```

IntegrateAlgebraic [A] time = 3.84, size = 61, normalized size = 1.45

$$
\frac{x\sqrt{\frac{a}{x^2}+b}\left(\sqrt{a+bx^2}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{\sqrt{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(a + b*x^2)/x^2],x]
```

```
[Out] (Sqrt[b + a/x^2]*x*(Sqrt[a + b*x^2] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[
a]]))/Sqrt[a + b*x^2]
```

fricas [A] time = 0.43, size = 108, normalized size = 2.57

$$
\left[x\sqrt{\frac{bx^2+a}{x^2}} + \frac{1}{2}\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{a}\,x\sqrt{\frac{bx^2+a}{x^2}}+2a}{x^2}\right), x\sqrt{\frac{bx^2+a}{x^2}} + \sqrt{-a}\arctan\left(\frac{\sqrt{-a}\,x\sqrt{\frac{bx^2+a}{x^2}}}{bx^2+a}\right)\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [x*sqrt((b*x^2 + a)/x^2) + 1/2*sqrt(a)*log(-(b*x^2 - 2*sqrt(a)*x*sqrt((b*x^
2 + a)/x^2) + 2*a)/x^2), x*sqrt((b*x^2 + a)/x^2) + sqrt(-a)*arctan(sqrt(-a)
*x*sqrt((b*x^2 + a)/x^2)/(b*x^2 + a))]
```

giac [B] time = 0.17, size = 69, normalized size = 1.64

$$
\frac{a\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}} + \sqrt{bx^2+a}\operatorname{sgn}(x) - \frac{\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}\right)\operatorname{sgn}(x)}{\sqrt{-a}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] a*arctan(sqrt(b*x^2 + a)/sqrt(-a))*sgn(x)/sqrt(-a) + sqrt(b*x^2 + a)*sgn(x)
 - (a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)
```

maple [A] time = 0.08, size = 61, normalized size = 1.45

$$\frac{\sqrt{\frac{b\,x^{2}+a}{x^{2}}}\,\left(-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)+\sqrt{b\,x^{2}+a}\right)x}{\sqrt{b\,x^{2}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/x^2)^(1/2),x)
```

```
[Out] ((b*x^2+a)/x^2)^(1/2)*x/(b*x^2+a)^(1/2)*((b*x^2+a)^(1/2)-a^(1/2)*ln(2*(a+(b
*x^2+a)^(1/2)*a^(1/2))/x))
```

maxima [A] time = 2.94, size = 53, normalized size = 1.26

$$\sqrt{b+\frac{a}{x^{2}}}\,x+\frac{1}{2}\,\sqrt{a}\,\log\left(\frac{\sqrt{b+\frac{a}{x^{2}}}\,x-\sqrt{a}}{\sqrt{b+\frac{a}{x^{2}}}\,x+\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b + a/x^2)*x + 1/2*sqrt(a)*log((sqrt(b + a/x^2)*x - sqrt(a))/(sqrt(b +
 a/x^2)*x + sqrt(a)))
```

mupad [B] time = 5.57, size = 55, normalized size = 1.31

$$x\,\sqrt{b+\frac{a}{x^{2}}}+\frac{\sqrt{a}\,\mathrm{asin}\left(\frac{\sqrt{a}\,1\mathrm{i}}{\sqrt{b}\,x}\right)\,\sqrt{b+\frac{a}{x^{2}}}\,1\mathrm{i}}{\sqrt{b}\,\sqrt{\frac{a}{b\,x^{2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)/x^2)^(1/2),x)
```

```
[Out] x*(b + a/x^2)^(1/2) + (a^(1/2)*asin((a^(1/2)*1i)/(b^(1/2)*x))*(b + a/x^2)^(
1/2)*1i)/(b^(1/2)*(a/(b*x^2) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{a+bx^{2}}{x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((a + b*x**2)/x**2), x)
```

3.287 $\int \sqrt{\frac{a+bx^3}{x^2}}\,dx$

Optimal. Leaf size=51

$$\frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx}}\right)$$

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1979, 2007, 2029, 206}

$$\frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x^3)/x^2],x]
```

```
[Out] (2*x*Sqrt[a/x^2 + b*x])/3 - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x]
)])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2007

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{a+bx^3}{x^2}}\,dx &= \int \sqrt{\frac{a}{x^2}+bx}\,dx\\ &= \frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}+a\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx}}\,dx\\ &= \frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{1}{3}(2a)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{1}{x\sqrt{\frac{a}{x^2}+bx}}\right)\\ &= \frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx}}\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 66, normalized size = 1.29

$$\frac{2x\sqrt{\frac{a}{x^2}+bx}\left(\sqrt{a+bx^3}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)}{3\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x^3)/x^2],x]
[Out] (2*x*Sqrt[a/x^2 + b*x]*(Sqrt[a + b*x^3] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]/S
qrt[a]]))/(3*Sqrt[a + b*x^3])
```

IntegrateAlgebraic [A] time = 4.03, size = 69, normalized size = 1.35

$$\frac{x\sqrt{\frac{a}{x^2}+bx}\left(\frac{2}{3}\sqrt{a+bx^3}-\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)}{\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(a + b*x^3)/x^2],x]
[Out] (x*Sqrt[a/x^2 + b*x]*((2*Sqrt[a + b*x^3])/3 - (2*Sqrt[a]*ArcTanh[Sqrt[a + b
*x^3]/Sqrt[a]])/3))/Sqrt[a + b*x^3]
```

fricas [A] time = 0.42, size = 104, normalized size = 2.04

$$\left[\frac{2}{3}x\sqrt{\frac{bx^3+a}{x^2}}+\frac{1}{3}\sqrt{a}\log\left(\frac{bx^3-2\sqrt{a}x\sqrt{\frac{bx^3+a}{x^2}}+2a}{x^3}\right),\frac{2}{3}x\sqrt{\frac{bx^3+a}{x^2}}+\frac{2}{3}\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{bx^3+a}{x^2}}}{a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)/x^2)^(1/2),x, algorithm="fricas")
[Out] [2/3*x*sqrt((b*x^3 + a)/x^2) + 1/3*sqrt(a)*log((b*x^3 - 2*sqrt(a)*x*sqrt((b
*x^3 + a)/x^2) + 2*a)/x^3), 2/3*x*sqrt((b*x^3 + a)/x^2) + 2/3*sqrt(-a)*arct
an(sqrt(-a)*x*sqrt((b*x^3 + a)/x^2)/a)]
```

giac [A] time = 0.19, size = 71, normalized size = 1.39

$$\frac{2a\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)\text{sgn}(x)}{3\sqrt{-a}}+\frac{2}{3}\sqrt{bx^3+a}\,\text{sgn}(x)-\frac{2\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}\right)\text{sgn}(x)}{3\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*a*arctan(sqrt(b*x^3 + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2/3*sqrt(b*x^3 + a
)*sgn(x) - 2/3*(a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(
-a)
```

maple [A] time = 0.08, size = 55, normalized size = 1.08

$$\frac{2\sqrt{\frac{b\,x^{3}+a}{x^{2}}}\,\left(-\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)+\sqrt{b\,x^{3}+a}\right)x}{3\sqrt{b\,x^{3}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)/x^2)^(1/2),x)
```

```
[Out] 2/3*((b*x^3+a)/x^2)^(1/2)*x*(-arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+(b*x
^3+a)^(1/2))/(b*x^3+a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{bx^3 + a}{x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x^3 + a)/x^2), x)
```

mupad [B] time = 5.34, size = 63, normalized size = 1.24

$$\frac{2\,x\,\sqrt{b\,x+\frac{a}{x^2}}}{3}+\frac{\sqrt{a}\,\mathrm{asin}\left(\frac{\sqrt{a}\,1{}\mathrm{i}}{\sqrt{b}\,x^{3/2}}\right)\,\sqrt{b\,x+\frac{a}{x^2}}\,2{}\mathrm{i}}{3\,\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{a}{b\,x^3}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)/x^2)^(1/2),x)
```

```
[Out] (2*x*(b*x + a/x^2)^(1/2))/3 + (a^(1/2)*asin((a^(1/2)*1i)/(b^(1/2)*x^(3/2)))
*(b*x + a/x^2)^(1/2)*2i)/(3*b^(1/2)*x^(1/2)*(a/(b*x^3) + 1)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)/x**2)**(1/2),x)
```

```
[Out] Timed out
```

3.288 $\int \sqrt{\frac{a+bx^n}{x^2}}\,dx$

Optimal. Leaf size=61

$$\frac{2x\sqrt{\frac{a}{x^2}+bx^{n-2}}}{n}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^{n-2}}}\right)}{n}$$

Rubi [A] time = 0.08, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1979, 2007, 2029, 206}

$$\frac{2x\sqrt{\frac{a}{x^2}+bx^{n-2}}}{n}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^{n-2}}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x^n)/x^2],x]
```

```
[Out] (2*x*Sqrt[a/x^2 + b*x^(-2 + n)])/n - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x
^2 + b*x^(-2 + n)])])/n
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2007

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \sqrt{\frac{a+bx^n}{x^2}}\,dx = \int \sqrt{\frac{a}{x^2}+bx^{-2+n}}\,dx$$
$$= \frac{2x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}{n} + a\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^{-2+n}}}\,dx$$
$$= \frac{2x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}{n} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}\right)}{n}$$
$$= \frac{2x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}{n} - \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}\right)}{n}$$

Mathematica [A] time = 0.02, size = 70, normalized size = 1.15

$$\frac{x\sqrt{\frac{a+bx^n}{x^2}}\left(2\sqrt{a+bx^n}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)\right)}{n\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x^n)/x^2],x]
[Out] (x*Sqrt[(a + b*x^n)/x^2]*(2*Sqrt[a + b*x^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*
x^n]/Sqrt[a]]))/(n*Sqrt[a + b*x^n])
```

IntegrateAlgebraic [A] time = 0.04, size = 73, normalized size = 1.20

$$\frac{x\sqrt{\frac{a+bx^n}{x^2}}\left(\frac{2\sqrt{a+bx^n}}{n}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{n}\right)}{\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(a + b*x^n)/x^2],x]
[Out] (x*Sqrt[(a + b*x^n)/x^2]*((2*Sqrt[a + b*x^n])/n - (2*Sqrt[a]*ArcTanh[Sqrt[a
 + b*x^n]/Sqrt[a]])/n))/Sqrt[a + b*x^n]
```

fricas [A] time = 0.42, size = 112, normalized size = 1.84

$$\left[\frac{2\,x\sqrt{\frac{bx^n+a}{x^2}}+\sqrt{a}\log\left(\frac{bx^n-2\,\sqrt{a}\,x\sqrt{\frac{bx^n+a}{x^2}}+2\,a}{x^n}\right)}{n}, \frac{2\left(x\sqrt{\frac{bx^n+a}{x^2}}+\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\,x\sqrt{\frac{bx^n+a}{x^2}}}{a}\right)\right)}{n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="fricas")
[Out] [(2*x*sqrt((b*x^n + a)/x^2) + sqrt(a)*log((b*x^n - 2*sqrt(a)*x*sqrt((b*x^n
+ a)/x^2) + 2*a)/x^n))/n, 2*(x*sqrt((b*x^n + a)/x^2) + sqrt(-a)*arctan(sqrt
(-a)*x*sqrt((b*x^n + a)/x^2)/a))/n]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{bx^n + a}{x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt((b*x^n + a)/x^2), x)
```

maple [A] time = 1.87, size = 74, normalized size = 1.21

$$-\frac{2\sqrt{\frac{b\,\mathrm{e}^{n\ln(x)}+a}{x^2}}\,\sqrt{a}\;x\,\mathrm{arctanh}\left(\frac{\sqrt{b\,\mathrm{e}^{n\ln(x)}+a}}{\sqrt{a}}\right)}{\sqrt{b\,\mathrm{e}^{n\ln(x)}+a}\;n}+\frac{2\sqrt{\frac{b\,\mathrm{e}^{n\ln(x)}+a}{x^2}}\;x}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^n+a)/x^2)^(1/2),x)
[Out] 2/n*((b*exp(n*ln(x))+a)/x^2)^(1/2)*x-2*a^(1/2)/n*arctanh((b*exp(n*ln(x))+a)
^(1/2)/a^(1/2))*((b*exp(n*ln(x))+a)/x^2)^(1/2)/(b*exp(n*ln(x))+a)^(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{bx^n + a}{x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((b*x^n + a)/x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{a + b\,x^n}{x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^n)/x^2)^(1/2),x)
[Out] int(((a + b*x^n)/x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{a + bx^n}{x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+b*x**n)/x**2)**(1/2),x)
[Out] Integral(sqrt((a + b*x**n)/x**2), x)
```

3.289 $\int \sqrt{\frac{-a+bx}{x^2}}\, dx$

Optimal. Leaf size=53

$$2x\sqrt{\frac{b}{x}-\frac{a}{x^2}}+2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{b}{x}-\frac{a}{x^2}}}\right)$$

Rubi [A] time = 0.08, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1979, 2007, 2013, 620, 203}

$$2x\sqrt{\frac{b}{x}-\frac{a}{x^2}}+2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{b}{x}-\frac{a}{x^2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-a + b*x)/x^2],x]
```

```
[Out] 2*Sqrt[-(a/x^2) + b/x]*x + 2*Sqrt[a]*ArcTan[Sqrt[a]/(Sqrt[-(a/x^2) + b/x]*x
)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2007

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx = \int \sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,dx$$
$$= 2\sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,x - a\int \frac{1}{\sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,x^2}\,dx$$
$$= 2\sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,x + a\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{bx-ax^2}}\,dx, x, \frac{1}{x}\right)$$
$$= 2\sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,x + (2a)\,\mathrm{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{1}{\sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,x}\right)$$
$$= 2\sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,x + 2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,x}\right)$$

Mathematica [A] time = 0.03, size = 66, normalized size = 1.25

$$\frac{2x\sqrt{\frac{bx-a}{x^2}}\left(\sqrt{bx-a}-\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\right)}{\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-a + b*x)/x^2],x]
```

```
[Out] (2*x*Sqrt[(-a + b*x)/x^2]*(Sqrt[-a + b*x] - Sqrt[a]*ArcTan[Sqrt[-a + b*x]/S
qrt[a]]))/Sqrt[-a + b*x]
```

IntegrateAlgebraic [A] time = 3.81, size = 67, normalized size = 1.26

$$\frac{x\sqrt{\frac{bx-a}{x^2}}\left(2\sqrt{bx-a}-2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\right)}{\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(-a + b*x)/x^2],x]
```

```
[Out] (x*Sqrt[(-a + b*x)/x^2]*(2*Sqrt[-a + b*x] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*x]
/Sqrt[a]]))/Sqrt[-a + b*x]
```

fricas [A] time = 0.40, size = 98, normalized size = 1.85

$$\left[2\,x\sqrt{\frac{bx-a}{x^2}}+\sqrt{-a}\,\log\left(\frac{bx-2\,\sqrt{-a}\,x\sqrt{\frac{bx-a}{x^2}}-2\,a}{x}\right), 2\,x\sqrt{\frac{bx-a}{x^2}}-2\,\sqrt{a}\,\arctan\left(\frac{x\sqrt{\frac{bx-a}{x^2}}}{\sqrt{a}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x-a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [2*x*sqrt((b*x - a)/x^2) + sqrt(-a)*log((b*x - 2*sqrt(-a)*x*sqrt((b*x - a)/
x^2) - 2*a)/x), 2*x*sqrt((b*x - a)/x^2) - 2*sqrt(a)*arctan(x*sqrt((b*x - a)
/x^2)/sqrt(a))]
```

giac [A] time = 0.16, size = 61, normalized size = 1.15

$$-2\,\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\mathrm{sgn}(x)+2\left(\sqrt{a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{a}}\right)-\sqrt{-a}\right)\mathrm{sgn}(x)+2\,\sqrt{bx-a}\,\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x-a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a))*sgn(x) + 2*(sqrt(a)*arctan(sqrt(-a
)/sqrt(a)) - sqrt(-a))*sgn(x) + 2*sqrt(b*x - a)*sgn(x)
```

maple [A] time = 0.08, size = 55, normalized size = 1.04

$$\frac{2\sqrt{\frac{bx-a}{x^2}}\left(-\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\sqrt{bx-a}\right)x}{\sqrt{bx-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x-a)/x^2)^(1/2),x)
```

```
[Out] 2*((b*x-a)/x^2)^(1/2)*x*(-a^(1/2)*arctan((b*x-a)^(1/2)/a^(1/2))+(b*x-a)^(1/
2))/(b*x-a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{bx-a}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x-a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x - a)/x^2), x)
```

mupad [B] time = 5.18, size = 67, normalized size = 1.26

$$2\,x\,\sqrt{\frac{b}{x}-\frac{a}{x^2}}+\frac{2\,\sqrt{a}\,\sqrt{x}\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)\,\sqrt{\frac{b}{x}-\frac{a}{x^2}}}{\sqrt{b}\,\sqrt{1-\frac{a}{b\,x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(a - b*x)/x^2)^(1/2),x)
```

```
[Out] 2*x*(b/x - a/x^2)^(1/2) + (2*a^(1/2)*x^(1/2)*asin(a^(1/2)/(b^(1/2)*x^(1/2))
)*(b/x - a/x^2)^(1/2))/(b^(1/2)*(1 - a/(b*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x-a)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((-a + b*x)/x**2), x)
```

3.290 $\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx$

Optimal. Leaf size=43

$$x\sqrt{b-\frac{a}{x^2}}+\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{b-\frac{a}{x^2}}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1972, 242, 277, 217, 203}

$$x\sqrt{b-\frac{a}{x^2}}+\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{b-\frac{a}{x^2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-a + b*x^2)/x^2],x]
```

```
[Out] Sqrt[b - a/x^2]*x + Sqrt[a]*ArcTan[Sqrt[a]/(Sqrt[b - a/x^2]*x)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] && !BinomialMatchQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx &= \int \sqrt{b-\frac{a}{x^2}}\,dx\\
&= -\text{Subst}\left(\int \frac{\sqrt{b-ax^2}}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{b-\frac{a}{x^2}}\,x + a\,\text{Subst}\left(\int \frac{1}{\sqrt{b-ax^2}}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{b-\frac{a}{x^2}}\,x + a\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{1}{\sqrt{b-\frac{a}{x^2}}\,x}\right)\\
&= \sqrt{b-\frac{a}{x^2}}\,x + \sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{b-\frac{a}{x^2}}\,x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 68, normalized size = 1.58

$$
x\sqrt{b-\frac{a}{x^2}} - \frac{\sqrt{a}\,x\sqrt{b-\frac{a}{x^2}}\,\tan^{-1}\left(\frac{\sqrt{bx^2-a}}{\sqrt{a}}\right)}{\sqrt{bx^2-a}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-a + b*x^2)/x^2],x]
```

```
[Out] Sqrt[b - a/x^2]*x - (Sqrt[a]*Sqrt[b - a/x^2]*x*ArcTan[Sqrt[-a + b*x^2]/Sqrt
[a]])/Sqrt[-a + b*x^2]
```

IntegrateAlgebraic [A] time = 3.95, size = 68, normalized size = 1.58

$$
\frac{x\sqrt{b-\frac{a}{x^2}}\left(\sqrt{bx^2-a} - \sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{bx^2-a}}{\sqrt{a}}\right)\right)}{\sqrt{bx^2-a}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(-a + b*x^2)/x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x*(Sqrt[-a + b*x^2] - Sqrt[a]*ArcTan[Sqrt[-a + b*x^2]/Sqrt
[a]]))/Sqrt[-a + b*x^2]
```

fricas [A] time = 0.42, size = 118, normalized size = 2.74

$$
\left[x\sqrt{\frac{bx^2-a}{x^2}} + \frac{1}{2}\sqrt{-a}\,\log\left(-\frac{bx^2 - 2\sqrt{-a}\,x\sqrt{\frac{bx^2-a}{x^2}} - 2a}{x^2}\right), x\sqrt{\frac{bx^2-a}{x^2}} + \sqrt{a}\,\arctan\left(\frac{\sqrt{a}\,x\sqrt{\frac{bx^2-a}{x^2}}}{bx^2-a}\right)\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2-a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [x*sqrt((b*x^2 - a)/x^2) + 1/2*sqrt(-a)*log(-(b*x^2 - 2*sqrt(-a)*x*sqrt((b*
x^2 - a)/x^2) - 2*a)/x^2), x*sqrt((b*x^2 - a)/x^2) + sqrt(a)*arctan(sqrt(a)
*x*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a))]
```

giac [A] time = 0.24, size = 63, normalized size = 1.47

$$
-\sqrt{a}\,\arctan\left(\frac{\sqrt{bx^2-a}}{\sqrt{a}}\right)\mathrm{sgn}(x) + \left(\sqrt{a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{a}}\right) - \sqrt{-a}\right)\mathrm{sgn}(x) + \sqrt{bx^2-a}\,\mathrm{sgn}(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2-a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(a)*arctan(sqrt(b*x^2 - a)/sqrt(a))*sgn(x) + (sqrt(a)*arctan(sqrt(-a)/
sqrt(a)) - sqrt(-a))*sgn(x) + sqrt(b*x^2 - a)*sgn(x)
```

maple [B] time = 0.09, size = 81, normalized size = 1.88

$$\frac{\sqrt{\frac{b\,x^2-a}{x^2}}\,\left(a\ln\left(\frac{-2a+2\sqrt{-a}\,\sqrt{b\,x^2-a}}{x}\right)+\sqrt{-a}\,\sqrt{b\,x^2-a}\right)x}{\sqrt{-a}\,\sqrt{b\,x^2-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2-a)/x^2)^(1/2),x)
```

```
[Out] ((b*x^2-a)/x^2)^(1/2)*x*((-a)^(1/2)*(b*x^2-a)^(1/2)+a*ln(2*((-a)^(1/2)*(b*x
^2-a)^(1/2)-a)/x))/(-a)^(1/2)/(b*x^2-a)^(1/2)
```

maxima [A] time = 2.99, size = 34, normalized size = 0.79

$$\sqrt{b-\frac{a}{x^2}}\,x-\sqrt{a}\,\arctan\left(\frac{\sqrt{b-\frac{a}{x^2}}\,x}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2-a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b - a/x^2)*x - sqrt(a)*arctan(sqrt(b - a/x^2)*x/sqrt(a))
```

mupad [B] time = 5.38, size = 54, normalized size = 1.26

$$x\,\sqrt{b-\frac{a}{x^2}}+\frac{\sqrt{a}\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)\,\sqrt{b-\frac{a}{x^2}}}{\sqrt{b}\,\sqrt{1-\frac{a}{b\,x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(a - b*x^2)/x^2)^(1/2),x)
```

```
[Out] x*(b - a/x^2)^(1/2) + (a^(1/2)*asin(a^(1/2)/(b^(1/2)*x))*(b - a/x^2)^(1/2))
/(b^(1/2)*(1 - a/(b*x^2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{-a+bx^2}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2-a)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((-a + b*x**2)/x**2), x)
```

3.291 $\int \sqrt{\frac{-a+bx^3}{x^2}}\, dx$

Optimal. Leaf size=53

$$\frac{2}{3}x\sqrt{bx-\frac{a}{x^2}}+\frac{2}{3}\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{bx-\frac{a}{x^2}}}\right)$$

Rubi [A] time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1979, 2007, 2029, 203}

$$\frac{2}{3}x\sqrt{bx-\frac{a}{x^2}}+\frac{2}{3}\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{bx-\frac{a}{x^2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-a + b*x^3)/x^2],x]
```

```
[Out] (2*x*Sqrt[-(a/x^2) + b*x])/3 + (2*Sqrt[a]*ArcTan[Sqrt[a]/(x*Sqrt[-(a/x^2) +
 b*x])])/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2007

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx &= \int \sqrt{-\frac{a}{x^2}+bx}\,dx \\
&= \frac{2}{3}x\sqrt{-\frac{a}{x^2}+bx} - a\int \frac{1}{x^2\sqrt{-\frac{a}{x^2}+bx}}\,dx \\
&= \frac{2}{3}x\sqrt{-\frac{a}{x^2}+bx} + \frac{1}{3}(2a)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{1}{x\sqrt{-\frac{a}{x^2}+bx}}\right) \\
&= \frac{2}{3}x\sqrt{-\frac{a}{x^2}+bx} + \frac{2}{3}\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{-\frac{a}{x^2}+bx}}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 73, normalized size = 1.38

$$\frac{2x\sqrt{bx-\frac{a}{x^2}}\left(\sqrt{bx^3-a}-\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bx^3-a}}{\sqrt{a}}\right)\right)}{3\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-a + b*x^3)/x^2],x]
```

```
[Out] (2*x*Sqrt[-(a/x^2) + b*x]*(Sqrt[-a + b*x^3] - Sqrt[a]*ArcTan[Sqrt[-a + b*x^
3]/Sqrt[a]]))/(3*Sqrt[-a + b*x^3])
```

IntegrateAlgebraic [A] time = 4.13, size = 76, normalized size = 1.43

$$\frac{x\sqrt{bx-\frac{a}{x^2}}\left(\frac{2}{3}\sqrt{bx^3-a}-\frac{2}{3}\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bx^3-a}}{\sqrt{a}}\right)\right)}{\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(-a + b*x^3)/x^2],x]
```

```
[Out] (x*Sqrt[-(a/x^2) + b*x]*((2*Sqrt[-a + b*x^3])/3 - (2*Sqrt[a]*ArcTan[Sqrt[-a
 + b*x^3]/Sqrt[a]])/3))/Sqrt[-a + b*x^3]
```

fricas [A] time = 0.40, size = 109, normalized size = 2.06

$$\left[\frac{2}{3}x\sqrt{\frac{bx^3-a}{x^2}}+\frac{1}{3}\sqrt{-a}\log\left(\frac{bx^3-2\sqrt{-a}\,x\sqrt{\frac{bx^3-a}{x^2}}-2a}{x^3}\right), \frac{2}{3}x\sqrt{\frac{bx^3-a}{x^2}}-\frac{2}{3}\sqrt{a}\arctan\left(\frac{x\sqrt{\frac{bx^3-a}{x^2}}}{\sqrt{a}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3-a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/3*x*sqrt((b*x^3 - a)/x^2) + 1/3*sqrt(-a)*log((b*x^3 - 2*sqrt(-a)*x*sqrt(
(b*x^3 - a)/x^2) - 2*a)/x^3), 2/3*x*sqrt((b*x^3 - a)/x^2) - 2/3*sqrt(a)*arc
tan(x*sqrt((b*x^3 - a)/x^2)/sqrt(a))]
```

giac [A] time = 0.19, size = 65, normalized size = 1.23

$$-\frac{2}{3}\sqrt{a}\arctan\left(\frac{\sqrt{bx^3-a}}{\sqrt{a}}\right)\mathrm{sgn}(x)+\frac{2}{3}\left(\sqrt{a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{a}}\right)-\sqrt{-a}\right)\mathrm{sgn}(x)+\frac{2}{3}\sqrt{bx^3-a}\,\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3-a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(a)*arctan(sqrt(b*x^3 - a)/sqrt(a))*sgn(x) + 2/3*(sqrt(a)*arctan(s
qrt(-a)/sqrt(a)) - sqrt(-a))*sgn(x) + 2/3*sqrt(b*x^3 - a)*sgn(x)
```

maple [A] time = 0.11, size = 73, normalized size = 1.38

$$\frac{2\sqrt{\frac{b\,x^3-a}{x^2}}\,\left(a\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3-a}}{\sqrt{-a}}\right)+\sqrt{b\,x^3-a}\,\sqrt{-a}\right)x}{3\sqrt{b\,x^3-a}\,\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3-a)/x^2)^(1/2),x)
```

```
[Out] 2/3*((b*x^3-a)/x^2)^(1/2)*x*((b*x^3-a)^(1/2)*(-a)^(1/2)+a*arctanh((b*x^3-a)
^(1/2)/(-a)^(1/2)))/(b*x^3-a)^(1/2)/(-a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{bx^3-a}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3-a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x^3 - a)/x^2), x)
```

mupad [B] time = 5.35, size = 63, normalized size = 1.19

$$\frac{2\,x\,\sqrt{b\,x-\frac{a}{x^2}}}{3}+\frac{2\,\sqrt{a}\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{3/2}}\right)\,\sqrt{b\,x-\frac{a}{x^2}}}{3\,\sqrt{b}\,\sqrt{x}\,\sqrt{1-\frac{a}{b\,x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(a - b*x^3)/x^2)^(1/2),x)
```

```
[Out] (2*x*(b*x - a/x^2)^(1/2))/3 + (2*a^(1/2)*asin(a^(1/2)/(b^(1/2)*x^(3/2)))*(b
*x - a/x^2)^(1/2))/(3*b^(1/2)*x^(1/2)*(1 - a/(b*x^3))^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3-a)/x**2)**(1/2),x)
```

```
[Out] Timed out
```

3.292 $\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx$

Optimal. Leaf size=63

$$\frac{2x\sqrt{bx^{n-2}-\frac{a}{x^2}}}{n}+\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{bx^{n-2}-\frac{a}{x^2}}}\right)}{n}$$

Rubi [A] time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1979, 2007, 2029, 203}

$$\frac{2x\sqrt{bx^{n-2}-\frac{a}{x^2}}}{n}+\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{bx^{n-2}-\frac{a}{x^2}}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-a + b*x^n)/x^2],x]
```

```
[Out] (2*x*Sqrt[-(a/x^2) + b*x^(-2 + n)])/n + (2*Sqrt[a]*ArcTan[Sqrt[a]/(x*Sqrt[-
(a/x^2) + b*x^(-2 + n)])])/n
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2007

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule Rubi steps

$$\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx = \int \sqrt{-\frac{a}{x^2}+bx^{-2+n}}\,dx$$
$$= \frac{2x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}{n} - a\int \frac{1}{x^2\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}\,dx$$
$$= \frac{2x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}{n} + \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{1}{x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}\right)}{n}$$
$$= \frac{2x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}{n} + \frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}\right)}{n}$$

Mathematica [A] time = 0.03, size = 78, normalized size = 1.24

$$\frac{x\sqrt{\frac{bx^n-a}{x^2}}\left(2\sqrt{bx^n-a}-2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{bx^n-a}}{\sqrt{a}}\right)\right)}{n\sqrt{bx^n-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-a + b*x^n)/x^2],x]
```

```
[Out] (x*Sqrt[(-a + b*x^n)/x^2]*(2*Sqrt[-a + b*x^n] - 2*Sqrt[a]*ArcTan[Sqrt[-a +
b*x^n]/Sqrt[a]]))/(n*Sqrt[-a + b*x^n])
```

IntegrateAlgebraic [A] time = 0.10, size = 81, normalized size = 1.29

$$\frac{x\sqrt{\frac{bx^n-a}{x^2}}\left(\frac{2\sqrt{bx^n-a}}{n}-\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{bx^n-a}}{\sqrt{a}}\right)}{n}\right)}{\sqrt{bx^n-a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(-a + b*x^n)/x^2],x]
```

```
[Out] (x*Sqrt[(-a + b*x^n)/x^2]*((2*Sqrt[-a + b*x^n])/n - (2*Sqrt[a]*ArcTan[Sqrt[
-a + b*x^n]/Sqrt[a]])/n))/Sqrt[-a + b*x^n]
```

fricas [A] time = 0.42, size = 118, normalized size = 1.87

$$\left[\frac{2\,x\sqrt{\frac{bx^n-a}{x^2}}+\sqrt{-a}\,\log\left(\frac{bx^n-2\,\sqrt{-a}\,x\sqrt{\frac{bx^n-a}{x^2}}-2\,a}{x^n}\right)}{n}, \frac{2\left(x\sqrt{\frac{bx^n-a}{x^2}}-\sqrt{a}\,\arctan\left(\frac{x\sqrt{\frac{bx^n-a}{x^2}}}{\sqrt{a}}\right)\right)}{n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+b*x^n)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [(2*x*sqrt((b*x^n - a)/x^2) + sqrt(-a)*log((b*x^n - 2*sqrt(-a)*x*sqrt((b*x^
n - a)/x^2) - 2*a)/x^n))/n, 2*(x*sqrt((b*x^n - a)/x^2) - sqrt(a)*arctan(x*s
qrt((b*x^n - a)/x^2)/sqrt(a)))/n]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{bx^n - a}{x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+b*x^n)/x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt((b*x^n - a)/x^2), x)
```

maple [A] time = 1.99, size = 105, normalized size = 1.67

$$-\frac{2\sqrt{\frac{b\,\mathrm{e}^{n\ln(x)}-a}{x^2}}\,\sqrt{a}\,x \arctan\left(\frac{\sqrt{b\,\mathrm{e}^{n\ln(x)}-a}}{\sqrt{a}}\right)}{\sqrt{b\,\mathrm{e}^{n\ln(x)}-a}\;n} - \frac{2\left(-b\,\mathrm{e}^{n\ln(x)}+a\right)\sqrt{\frac{b\,\mathrm{e}^{n\ln(x)}-a}{x^2}}\;x}{\left(b\,\mathrm{e}^{n\ln(x)}-a\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^n-a)/x^2)^(1/2),x)
[Out] -2*(a-b*exp(n*ln(x)))/n/(b*exp(n*ln(x))-a)*((b*exp(n*ln(x))-a)/x^2)^(1/2)*x
-2*a^(1/2)/n*arctan((b*exp(n*ln(x))-a)^(1/2)/a^(1/2))*((b*exp(n*ln(x))-a)/x
^2)^(1/2)/(b*exp(n*ln(x))-a)^(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{bx^n - a}{x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+b*x^n)/x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((b*x^n - a)/x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{-\frac{a - b\,x^n}{x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(a - b*x^n)/x^2)^(1/2),x)
[Out] int((-(a - b*x^n)/x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{-a + bx^n}{x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+b*x**n)/x**2)**(1/2),x)
[Out] Integral(sqrt((-a + b*x**n)/x**2), x)
```

3.293 $\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx$

Optimal. Leaf size=62

$$\frac{2x^{-j/2}(cx)^{j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{\sqrt{a}\,c(j-n)}$$

Rubi [A] time = 0.11, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2031, 2029, 206}

$$\frac{2x^{-j/2}(cx)^{j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{\sqrt{a}\,c(j-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 + j/2)/Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*(c*x)^(j/2)*ArcTanh[(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(Sqrt[a]*c*(
j - n)*x^(j/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx &= \frac{\left(x^{-j/2}(cx)^{j/2}\right)\int \frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx}{c}\\
&= \frac{\left(2x^{-j/2}(cx)^{j/2}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}\\
&= \frac{2x^{-j/2}(cx)^{j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{\sqrt{a}\,c(j-n)}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 98, normalized size = 1.58

$$\frac{2\sqrt{b}\,(cx)^{j/2}x^{\frac{n-j}{2}}\sqrt{\frac{ax^{j-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{j-n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}\,c(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 + j/2)/Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*Sqrt[b]*x^((-j + n)/2)*(c*x)^(j/2)*Sqrt[1 + (a*x^(j - n))/b]*ArcSinh[(Sq
rt[a]*x^((j - n)/2))/Sqrt[b]])/(Sqrt[a]*c*(j - n)*Sqrt[a*x^j + b*x^n])
```

IntegrateAlgebraic [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c*x)^(-1 + j/2)/Sqrt[a*x^j + b*x^n],x]
```

```
[Out] Defer[IntegrateAlgebraic][(c*x)^(-1 + j/2)/Sqrt[a*x^j + b*x^n], x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{1}{2}j-1}}{\sqrt{ax^j+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(1/2*j - 1)/sqrt(a*x^j + b*x^n), x)
```

maple [F] time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{j}{2}-1}}{\sqrt{a\,x^j+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{1}{2}j-1}}{\sqrt{ax^j+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate((c*x)^(1/2*j - 1)/sqrt(a*x^j + b*x^n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^{\frac{j}{2}-1}}{\sqrt{a\,x^j+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(j/2 - 1)/(a*x^j + b*x^n)^(1/2),x)
[Out] int((c*x)^(j/2 - 1)/(a*x^j + b*x^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{j}{2}-1}}{\sqrt{ax^j+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1+1/2*j)/(a*x**j+b*x**n)**(1/2),x)
[Out] Integral((c*x)**(j/2 - 1)/sqrt(a*x**j + b*x**n), x)
```

3.294 $\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx$

Optimal. Leaf size=53

$$\frac{2\sqrt{cx}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{\sqrt{a}\,(3-n)\sqrt{x}}$$

Rubi [A] time = 0.10, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2031, 2029, 206}

$$\frac{2\sqrt{cx}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{\sqrt{a}\,(3-n)\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/Sqrt[a*x^3 + b*x^n],x]
```

```
[Out] (2*Sqrt[c*x]*ArcTanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(Sqrt[a]*(3 - n)*Sqrt[x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx &= \frac{\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{ax^3+bx^n}}\,dx}{\sqrt{x}}\\ &= \frac{\left(2\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{(3-n)\sqrt{x}}\\ &= \frac{2\sqrt{cx}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{\sqrt{a}\,(3-n)\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.17, size = 89, normalized size = 1.68

$$-\frac{2\sqrt{b}\,\sqrt{cx}\,x^{\frac{n-1}{2}}\sqrt{\frac{ax^{3-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}\,(n-3)\sqrt{ax^3+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/Sqrt[a*x^3 + b*x^n],x]
```

```
[Out] (-2*Sqrt[b]*x^((-1 + n)/2)*Sqrt[c*x]*Sqrt[1 + (a*x^(3 - n))/b]*ArcSinh[(Sqr
t[a]*x^(3/2 - n/2))/Sqrt[b]])/(Sqrt[a]*(-3 + n)*Sqrt[a*x^3 + b*x^n])
```

IntegrateAlgebraic [F] time = 2.05, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[c*x]/Sqrt[a*x^3 + b*x^n],x]
```

```
[Out] Defer[IntegrateAlgebraic][Sqrt[c*x]/Sqrt[a*x^3 + b*x^n], x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/sqrt(a*x^3 + b*x^n), x)
```

maple [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\sqrt{a\,x^3+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/sqrt(a*x^3 + b*x^n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x}}{\sqrt{b\,x^n+a\,x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(b*x^n + a*x^3)^(1/2),x)
[Out] int((c*x)^(1/2)/(b*x^n + a*x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(a*x**3+b*x**n)**(1/2),x)
[Out] Integral(sqrt(c*x)/sqrt(a*x**3 + b*x**n), x)
```

3.295 $\int \frac{1}{\sqrt{ax^2+bx^n}} \, dx$

Optimal. Leaf size=37

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{\sqrt{a}\,(2-n)}$$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2008, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{\sqrt{a}\,(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x^2 + b*x^n],x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^n]])/(Sqrt[a]*(2 - n))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{ax^2+bx^n}} \, dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{1-ax^2} \, dx, x, \frac{x}{\sqrt{ax^2+bx^n}}\right)}{2-n}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{\sqrt{a}\,(2-n)}\end{aligned}$$

Mathematica [B] time = 0.07, size = 78, normalized size = 2.11

$$-\frac{2\sqrt{b}\,x^{n/2}\sqrt{\frac{ax^{2-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{1-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}\,(n-2)\sqrt{ax^2+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x^2 + b*x^n],x]
```

```
[Out] (-2*Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(2 - n))/b]*ArcSinh[(Sqrt[a]*x^(1 - n/2))
/Sqrt[b]])/(Sqrt[a]*(-2 + n)*Sqrt[a*x^2 + b*x^n])
```

IntegrateAlgebraic [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2+bx^n}} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/Sqrt[a*x^2 + b*x^n],x]
[Out] Defer[IntegrateAlgebraic][1/Sqrt[a*x^2 + b*x^n], x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2 + bx^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(a*x^2 + b*x^n), x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\, x^2 + b\, x^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2+b*x^n)^(1/2),x)
[Out] int(1/(a*x^2+b*x^n)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2 + bx^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(a*x^2 + b*x^n), x)
```

mupad [B] time = 5.39, size = 67, normalized size = 1.81

$$\frac{\sqrt{b}\, x^{n/2}\, \mathrm{asin}\left(\frac{\sqrt{a}\, x^{1-\frac{n}{2}}\, 1\mathrm{i}}{\sqrt{b}}\right)\, \sqrt{\frac{a\, x^{2-n}}{b}+1}\, 1\mathrm{i}}{\sqrt{a}\, \left(\frac{n}{2}-1\right)\, \sqrt{b\, x^n+a\, x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n + a*x^2)^(1/2),x)
[Out] (b^(1/2)*x^(n/2)*asin((a^(1/2)*x^(1 - n/2)*1i)/b^(1/2))*((a*x^(2 - n))/b +
1)^(1/2)*1i)/(a^(1/2)*(n/2 - 1)*(b*x^n + a*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2 + bx^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**2+b*x**n)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**2 + b*x**n), x)
```

3.296 $\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx$

Optimal. Leaf size=51

$$\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{\sqrt{a}\,(1-n)\sqrt{cx}}$$

Rubi [A] time = 0.09, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2031, 2029, 206}

$$\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{\sqrt{a}\,(1-n)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*Sqrt[a*x + b*x^n]),x]
```

```
[Out] (2*Sqrt[x]*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(Sqrt[a]*(1 - n)*S
qrt[c*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx &= \frac{\sqrt{x}\int \frac{1}{\sqrt{x}\sqrt{ax+bx^n}}\,dx}{\sqrt{cx}}\\
&= \frac{\left(2\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{(1-n)\sqrt{cx}}\\
&= \frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{\sqrt{a}\,(1-n)\sqrt{cx}}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 87, normalized size = 1.71

$$-\frac{2\sqrt{b}\,x^{\frac{n+1}{2}}\sqrt{\frac{ax^{1-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}\,(n-1)\sqrt{cx}\sqrt{ax+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*Sqrt[a*x + b*x^n]),x]
```

```
[Out] (-2*Sqrt[b]*x^((1 + n)/2)*Sqrt[1 + (a*x^(1 - n))/b]*ArcSinh[(Sqrt[a]*x^(1/2
 - n/2))/Sqrt[b]])/(Sqrt[a]*(-1 + n)*Sqrt[c*x]*Sqrt[a*x + b*x^n])
```

IntegrateAlgebraic [F] time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx}\,\sqrt{ax+bx^n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(Sqrt[c*x]*Sqrt[a*x + b*x^n]),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/(Sqrt[c*x]*Sqrt[a*x + b*x^n]), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+bx^n}\,\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^n)*sqrt(c*x)), x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx}\,\sqrt{ax+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x)
```

```
[Out] int(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+bx^n}\,\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^n)*sqrt(c*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{c\,x}\,\sqrt{b\,x^n+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/2)*(b*x^n + a*x)^(1/2)),x)
[Out] int(1/((c*x)^(1/2)*(b*x^n + a*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx}\,\sqrt{ax+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(a*x+b*x**n)**(1/2),x)
[Out] Integral(1/(sqrt(c*x)*sqrt(a*x + b*x**n)), x)
```

3.297 $\int \frac{1}{cx\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=31

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}\,cn}$$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {12, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}\,cn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(c*x*Sqrt[a + b*x^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*c*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{cx\sqrt{a+bx^n}}\,dx = \frac{\int \frac{1}{x\sqrt{a+bx^n}}\,dx}{c}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{cn}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{bcn}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}\,cn}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}\,cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(c*x*Sqrt[a + b*x^n]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*c*n)
```

IntegrateAlgebraic [A] time = 0.05, size = 31, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}\,cn}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(c*x*Sqrt[a + b*x^n]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*c*n)
```

fricas [A] time = 0.42, size = 76, normalized size = 2.45

$$\left[\frac{\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)}{\sqrt{a}\,cn}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right)}{acn}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x^n)^(1/2),x, algorithm="fricas")
[Out] [log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n)/(sqrt(a)*c*n), 2*sqrt(-
a)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a)/(a*c*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^n+a}\,cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^n + a)*c*x), x)
```

maple [A] time = 0.05, size = 26, normalized size = 0.84

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{n}+a}}{\sqrt{a}}\right)}{\sqrt{a}\;c n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/c/x/(b*x^n+a)^(1/2),x)
[Out] -2*arctanh((b*x^n+a)^(1/2)/a^(1/2))/c/n/a^(1/2)
```

maxima [A] time = 2.96, size = 42, normalized size = 1.35

$$\frac{\log\left(\frac{\sqrt{bx^{n}+a}-\sqrt{a}}{\sqrt{bx^{n}+a}+\sqrt{a}}\right)}{\sqrt{a}\,cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x^n)^(1/2),x, algorithm="maxima")
[Out] log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/(sqrt(a)*c*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{c\,x\,\sqrt{a+b\,x^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x*(a + b*x^n)^(1/2)),x)
[Out] int(1/(c*x*(a + b*x^n)^(1/2)), x)
```

sympy [A] time = 1.98, size = 27, normalized size = 0.87

$$-\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{a}\,x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}\,cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x**n)**(1/2),x)
[Out] -2*asinh(sqrt(a)*x**(-n/2)/sqrt(b))/(sqrt(a)*c*n)
```

3.298 $\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx$

Optimal. Leaf size=54

$$-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{\sqrt{a}\,c(n+1)\sqrt{cx}}$$

Rubi [A] time = 0.11, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2031, 2029, 206}

$$-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{\sqrt{a}\,c(n+1)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*Sqrt[a/x + b*x^n]),x]
```

```
[Out] (-2*Sqrt[x]*ArcTanh[Sqrt[a]/(Sqrt[x]*Sqrt[a/x + b*x^n])])/(Sqrt[a]*c*(1 + n
)*Sqrt[c*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx &= \frac{\sqrt{x}\int \frac{1}{x^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx}{c\sqrt{cx}}\\
&= -\frac{\left(2\sqrt{x}\right)\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{c(1+n)\sqrt{cx}}\\
&= -\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{\sqrt{a}\,c(1+n)\sqrt{cx}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 68, normalized size = 1.26

$$-\frac{2x\sqrt{a+bx^{n+1}}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+1}}}{\sqrt{a}}\right)}{\sqrt{a}(n+1)(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*Sqrt[a/x + b*x^n]),x]
```

```
[Out] (-2*x*Sqrt[a + b*x^(1 + n)]*ArcTanh[Sqrt[a + b*x^(1 + n)]/Sqrt[a]])/(Sqrt[a
]*(1 + n)*(c*x)^(3/2)*Sqrt[a/x + b*x^n])
```

IntegrateAlgebraic [A] time = 1.12, size = 70, normalized size = 1.30

$$-\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+1}}}{\sqrt{a}}\right)}{\sqrt{a}c^2(n+1)\sqrt{a+bx^{n+1}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x)^(3/2)*Sqrt[a/x + b*x^n]),x]
```

```
[Out] (-2*Sqrt[c*x]*Sqrt[a/x + b*x^n]*ArcTanh[Sqrt[a + b*x^(1 + n)]/Sqrt[a]])/(Sq
rt[a]*c^2*(1 + n)*Sqrt[a + b*x^(1 + n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^n+\frac{a}{x}}\,(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^n + a/x)*(c*x)^(3/2)), x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{3}{2}}\sqrt{b\,x^n+\frac{a}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(b*x^n+a/x)^(1/2),x)
```

```
[Out] int(1/(c*x)^(3/2)/(b*x^n+a/x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^n + \frac{a}{x}}\,(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^n + a/x)*(c*x)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(c\,x)^{3/2}\,\sqrt{b\,x^n + \frac{a}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/2)*(b*x^n + a/x)^(1/2)),x)
[Out] int(1/((c*x)^(3/2)*(b*x^n + a/x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{3}{2}}\,\sqrt{\frac{a}{x} + bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(a/x+b*x**n)**(1/2),x)
[Out] Integral(1/((c*x)**(3/2)*sqrt(a/x + b*x**n)), x)
```

3.299 $\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx$

Optimal. Leaf size=40

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{\sqrt{a}\,c^2(n+2)}$$

Rubi [A] time = 0.07, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {12, 2029, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{\sqrt{a}\,c^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(c^2*x^2*Sqrt[a/x^2 + b*x^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x^n])])/(Sqrt[a]*c^2*(2 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx &= \frac{\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx}{c^2}\\
&= -\frac{2\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{c^2(2+n)}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{\sqrt{a}\,c^2(2+n)}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 1.65

$$-\frac{2\sqrt{a+bx^{n+2}}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+2}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^2(n+2)x\sqrt{\frac{a}{x^2}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(c^2*x^2*Sqrt[a/x^2 + b*x^n]),x]
```

```
[Out] (-2*Sqrt[a + b*x^(2 + n)]*ArcTanh[Sqrt[a + b*x^(2 + n)]/Sqrt[a]])/(Sqrt[a]*
c^2*(2 + n)*x*Sqrt[a/x^2 + b*x^n])
```

IntegrateAlgebraic [A] time = 0.11, size = 64, normalized size = 1.60

$$-\frac{2x\sqrt{\frac{a}{x^2}+bx^n}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+2}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^2(n+2)\sqrt{a+bx^{n+2}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(c^2*x^2*Sqrt[a/x^2 + b*x^n]),x]
```

```
[Out] (-2*x*Sqrt[a/x^2 + b*x^n]*ArcTanh[Sqrt[a + b*x^(2 + n)]/Sqrt[a]])/(Sqrt[a]*
c^2*(2 + n)*Sqrt[a + b*x^(2 + n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^n+\frac{a}{x^2}}\,c^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^n + a/x^2)*c^2*x^2), x)
```

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,x^n+\frac{a}{x^2}}\,c^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/c^2/x^2/(b*x^n+a/x^2)^(1/2),x)
```

```
[Out] int(1/c^2/x^2/(b*x^n+a/x^2)^(1/2),x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{bx^n+\frac{a}{x^2}}\,x^2}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^n + a/x^2)*x^2), x)/c^2
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{c^2\,x^2\,\sqrt{b\,x^n+\frac{a}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^2*(b*x^n + a/x^2)^(1/2)),x)
[Out] int(1/(c^2*x^2*(b*x^n + a/x^2)^(1/2)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c**2/x**2/(a/x**2+b*x**n)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(a/x**2 + b*x**n)), x)/c**2
```

3.300 $\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx$

Optimal. Leaf size=54

$$-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{\sqrt{a}\,c^2(n+3)\sqrt{cx}}$$

Rubi [A] time = 0.14, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2031, 2029, 206}

$$-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{\sqrt{a}\,c^2(n+3)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*Sqrt[a/x^3 + b*x^n]),x]
```

```
[Out] (-2*Sqrt[x]*ArcTanh[Sqrt[a]/(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/(Sqrt[a]*c^2*(3
 + n)*Sqrt[c*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx &= \frac{\sqrt{x}\int \frac{1}{x^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx}{c^2\sqrt{cx}}\\
&= -\frac{\left(2\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{c^2(3+n)\sqrt{cx}}\\
&= -\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{\sqrt{a}\,c^2(3+n)\sqrt{cx}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 68, normalized size = 1.26

$$-\frac{2x\sqrt{a+bx^{n+3}}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+3}}}{\sqrt{a}}\right)}{\sqrt{a}\,(n+3)(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*Sqrt[a/x^3 + b*x^n]),x]
```

```
[Out] (-2*x*Sqrt[a + b*x^(3 + n)]*ArcTanh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]])/(Sqrt[a
]*(3 + n)*(c*x)^(5/2)*Sqrt[a/x^3 + b*x^n])
```

IntegrateAlgebraic [A] time = 1.20, size = 70, normalized size = 1.30

$$-\frac{2\sqrt{a+bx^{n+3}}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+3}}}{\sqrt{a}}\right)}{\sqrt{a}\,c(n+3)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x)^(5/2)*Sqrt[a/x^3 + b*x^n]),x]
```

```
[Out] (-2*Sqrt[a + b*x^(3 + n)]*ArcTanh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]])/(Sqrt[a]*
c*(3 + n)*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^n+\frac{a}{x^3}}\,(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^n + a/x^3)*(c*x)^(5/2)), x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{5}{2}}\sqrt{b\,x^n+\frac{a}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(b*x^n+a/x^3)^(1/2),x)
```

```
[Out] int(1/(c*x)^(5/2)/(b*x^n+a/x^3)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^n + \frac{a}{x^3}}\,(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^n + a/x^3)*(c*x)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(c\,x)^{5/2}\,\sqrt{b\,x^n + \frac{a}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(b*x^n + a/x^3)^(1/2)),x)
[Out] int(1/((c*x)^(5/2)*(b*x^n + a/x^3)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(a/x**3+b*x**n)**(1/2),x)
[Out] Timed out
```

3.301 $\int \frac{(cx)^{-1+\frac{3j}{2}}}{\left(ax^j+bx^n\right)^{3/2}}\,dx$

Optimal. Leaf size=107

$$\frac{2x^{-3j/2}(cx)^{3j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{a^{3/2}c(j-n)}-\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}}$$

Rubi [A] time = 0.19, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2031, 2030, 2029, 206}

$$\frac{2x^{-3j/2}(cx)^{3j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{a^{3/2}c(j-n)}-\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 + (3*j)/2)/(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (-2*(c*x)^((3*j)/2))/(a*c*(j - n)*x^j*Sqrt[a*x^j + b*x^n]) + (2*(c*x)^((3*j
)/2)*ArcTanh[(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(a^(3/2)*c*(j - n)*x^(
(3*j)/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int
egerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{\left(ax^j+bx^n\right)^{3/2}}\,dx = \frac{\left(x^{-3j/2}(cx)^{3j/2}\right)\int \frac{x^{-1+\frac{3j}{2}}}{\left(ax^j+bx^n\right)^{3/2}}\,dx}{c}$$

$$= -\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}} + \frac{\left(x^{-3j/2}(cx)^{3j/2}\right)\int \frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx}{ac}$$

$$= -\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}} + \frac{\left(2x^{-3j/2}(cx)^{3j/2}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{ac(j-n)}$$

$$= -\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}} + \frac{2x^{-3j/2}(cx)^{3j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{a^{3/2}c(j-n)}$$

Mathematica [A] time = 0.20, size = 117, normalized size = 1.09

$$-\frac{2x^{-3j/2}(cx)^{3j/2}\left(\sqrt{a}\,x^{j/2}-\sqrt{b}\,x^{n/2}\sqrt{\frac{ax^{j-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{j-n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}c(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 + (3*j)/2)/(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (-2*(c*x)^((3*j)/2)*(Sqrt[a]*x^(j/2) - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(j - n
))/b]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[b]]))/(a^(3/2)*c*(j - n)*x^((3*j
)/2)*Sqrt[a*x^j + b*x^n])
```

IntegrateAlgebraic [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{\left(ax^j+bx^n\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c*x)^(-1 + (3*j)/2)/(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(c*x)^(-1 + (3*j)/2)/(a*x^j + b*x^n)^(3/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}j-1}}{\left(ax^j+bx^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate((c*x)^(3/2*j - 1)/(a*x^j + b*x^n)^(3/2), x)
```

maple [F] time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3j}{2}-1}}{\left(a\,x^{j}+b\,x^{n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x)
[Out] int((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}j-1}}{\left(ax^{j}+bx^{n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x)^(3/2*j - 1)/(a*x^j + b*x^n)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{\frac{3\,j}{2}-1}}{\left(a\,x^{j}+b\,x^{n}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^((3*j)/2 - 1)/(a*x^j + b*x^n)^(3/2),x)
[Out] int((c*x)^((3*j)/2 - 1)/(a*x^j + b*x^n)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3j}{2}-1}}{\left(ax^{j}+bx^{n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1+3/2*j)/(a*x**j+b*x**n)**(3/2),x)
[Out] Integral((c*x)**(3*j/2 - 1)/(a*x**j + b*x**n)**(3/2), x)
```

3.302 $\int \frac{(cx)^{7/2}}{\left(ax^3+bx^n\right)^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{2c^3\sqrt{cx}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{a^{3/2}(3-n)\sqrt{x}}-\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}$$

Rubi [A] time = 0.16, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2030, 2031, 2029, 206}

$$\frac{2c^3\sqrt{cx}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{a^{3/2}(3-n)\sqrt{x}}-\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2),x]
```

```
[Out] (-2*c^2*(c*x)^(3/2))/(a*(3 - n)*Sqrt[a*x^3 + b*x^n]) + (2*c^3*Sqrt[c*x]*Arc
Tanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(a^(3/2)*(3 - n)*Sqrt[x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int
egerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{(cx)^{7/2}}{\left(ax^3+bx^n\right)^{3/2}}\,dx = -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}} + \frac{c^3\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx}{a}$$

$$= -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}} + \frac{\left(c^3\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{ax^3+bx^n}}\,dx}{a\sqrt{x}}$$

$$= -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}} + \frac{\left(2c^3\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{a(3-n)\sqrt{x}}$$

$$= -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}} + \frac{2c^3\sqrt{cx}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{a^{3/2}(3-n)\sqrt{x}}$$

Mathematica [A] time = 0.21, size = 109, normalized size = 1.16

$$\frac{2c^3\sqrt{cx}\left(\sqrt{a}\,x^{3/2}-\sqrt{b}\,x^{n/2}\sqrt{\frac{ax^{3-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}(n-3)\sqrt{x}\,\sqrt{ax^3+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2),x]
```

```
[Out] (2*c^3*Sqrt[c*x]*(Sqrt[a]*x^(3/2) - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(3 - n))/
b]*ArcSinh[(Sqrt[a]*x^(3/2 - n/2))/Sqrt[b]]))/(a^(3/2)*(-3 + n)*Sqrt[x]*Sqr
t[a*x^3 + b*x^n])
```

IntegrateAlgebraic [F] time = 7.60, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{7/2}}{\left(ax^3+bx^n\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{2}}}{\left(ax^3+bx^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate((c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2), x)
```

maple [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{2}}}{\left(a\,x^{3}+b\,x^{n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x)
[Out] int((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{2}}}{\left(ax^{3}+bx^{n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{7/2}}{\left(b\,x^{n}+a\,x^{3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(b*x^n + a*x^3)^(3/2),x)
[Out] int((c*x)^(7/2)/(b*x^n + a*x^3)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)/(a*x**3+b*x**n)**(3/2),x)
[Out] Timed out
```

3.303 $\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2c^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{a^{3/2}(2-n)}-\frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}}$$

Rubi [A] time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 2030, 2008, 206}

$$\frac{2c^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{a^{3/2}(2-n)}-\frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(c^2*x^2)/(a*x^2 + b*x^n)^(3/2),x]
```

```
[Out] (-2*c^2*x)/(a*(2 - n)*Sqrt[a*x^2 + b*x^n]) + (2*c^2*ArcTanh[(Sqrt[a]*x)/Sqr
t[a*x^2 + b*x^n]])/(a^(3/2)*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int
egerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{c^2x^2}{\left(ax^2+bx^n\right)^{3/2}}\,dx = c^2\int \frac{x^2}{\left(ax^2+bx^n\right)^{3/2}}\,dx$$
$$= -\frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}} + \frac{c^2\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx}{a}$$
$$= -\frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}} + \frac{(2c^2)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^n}}\right)}{a(2-n)}$$
$$= -\frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}} + \frac{2c^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{a^{3/2}(2-n)}$$

Mathematica [A] time = 0.15, size = 91, normalized size = 1.26

$$\frac{2c^2\left(\sqrt{a}\,x-\sqrt{b}\,x^{n/2}\sqrt{\frac{ax^{2-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{1-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}(n-2)\sqrt{ax^2+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c^2*x^2)/(a*x^2 + b*x^n)^(3/2),x]
```

```
[Out] (2*c^2*(Sqrt[a]*x - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(2 - n))/b]*ArcSinh[(Sqrt
[a]*x^(1 - n/2))/Sqrt[b]]))/(a^(3/2)*(-2 + n)*Sqrt[a*x^2 + b*x^n])
```

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{c^2x^2}{\left(ax^2+bx^n\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c^2*x^2)/(a*x^2 + b*x^n)^(3/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(c^2*x^2)/(a*x^2 + b*x^n)^(3/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^2/(a*x^2+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c^2x^2}{\left(ax^2+bx^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^2/(a*x^2+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(c^2*x^2/(a*x^2 + b*x^n)^(3/2), x)
```

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{c^2x^2}{\left(a\,x^2+b\,x^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c^2*x^2/(a*x^2+b*x^n)^(3/2),x)
[Out] int(c^2*x^2/(a*x^2+b*x^n)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2 \int \frac{x^2}{\left(ax^2+bx^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^2/(a*x^2+b*x^n)^(3/2),x, algorithm="maxima")
[Out] c^2*integrate(x^2/(a*x^2 + b*x^n)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c^2\,x^2}{\left(b\,x^n+a\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2)/(b*x^n + a*x^2)^(3/2),x)
[Out] int((c^2*x^2)/(b*x^n + a*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2 \int \frac{x^2}{ax^2\sqrt{ax^2+bx^n}+bx^n\sqrt{ax^2+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c**2*x**2/(a*x**2+b*x**n)**(3/2),x)
[Out] c**2*Integral(x**2/(a*x**2*sqrt(a*x**2 + b*x**n) + b*x**n*sqrt(a*x**2 + b*x
**n)), x)
```

3.304 $\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{2c\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a^{3/2}(1-n)\sqrt{cx}}-\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}$$

Rubi [A] time = 0.14, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2030, 2031, 2029, 206}

$$\frac{2c\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a^{3/2}(1-n)\sqrt{cx}}-\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a*x + b*x^n)^(3/2),x]
```

```
[Out] (-2*Sqrt[c*x])/(a*(1 - n)*Sqrt[a*x + b*x^n]) + (2*c*Sqrt[x]*ArcTanh[(Sqrt[a
]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(a^(3/2)*(1 - n)*Sqrt[c*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int
egerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx = -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}} + \frac{c\int \frac{1}{\sqrt{cx}\,\sqrt{ax+bx^n}}\,dx}{a}$$
$$= -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}} + \frac{\left(c\sqrt{x}\right)\int \frac{1}{\sqrt{x}\,\sqrt{ax+bx^n}}\,dx}{a\sqrt{cx}}$$
$$= -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}} + \frac{\left(2c\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a(1-n)\sqrt{cx}}$$
$$= -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}} + \frac{2c\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a^{3/2}(1-n)\sqrt{cx}}$$

Mathematica [A] time = 0.20, size = 104, normalized size = 1.22

$$\frac{2\sqrt{cx}\left(\sqrt{a}\,\sqrt{x}-\sqrt{b}\,x^{n/2}\sqrt{\frac{ax^{1-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}(n-1)\sqrt{x}\,\sqrt{ax+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a*x + b*x^n)^(3/2),x]
```

```
[Out] (2*Sqrt[c*x]*(Sqrt[a]*Sqrt[x] - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(1 - n))/b]*A
rcSinh[(Sqrt[a]*x^(1/2 - n/2))/Sqrt[b]]))/(a^(3/2)*(-1 + n)*Sqrt[x]*Sqrt[a*
x + b*x^n])
```

IntegrateAlgebraic [F] time = 1.43, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[c*x]/(a*x + b*x^n)^(3/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][Sqrt[c*x]/(a*x + b*x^n)^(3/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(a*x + b*x^n)^(3/2), x)
```

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{(ax + b\,x^{n})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x)
[Out] int((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{(ax + bx^{n})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x)/(a*x + b*x^n)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x}}{{\left(b\,x^n+a\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(b*x^n + a*x)^(3/2),x)
[Out] int((c*x)^(1/2)/(b*x^n + a*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{(ax + bx^{n})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(a*x+b*x**n)**(3/2),x)
[Out] Integral(sqrt(c*x)/(a*x + b*x**n)**(3/2), x)
```

3.305 $\int \frac{1}{cx(a+bx^n)^{3/2}} dx$

Optimal. Leaf size=54

$$\frac{2}{acn\sqrt{a+bx^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}cn}$$

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {12, 266, 51, 63, 208}

$$\frac{2}{acn\sqrt{a+bx^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}cn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(c*x*(a + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*c*n*Sqrt[a + b*x^n]) - (2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(a^(3/2)*c
*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{cx(a+bx^n)^{3/2}}\,dx = \frac{\int \frac{1}{x(a+bx^n)^{3/2}}\,dx}{c}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^n\right)}{cn}$$

$$= \frac{2}{acn\sqrt{a+bx^n}} + \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{acn}$$

$$= \frac{2}{acn\sqrt{a+bx^n}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{abcn}$$

$$= \frac{2}{acn\sqrt{a+bx^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}cn}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.74

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^n}{a}+1\right)}{acn\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(c*x*(a + b*x^n)^(3/2)),x]
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^n)/a])/(a*c*n*Sqrt[a + b*x^n])
```

IntegrateAlgebraic [A] time = 0.07, size = 54, normalized size = 1.00

$$\frac{2}{acn\sqrt{a+bx^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}cn}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(c*x*(a + b*x^n)^(3/2)),x]
[Out] 2/(a*c*n*Sqrt[a + b*x^n]) - (2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(a^(3/2)*c
*n)
```

fricas [A] time = 0.42, size = 148, normalized size = 2.74

$$\left[\frac{\left(\sqrt{a}\,bx^n + a^{\frac{3}{2}}\right)\log\left(\frac{bx^n - 2\sqrt{bx^n+a}\,\sqrt{a}+2a}{x^n}\right) + 2\sqrt{bx^n+a}\,a}{a^2bcnx^n + a^3cn}, \frac{2\left(\left(\sqrt{-a}\,bx^n + \sqrt{-a}\,a\right)\arctan\left(\frac{\sqrt{bx^n+a}\,\sqrt{-a}}{a}\right) + \sqrt{bx^n+a}\,a\right)}{a^2bcnx^n + a^3cn}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x^n)^(3/2),x, algorithm="fricas")
[Out] [((sqrt(a)*b*x^n + a^(3/2))*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x
^n) + 2*sqrt(b*x^n + a)*a)/(a^2*b*c*n*x^n + a^3*c*n), 2*((sqrt(-a)*b*x^n +
sqrt(-a)*a)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a) + sqrt(b*x^n + a)*a)/(a^2*b*
c*n*x^n + a^3*c*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n+a)^{\frac{3}{2}}cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^(3/2)*c*x), x)
```

maple [A] time = 0.05, size = 42, normalized size = 0.78

$$\frac{-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{n}+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{b\,x^{n}+a}\;a}}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/c/x/(b*x^n+a)^(3/2),x)
```

```
[Out] 1/c/n*(-2/a^(3/2)*arctanh((b*x^n+a)^(1/2)/a^(1/2))+2/(b*x^n+a)^(1/2)/a)
```

maxima [A] time = 3.06, size = 61, normalized size = 1.13

$$\frac{\frac{\log\left(\frac{\sqrt{bx^{n}+a}-\sqrt{a}}{\sqrt{bx^{n}+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}n}+\frac{2}{\sqrt{bx^{n}+a}\,an}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] (log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/(a^(3/2)*n) +
 2/(sqrt(b*x^n + a)*a*n))/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{c\,x\,(a+b\,x^{n})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x*(a + b*x^n)^(3/2)),x)
```

```
[Out] int(1/(c*x*(a + b*x^n)^(3/2)), x)
```

sympy [B] time = 3.72, size = 185, normalized size = 3.43

$$\frac{\frac{2a^{3}\sqrt{1+\frac{bx^{n}}{a}}}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^{n}}+\frac{a^{3}\log\left(\frac{bx^{n}}{a}\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^{n}}-\frac{2a^{3}\log\left(\sqrt{1+\frac{bx^{n}}{a}}+1\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^{n}}+\frac{a^{2}bx^{n}\log\left(\frac{bx^{n}}{a}\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^{n}}-\frac{2a^{2}bx^{n}\log\left(\sqrt{1+\frac{bx^{n}}{a}}+1\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^{n}}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x**n)**(3/2),x)
```

```
[Out] (2*a**3*sqrt(1 + b*x**n/a)/(a**(9/2)*n + a**(7/2)*b*n*x**n) + a**3*log(b*x*
*n/a)/(a**(9/2)*n + a**(7/2)*b*n*x**n) - 2*a**3*log(sqrt(1 + b*x**n/a) + 1)
/(a**(9/2)*n + a**(7/2)*b*n*x**n) + a**2*b*x**n*log(b*x**n/a)/(a**(9/2)*n +
 a**(7/2)*b*n*x**n) - 2*a**2*b*x**n*log(sqrt(1 + b*x**n/a) + 1)/(a**(9/2)*n
 + a**(7/2)*b*n*x**n))/c
```

3.306 $\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{2}{ac^2(n+1)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{a^{3/2}c^2(n+1)\sqrt{cx}}$$

Rubi [A] time = 0.19, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2030, 2031, 2029, 206}

$$\frac{2}{ac^2(n+1)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{a^{3/2}c^2(n+1)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a/x + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*c^2*(1 + n)*Sqrt[c*x]*Sqrt[a/x + b*x^n]) - (2*Sqrt[x]*ArcTanh[Sqrt[a]/
(Sqrt[x]*Sqrt[a/x + b*x^n])])/(a^(3/2)*c^2*(1 + n)*Sqrt[c*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int
egerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx = \frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}} + \frac{\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx}{ac}$$

$$= \frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}} + \frac{\sqrt{x}\int \frac{1}{x^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx}{ac^2\sqrt{cx}}$$

$$= \frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}} - \frac{\left(2\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{ac^2(1+n)\sqrt{cx}}$$

$$= \frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}} - \frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{a^{3/2}c^2(1+n)\sqrt{cx}}$$

Mathematica [C] time = 0.05, size = 55, normalized size = 0.61

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^{n+1}}{a}+1\right)}{ac^2(n+1)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a/x + b*x^n)^(3/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^(1 + n))/a])/(a*c^2*(1 + n)*Sqr
t[c*x]*Sqrt[a/x + b*x^n])
```

IntegrateAlgebraic [A] time = 1.54, size = 107, normalized size = 1.19

$$\frac{\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}\left(\frac{2}{ac^{5/2}(n+1)\sqrt{a+bx^{n+1}}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+1}}}{\sqrt{a}}\right)}{a^{3/2}c^{5/2}(n+1)}\right)}{\sqrt{c}\sqrt{a+bx^{n+1}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x)^(5/2)*(a/x + b*x^n)^(3/2)),x]
```

```
[Out] (Sqrt[c*x]*Sqrt[a/x + b*x^n]*(2/(a*c^(5/2)*(1 + n)*Sqrt[a + b*x^(1 + n)]) -
 (2*ArcTanh[Sqrt[a + b*x^(1 + n)]/Sqrt[a]])/(a^(3/2)*c^(5/2)*(1 + n))))/(Sq
rt[c]*Sqrt[a + b*x^(1 + n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^n + \frac{a}{x}\right)^{\frac{3}{2}} (cx)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a/x)^(3/2)*(c*x)^(5/2)), x)
```

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{5}{2}} \left(b\, x^n + \frac{a}{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(b*x^n+a/x)^(3/2),x)
[Out] int(1/(c*x)^(5/2)/(b*x^n+a/x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^n + \frac{a}{x}\right)^{\frac{3}{2}} (cx)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^n + a/x)^(3/2)*(c*x)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\, x)^{5/2} \left(b\, x^n + \frac{a}{x}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(b*x^n + a/x)^(3/2)),x)
[Out] int(1/((c*x)^(5/2)*(b*x^n + a/x)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(a/x+b*x**n)**(3/2),x)
[Out] Timed out
```

3.307 $\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2}{ac^4(n+2)x\sqrt{\frac{a}{x^2}+bx^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{a^{3/2}c^4(n+2)}$$

Rubi [A] time = 0.16, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 2030, 2029, 206}

$$\frac{2}{ac^4(n+2)x\sqrt{\frac{a}{x^2}+bx^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{a^{3/2}c^4(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(c^4*x^4*(a/x^2 + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*c^4*(2 + n)*x*Sqrt[a/x^2 + b*x^n]) - (2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2
+ b*x^n])])/(a^(3/2)*c^4*(2 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int
egerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx = \frac{\int \frac{1}{x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx}{c^4}$$

$$= \frac{2}{ac^4(2+n)x\sqrt{\frac{a}{x^2}+bx^n}} + \frac{\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx}{ac^4}$$

$$= \frac{2}{ac^4(2+n)x\sqrt{\frac{a}{x^2}+bx^n}} - \frac{2\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{ac^4(2+n)}$$

$$= \frac{2}{ac^4(2+n)x\sqrt{\frac{a}{x^2}+bx^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{a^{3/2}c^4(2+n)}$$

Mathematica [C] time = 0.04, size = 51, normalized size = 0.71

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^{n+2}}{a}+1\right)}{ac^4(n+2)x\sqrt{\frac{a}{x^2}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(c^4*x^4*(a/x^2 + b*x^n)^(3/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^(2 + n))/a])/(a*c^4*(2 + n)*x*S
qrt[a/x^2 + b*x^n])
```

IntegrateAlgebraic [A] time = 0.13, size = 92, normalized size = 1.28

$$\frac{x\sqrt{\frac{a}{x^2}+bx^n}\left(\frac{2}{ac^4(n+2)\sqrt{a+bx^{n+2}}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+2}}}{\sqrt{a}}\right)}{a^{3/2}c^4(n+2)}\right)}{\sqrt{a+bx^{n+2}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(c^4*x^4*(a/x^2 + b*x^n)^(3/2)),x]
```

```
[Out] (x*Sqrt[a/x^2 + b*x^n]*(2/(a*c^4*(2 + n)*Sqrt[a + b*x^(2 + n)]) - (2*ArcTan
h[Sqrt[a + b*x^(2 + n)]/Sqrt[a]])/(a^(3/2)*c^4*(2 + n))))/Sqrt[a + b*x^(2 +
 n)]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^n + \frac{a}{x^2}\right)^{\frac{3}{2}} c^4 x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a/x^2)^(3/2)*c^4*x^4), x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^n + \frac{a}{x^2}\right)^{\frac{3}{2}} c^4 x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/c^4/x^4/(b*x^n+a/x^2)^(3/2),x)
[Out] int(1/c^4/x^4/(b*x^n+a/x^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\left(bx^n+\frac{a}{x^2}\right)^{\frac{3}{2}} x^4}\, dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^n + a/x^2)^(3/2)*x^4), x)/c^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{c^4\, x^4 \left(b\,x^n + \frac{a}{x^2}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^4*x^4*(b*x^n + a/x^2)^(3/2)),x)
[Out] int(1/(c^4*x^4*(b*x^n + a/x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{ax^2\sqrt{\frac{a}{x^2}+bx^n}+bx^4x^n\sqrt{\frac{a}{x^2}+bx^n}}\, dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c**4/x**4/(a/x**2+b*x**n)**(3/2),x)
[Out] Integral(1/(a*x**2*sqrt(a/x**2 + b*x**n) + b*x**4*x**n*sqrt(a/x**2 + b*x**n
)), x)/c**4
```

3.308 $\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{2}{ac^4(n+3)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{a^{3/2}c^5(n+3)\sqrt{cx}}$$

Rubi [A] time = 0.22, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2030, 2031, 2029, 206}

$$\frac{2}{ac^4(n+3)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{a^{3/2}c^5(n+3)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(11/2)*(a/x^3 + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*c^4*(3 + n)*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n]) - (2*Sqrt[x]*ArcTanh[Sqrt
[a]/(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/(a^(3/2)*c^5*(3 + n)*Sqrt[c*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int
egerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx = \frac{2}{ac^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}+\frac{\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx}{ac^3}$$

$$= \frac{2}{ac^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}+\frac{\sqrt{x}\int \frac{1}{x^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx}{ac^5\sqrt{cx}}$$

$$= \frac{2}{ac^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}-\frac{\left(2\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{ac^5(3+n)\sqrt{cx}}$$

$$= \frac{2}{ac^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{a^{3/2}c^5(3+n)\sqrt{cx}}$$

Mathematica [C] time = 0.05, size = 55, normalized size = 0.61

$$\frac{2\,_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^{n+3}}{a}+1\right)}{ac^4(n+3)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(11/2)*(a/x^3 + b*x^n)^(3/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^(3 + n))/a])/(a*c^4*(3 + n)*(c*
x)^(3/2)*Sqrt[a/x^3 + b*x^n])
```

IntegrateAlgebraic [A] time = 1.64, size = 107, normalized size = 1.19

$$\frac{(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}\left(\frac{2}{ac^{11/2}(n+3)\sqrt{a+bx^{n+3}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+3}}}{\sqrt{a}}\right)}{a^{3/2}c^{11/2}(n+3)}\right)}{c^{3/2}\sqrt{a+bx^{n+3}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x)^(11/2)*(a/x^3 + b*x^n)^(3/2)),x]
```

```
[Out] ((c*x)^(3/2)*Sqrt[a/x^3 + b*x^n]*(2/(a*c^(11/2)*(3 + n)*Sqrt[a + b*x^(3 + n
)]) - (2*ArcTanh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]])/(a^(3/2)*c^(11/2)*(3 + n))
))/(c^(3/2)*Sqrt[a + b*x^(3 + n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^n + \frac{a}{x^3}\right)^{\frac{3}{2}} (cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a/x^3)^(3/2)*(c*x)^(11/2)), x)
```

maple [F] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{11}{2}} \left(b\,x^n + \frac{a}{x^3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(11/2)/(b*x^n+a/x^3)^(3/2),x)
[Out] int(1/(c*x)^(11/2)/(b*x^n+a/x^3)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^n + \frac{a}{x^3}\right)^{\frac{3}{2}} (cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^n + a/x^3)^(3/2)*(c*x)^(11/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{11/2}\,\left(b\,x^n + \frac{a}{x^3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(11/2)*(b*x^n + a/x^3)^(3/2)),x)
[Out] int(1/((c*x)^(11/2)*(b*x^n + a/x^3)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(11/2)/(a/x**3+b*x**n)**(3/2),x)
[Out] Timed out
```

3.309 $\int \frac{1}{c^7 x^7 \left(\frac{a}{x^4}+b x^n\right)^{3/2}} \, dx$

Optimal. Leaf size=72

$$\frac{2}{a c^7 (n+4) x^2 \sqrt{\frac{a}{x^4}+b x^n}}-\frac{2 \tanh^{-1}\left(\frac{\sqrt{a}}{x^2 \sqrt{\frac{a}{x^4}+b x^n}}\right)}{a^{3/2} c^7 (n+4)}$$

Rubi [A] time = 0.15, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 2030, 2029, 206}

$$\frac{2}{a c^7 (n+4) x^2 \sqrt{\frac{a}{x^4}+b x^n}}-\frac{2 \tanh^{-1}\left(\frac{\sqrt{a}}{x^2 \sqrt{\frac{a}{x^4}+b x^n}}\right)}{a^{3/2} c^7 (n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(c^7*x^7*(a/x^4 + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*c^7*(4 + n)*x^2*Sqrt[a/x^4 + b*x^n]) - (2*ArcTanh[Sqrt[a]/(x^2*Sqrt[a/
x^4 + b*x^n])])/(a^(3/2)*c^7*(4 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int
egerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx = \frac{\int \frac{1}{x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx}{c^7}$$
$$= \frac{2}{ac^7(4+n)x^2\sqrt{\frac{a}{x^4}+bx^n}} + \frac{\int \frac{1}{x^3\sqrt{\frac{a}{x^4}+bx^n}}\,dx}{ac^7}$$
$$= \frac{2}{ac^7(4+n)x^2\sqrt{\frac{a}{x^4}+bx^n}} - \frac{2\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{ac^7(4+n)}$$
$$= \frac{2}{ac^7(4+n)x^2\sqrt{\frac{a}{x^4}+bx^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{a^{3/2}c^7(4+n)}$$

Mathematica [C] time = 0.07, size = 51, normalized size = 0.71

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^{n+4}}{a}+1\right)}{ac^7(n+4)x^2\sqrt{\frac{a}{x^4}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(c^7*x^7*(a/x^4 + b*x^n)^(3/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^(4 + n))/a])/(a*c^7*(4 + n)*x^2
*Sqrt[a/x^4 + b*x^n])
```

IntegrateAlgebraic [A] time = 0.14, size = 94, normalized size = 1.31

$$\frac{x^2\sqrt{\frac{a}{x^4}+bx^n}\left(\frac{2}{ac^7(n+4)\sqrt{a+bx^{n+4}}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+4}}}{\sqrt{a}}\right)}{a^{3/2}c^7(n+4)}\right)}{\sqrt{a+bx^{n+4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(c^7*x^7*(a/x^4 + b*x^n)^(3/2)),x]
```

```
[Out] (x^2*Sqrt[a/x^4 + b*x^n]*(2/(a*c^7*(4 + n)*Sqrt[a + b*x^(4 + n)]) - (2*ArcT
anh[Sqrt[a + b*x^(4 + n)]/Sqrt[a]])/(a^(3/2)*c^7*(4 + n))))/Sqrt[a + b*x^(4
 + n)]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^n + \frac{a}{x^4}\right)^{\frac{3}{2}} c^7 x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a/x^4)^(3/2)*c^7*x^7), x)
```

maple [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\, x^n + \frac{a}{x^4}\right)^{\frac{3}{2}} c^7 x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/c^7/x^7/(b*x^n+a/x^4)^(3/2),x)
[Out] int(1/c^7/x^7/(b*x^n+a/x^4)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\left(bx^n + \frac{a}{x^4}\right)^{\frac{3}{2}} x^7}\, dx}{c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^n + a/x^4)^(3/2)*x^7), x)/c^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{c^7\, x^7\, \left(b\, x^n + \frac{a}{x^4}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^7*x^7*(b*x^n + a/x^4)^(3/2)),x)
[Out] int(1/(c^7*x^7*(b*x^n + a/x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{ax^3 \sqrt{\frac{a}{x^4} + bx^n} + bx^7 x^n \sqrt{\frac{a}{x^4} + bx^n}}\, dx}{c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c**7/x**7/(a/x**4+b*x**n)**(3/2),x)
[Out] Integral(1/(a*x**3*sqrt(a/x**4 + b*x**n) + b*x**7*x**n*sqrt(a/x**4 + b*x**n
)), x)/c**7
```

3.310 $\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx$

Optimal. Leaf size=32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x}+bx^2}}\right)}{3\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1979, 2008, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x}+bx^2}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a + b*x^3)/x],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a/x + b*x^2]])/(3*Sqrt[b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx &= \int \frac{1}{\sqrt{\frac{a}{x}+bx^2}}\,dx \\
&= \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x}+bx^2}}\right) \\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x}+bx^2}}\right)}{3\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 63, normalized size = 1.97

$$\frac{2\sqrt{a+bx^3}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a+bx^3}}\right)}{3\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{a+bx^3}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a + b*x^3)/x],x]
```

```
[Out] (2*Sqrt[a + b*x^3]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]])/(3*Sqrt[b]*S
qrt[x]*Sqrt[(a + b*x^3)/x])
```

IntegrateAlgebraic [A] time = 0.36, size = 34, normalized size = 1.06

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{\frac{a+bx^3}{x}}}{\sqrt{b}\,x}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[(a + b*x^3)/x],x]
```

```
[Out] (2*ArcTanh[Sqrt[(a + b*x^3)/x]/(Sqrt[b]*x)])/(3*Sqrt[b])
```

fricas [A] time = 0.57, size = 102, normalized size = 3.19

$$\left[\frac{\log\left(-8\,b^2x^6-8\,abx^3-a^2-4\left(2\,bx^5+ax^2\right)\sqrt{b}\,\sqrt{\frac{bx^3+a}{x}}\right)}{6\,\sqrt{b}},\,-\frac{\sqrt{-b}\,\arctan\left(\frac{2\,\sqrt{-b}\,x^2\sqrt{\frac{bx^3+a}{x}}}{2\,bx^3+a}\right)}{3\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log(-8*b^2*x^6 - 8*a*b*x^3 - a^2 - 4*(2*b*x^5 + a*x^2)*sqrt(b)*sqrt((b
*x^3 + a)/x))/sqrt(b), -1/3*sqrt(-b)*arctan(2*sqrt(-b)*x^2*sqrt((b*x^3 + a)
/x)/(2*b*x^3 + a))/b]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

maple [C] time = 0.57, size = 477, normalized size = 14.91

$$-\frac{4\left(b\,x^3+a\right)\left(i\sqrt{3}-1\right)\sqrt{-\frac{\left(i\sqrt{3}-3\right)bx}{\left(i\sqrt{3}-1\right)\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)}}\,\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)^2\sqrt{\frac{2bx+i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}+\left(-a\,b^2\right)^{\frac{1}{3}}}{\left(1+i\sqrt{3}\right)\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)}}\,\sqrt{\frac{-2bx+i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}-\left(-a\,b^2\right)^{\frac{1}{3}}}{\left(i\sqrt{3}-1\right)\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)}}\,\left(\mathrm{EllipticF}\left(\sqrt{-\frac{\left(i\sqrt{3}-3\right)bx}{\left(i\sqrt{3}-1\right)\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)}},\sqrt{\frac{\left(i\sqrt{3}+3\right)\left(i\sqrt{3}-1\right)}{\left(1+i\sqrt{3}\right)\left(i\sqrt{3}-3\right)}}\right)-\mathrm{EllipticPi}\left(\sqrt{-\frac{\left(i\sqrt{3}-3\right)bx}{\left(i\sqrt{3}-1\right)\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)}},\frac{i\sqrt{3}-1}{i\sqrt{3}-3},\sqrt{\frac{\left(i\sqrt{3}+3\right)\left(i\sqrt{3}-1\right)}{\left(1+i\sqrt{3}\right)\left(i\sqrt{3}-3\right)}}\right)\right)}{\sqrt{\frac{b\,x^3+a}{x}}\,\sqrt{\left(b\,x^3+a\right)x}\,\left(i\sqrt{3}-3\right)\sqrt{\frac{\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)\left(2bx+i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}+\left(-a\,b^2\right)^{\frac{1}{3}}\right)\left(-2bx+i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}-\left(-a\,b^2\right)^{\frac{1}{3}}\right)x}{b^2}}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^3+a)/x)^(1/2),x)
```

```
[Out] -4*(b*x^3+a)*(I*3^(1/2)-1)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/
3))*b*x)^(1/2)*(-b*x+(-a*b^2)^(1/3))^2*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a
*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*
(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)/b
^2*(EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2
),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))-Elliptic
Pi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),(I*3^(1/2
)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2)))/((b*x^3+a)/x)^(1/2)/((b*x^3+a)*x)^(1/2)/(I*3^(1/2)-3)/((-b*x+(-a*
b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/
2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/b^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{bx^3+a}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((b*x^3 + a)/x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{b\,x^3+a}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)/x)^(1/2),x)
```

```
[Out] int(1/((a + b*x^3)/x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**3+a)/x)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a + b*x**3)/x), x)
```

3.311 $\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx$

Optimal. Leaf size=32

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^2}+bx^2}}\right)}{2\sqrt{b}}$$

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1979, 2008, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^2}+bx^2}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a + b*x^4)/x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a/x^2 + b*x^2]]/(2*Sqrt[b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx &= \int \frac{1}{\sqrt{\frac{a}{x^2}+bx^2}}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x^2}+bx^2}}\right)\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^2}+bx^2}}\right)}{2\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 1.84

$$\frac{\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}x\sqrt{\frac{a+bx^4}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a + b*x^4)/x^2],x]
```

```
[Out] (Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]*x*Sqrt[
(a + b*x^4)/x^2])
```

IntegrateAlgebraic [A] time = 3.54, size = 58, normalized size = 1.81

$$\frac{x\sqrt{\frac{a+bx^4}{x^2}}\log\left(\sqrt{a+bx^4}+\sqrt{b}x^2\right)}{2\sqrt{b}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[(a + b*x^4)/x^2],x]
```

```
[Out] (x*Sqrt[(a + b*x^4)/x^2]*Log[Sqrt[b]*x^2 + Sqrt[a + b*x^4]])/(2*Sqrt[b]*Sqr
t[a + b*x^4])
```

fricas [A] time = 0.42, size = 80, normalized size = 2.50

$$\left[\frac{\log\left(-2\,bx^4-2\,\sqrt{b}\,x^3\sqrt{\frac{bx^4+a}{x^2}}-a\right)}{4\,\sqrt{b}},-\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\,x^3\sqrt{\frac{bx^4+a}{x^2}}}{bx^4+a}\right)}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4+a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-2*b*x^4 - 2*sqrt(b)*x^3*sqrt((b*x^4 + a)/x^2) - a)/sqrt(b), -1/2*
sqrt(-b)*arctan(sqrt(-b)*x^3*sqrt((b*x^4 + a)/x^2)/(b*x^4 + a))/b]
```

giac [A] time = 0.21, size = 40, normalized size = 1.25

$$\frac{\log(|a|)\,\mathrm{sgn}(x)}{4\,\sqrt{b}}-\frac{\log\left(\left|-\sqrt{b}\,x^2+\sqrt{bx^4+a}\right|\right)}{2\,\sqrt{b}\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(a))*sgn(x)/sqrt(b) - 1/2*log(abs(-sqrt(b)*x^2 + sqrt(b*x^4 + a)
))/(sqrt(b)*sgn(x))
```

maple [A] time = 0.19, size = 49, normalized size = 1.53

$$\frac{\sqrt{b\,x^4+a}\,\ln\left(\sqrt{b}\,x^2+\sqrt{b\,x^4+a}\right)}{2\sqrt{\frac{b\,x^4+a}{x^2}}\,\sqrt{b}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^4+a)/x^2)^(1/2),x)
```

```
[Out] 1/2/((b*x^4+a)/x^2)^(1/2)/x*(b*x^4+a)^(1/2)*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))
/b^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b \int \frac{x^5}{\left(bx^4 + a\right)^{\frac{3}{2}}}\, dx + \frac{x^2}{2 \sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4+a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b*integrate(x^5/(b*x^4 + a)^(3/2), x) + 1/2*x^2/sqrt(b*x^4 + a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{b\,x^4+a}{x^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)/x^2)^(1/2),x)
```

```
[Out] int(1/((a + b*x^4)/x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**4+a)/x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a + b*x**4)/x**2), x)
```

3.312 $\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}} \, dx$

Optimal. Leaf size=32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^3}+bx^2}}\right)}{5\sqrt{b}}$$

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1979, 2008, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^3}+bx^2}}\right)}{5\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a + b*x^5)/x^3],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a/x^3 + b*x^2]])/(5*Sqrt[b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}} \, dx &= \int \frac{1}{\sqrt{\frac{a}{x^3}+bx^2}} \, dx \\
&= \frac{2}{5}\text{Subst}\left(\int \frac{1}{1-bx^2} \, dx, x, \frac{x}{\sqrt{\frac{a}{x^3}+bx^2}}\right) \\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^3}+bx^2}}\right)}{5\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 63, normalized size = 1.97

$$\frac{2\sqrt{a+bx^5}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{a+bx^5}}\right)}{5\sqrt{b}\,x^{3/2}\sqrt{\frac{a+bx^5}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a + b*x^5)/x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^5]*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a + b*x^5]])/(5*Sqrt[b]*x
^(3/2)*Sqrt[(a + b*x^5)/x^3])
```

IntegrateAlgebraic [A] time = 75.15, size = 64, normalized size = 2.00

$$\frac{2x^{3/2}\sqrt{\frac{a+bx^5}{x^3}}\,\log\left(\sqrt{a+bx^5}+\sqrt{b}\,x^{5/2}\right)}{5\sqrt{b}\,\sqrt{a+bx^5}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[(a + b*x^5)/x^3],x]
```

```
[Out] (2*x^(3/2)*Sqrt[(a + b*x^5)/x^3]*Log[Sqrt[b]*x^(5/2) + Sqrt[a + b*x^5]])/(5
*Sqrt[b]*Sqrt[a + b*x^5])
```

fricas [A] time = 0.93, size = 102, normalized size = 3.19

$$\left[\frac{\log\left(-8\,b^2x^{10}-8\,abx^5-a^2-4\left(2\,bx^9+ax^4\right)\sqrt{b}\,\sqrt{\frac{bx^5+a}{x^3}}\right)}{10\,\sqrt{b}},-\frac{\sqrt{-b}\,\arctan\left(\frac{2\,\sqrt{-b}\,x^4\sqrt{\frac{bx^5+a}{x^3}}}{2\,bx^5+a}\right)}{5\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^5+a)/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/10*log(-8*b^2*x^10 - 8*a*b*x^5 - a^2 - 4*(2*b*x^9 + a*x^4)*sqrt(b)*sqrt(
(b*x^5 + a)/x^3))/sqrt(b), -1/5*sqrt(-b)*arctan(2*sqrt(-b)*x^4*sqrt((b*x^5
+ a)/x^3)/(2*b*x^5 + a))/b]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^5+a)/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(t_nostep)]Warning, integration of
abs or sign assumes constant sign by intervals (correct if the argument is
real):Check [abs(x)]Limit: Max order reached or unable to make series expan
sion Error: Bad Argument Value
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{b\,x^{5}+a}{x^{3}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^5+a)/x^3)^(1/2),x)
[Out] int(1/((b*x^5+a)/x^3)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{bx^{5}+a}{x^{3}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^5+a)/x^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt((b*x^5 + a)/x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{b\,x^{5}+a}{x^{3}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^5)/x^3)^(1/2),x)
[Out] int(1/((a + b*x^5)/x^3)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**5+a)/x**3)**(1/2),x)
[Out] Timed out
```

3.313 $\int \frac{1}{\sqrt{x^{2-n}(a+bx^n)}} \, dx$

Optimal. Leaf size=37

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^{2-n}+bx^2}}\right)}{\sqrt{b}\,n}$$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1979, 2008, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^{2-n}+bx^2}}\right)}{\sqrt{b}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^(2 - n)*(a + b*x^n)],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^(2 - n)]])/(Sqrt[b]*n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x^{2-n}\,(a+bx^n)}} \, dx &= \int \frac{1}{\sqrt{bx^2+ax^{2-n}}} \, dx \\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{1-bx^2} \, dx, x, \frac{x}{\sqrt{bx^2+ax^{2-n}}}\right)}{n} \\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+ax^{2-n}}}\right)}{\sqrt{b}\,n}
\end{aligned}$$

Mathematica [B] time = 0.05, size = 76, normalized size = 2.05

$$\frac{2\sqrt{a}\,x^{1-\frac{n}{2}}\sqrt{\frac{bx^n}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a}}\right)}{\sqrt{b}\,n\sqrt{x^{2-n}\,(a+bx^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^(2 - n)*(a + b*x^n)],x]
```

```
[Out] (2*Sqrt[a]*x^(1 - n/2)*Sqrt[1 + (b*x^n)/a]*ArcSinh[(Sqrt[b]*x^(n/2))/Sqrt[a
]])/(Sqrt[b]*n*Sqrt[x^(2 - n)*(a + b*x^n)])
```

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^{2-n}\left(a+bx^{n}\right)}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/Sqrt[x^(2 - n)*(a + b*x^n)],x]
```

```
[Out] Defer[IntegrateAlgebraic][1/Sqrt[x^(2 - n)*(a + b*x^n)], x]
```

fricas [A] time = 0.43, size = 102, normalized size = 2.76

$$\left[\frac{\log\left(\frac{2\,bxx^{n}+ax+2\,\sqrt{b}\,x^{n}\sqrt{\frac{bx^{2}x^{n}+ax^{2}}{x^{n}}}}{x}\right)}{\sqrt{b}\,n}, -\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,\sqrt{\frac{bx^{2}x^{n}+ax^{2}}{x^{n}}}}{bx}\right)}{bn}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a+b*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] [log((2*b*x*x^n + a*x + 2*sqrt(b)*x^n*sqrt((b*x^2*x^n + a*x^2)/x^n))/x)/(sq
rt(b)*n), -2*sqrt(-b)*arctan(sqrt(-b)*sqrt((b*x^2*x^n + a*x^2)/x^n)/(b*x))/
(b*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(bx^{n}+a\right)x^{-n+2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a+b*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((b*x^n + a)*x^(-n + 2)), x)
```

maple [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(b\,x^{n}+a\right)x^{-n+2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2-n)*(b*x^n+a))^(1/2),x)
```

```
[Out] int(1/(x^(2-n)*(b*x^n+a))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(bx^{n}+a\right)x^{-n+2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a+b*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((b*x^n + a)*x^(-n + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{x^{2-n}\,\left(a+b\,x^n\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2 - n)*(a + b*x^n))^(1/2),x)
[Out] int(1/(x^(2 - n)*(a + b*x^n))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^{2-n}\left(a+bx^n\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(2-n)*(a+b*x**n))**(1/2),x)
[Out] Integral(1/sqrt(x**(2 - n)*(a + b*x**n)), x)
```

3.314 $\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx$

Optimal. Leaf size=33

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x}-bx^2}}\right)}{3\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1979, 2008, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x}-bx^2}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a - b*x^3)/x],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[a/x - b*x^2]])/(3*Sqrt[b])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx &= \int \frac{1}{\sqrt{\frac{a}{x}-bx^2}}\,dx\\ &= \frac{2}{3}\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x}-bx^2}}\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x}-bx^2}}\right)}{3\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 2.00

$$\frac{2\sqrt{a-bx^3}\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a-bx^3}}\right)}{3\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{a-bx^3}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a - b*x^3)/x],x]
```

```
[Out] (2*Sqrt[a - b*x^3]*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a - b*x^3]])/(3*Sqrt[b]*Sq
rt[x]*Sqrt[(a - b*x^3)/x])
```

IntegrateAlgebraic [A] time = 0.38, size = 35, normalized size = 1.06

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{\frac{a-bx^3}{x}}}{\sqrt{b}\,x}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[(a - b*x^3)/x],x]
```

```
[Out] (-2*ArcTan[Sqrt[(a - b*x^3)/x]/(Sqrt[b]*x)])/(3*Sqrt[b])
```

fricas [A] time = 0.54, size = 111, normalized size = 3.36

$$\left[-\frac{\sqrt{-b}\,\log\left(-8\,b^2x^6+8\,abx^3-a^2+4\left(2\,bx^5-ax^2\right)\sqrt{-b}\,\sqrt{-\frac{bx^3-a}{x}}\right)}{6\,b}, -\frac{\arctan\left(\frac{2\,\sqrt{b}\,x^2\sqrt{-\frac{bx^3-a}{x}}}{2\,bx^3-a}\right)}{3\,\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^3+a)/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*sqrt(-b)*log(-8*b^2*x^6 + 8*a*b*x^3 - a^2 + 4*(2*b*x^5 - a*x^2)*sqrt(
-b)*sqrt(-(b*x^3 - a)/x))/b, -1/3*arctan(2*sqrt(b)*x^2*sqrt(-(b*x^3 - a)/x)
/(2*b*x^3 - a))/sqrt(b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^3+a)/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Undef/Unsigned Inf encountered in limitLimit: Max order reached or unable t
o make series expansion Error: Bad Argument Value
```

maple [C] time = 6.22, size = 471, normalized size = 14.27

$$\frac{4\left(bx^3-a\right)\left(1+i\sqrt{3}\right)\sqrt{-\frac{\left(i\sqrt{3}+3\right)bx}{\left(1+i\sqrt{3}\right)\left(-bx+\left(ab^2\right)^{\frac{1}{3}}\right)}}\left(-bx+\left(ab^2\right)^{\frac{1}{3}}\right)^2\sqrt{\frac{-2bx+i\sqrt{3}\,\left(ab^2\right)^{\frac{1}{3}}-\left(ab^2\right)^{\frac{1}{3}}}{\left(i\sqrt{3}-1\right)\left(-bx+\left(ab^2\right)^{\frac{1}{3}}\right)}}\sqrt{\frac{2bx+i\sqrt{3}\,\left(ab^2\right)^{\frac{1}{3}}+\left(ab^2\right)^{\frac{1}{3}}}{\left(1+i\sqrt{3}\right)\left(-bx+\left(ab^2\right)^{\frac{1}{3}}\right)}}\left(\mathrm{EllipticF}\left(\sqrt{-\frac{\left(i\sqrt{3}+3\right)bx}{\left(1+i\sqrt{3}\right)\left(-bx+\left(ab^2\right)^{\frac{1}{3}}\right)}},\sqrt{\frac{\left(i\sqrt{3}-3\right)\left(1+i\sqrt{3}\right)}{\left(i\sqrt{3}-1\right)\left(i\sqrt{3}+3\right)}}\right)-\mathrm{EllipticPi}\left(\sqrt{-\frac{\left(i\sqrt{3}+3\right)bx}{\left(1+i\sqrt{3}\right)\left(-bx+\left(ab^2\right)^{\frac{1}{3}}\right)}},\frac{1+i\sqrt{3}}{i\sqrt{3}+3},\sqrt{\frac{\left(i\sqrt{3}-3\right)\left(1+i\sqrt{3}\right)}{\left(i\sqrt{3}-1\right)\left(i\sqrt{3}+3\right)}}\right)\right)}{\sqrt{-\frac{bx^3-a}{x}}\,\sqrt{-\left(bx^3-a\right)x}\,\left(i\sqrt{3}+3\right)\sqrt{-\frac{\left(-bx+\left(ab^2\right)^{\frac{1}{3}}\right)\left(-2bx+i\sqrt{3}\,\left(ab^2\right)^{\frac{1}{3}}-\left(ab^2\right)^{\frac{1}{3}}\right)\left(2bx+i\sqrt{3}\,\left(ab^2\right)^{\frac{1}{3}}+\left(ab^2\right)^{\frac{1}{3}}\right)x}{b^2}}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-b*x^3+a)/x)^(1/2),x)
```

```
[Out] 4*(b*x^3-a)*(1+I*3^(1/2))*(-(I*3^(1/2)+3)*x*b/(1+I*3^(1/2))/(-b*x+(a*b^2)^(
1/3)))^(1/2)*(-b*x+(a*b^2)^(1/3))^2*((I*3^(1/2)*(a*b^2)^(1/3)-2*b*x-(a*b^2)
^(1/3))/(I*3^(1/2)-1)/(-b*x+(a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(a*b^2)^(1/3)
+2*b*x+(a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(a*b^2)^(1/3)))^(1/2)/b^2*(Ellipt
icF((-(I*3^(1/2)+3)*x*b/(1+I*3^(1/2))/(-b*x+(a*b^2)^(1/3)))^(1/2),((I*3^(1/
2)-3)*(1+I*3^(1/2))/(I*3^(1/2)-1)/(I*3^(1/2)+3))^(1/2))-EllipticPi((-(I*3^(
1/2)+3)*x*b/(1+I*3^(1/2))/(-b*x+(a*b^2)^(1/3)))^(1/2),(1+I*3^(1/2))/(I*3^(1
/2)+3),((I*3^(1/2)-3)*(1+I*3^(1/2))/(I*3^(1/2)-1)/(I*3^(1/2)+3))^(1/2)))/(-
(b*x^3-a)/x)^(1/2)/(-(b*x^3-a)*x)^(1/2)/(I*3^(1/2)+3)/(-1/b^2*x*(-b*x+(a*b^
2)^(1/3))*(I*3^(1/2)*(a*b^2)^(1/3)-2*b*x-(a*b^2)^(1/3))*(I*3^(1/2)*(a*b^2)^
(1/3)+2*b*x+(a*b^2)^(1/3)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\frac{bx^3-a}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^3+a)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-(b*x^3 - a)/x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{a-b\,x^3}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^3)/x)^(1/2),x)
```

```
[Out] int(1/((a - b*x^3)/x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x**3+a)/x)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a - b*x**3)/x), x)
```

3.315 $\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx$

Optimal. Leaf size=33

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^2}-bx^2}}\right)}{2\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1979, 2008, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^2}-bx^2}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a - b*x^4)/x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a/x^2 - b*x^2]]/(2*Sqrt[b])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx &= \int \frac{1}{\sqrt{\frac{a}{x^2}-bx^2}}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x^2}-bx^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^2}-bx^2}}\right)}{2\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 1.88

$$\frac{\sqrt{a-bx^4}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}x\sqrt{\frac{a-bx^4}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a - b*x^4)/x^2],x]
```

```
[Out] (Sqrt[a - b*x^4]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/(2*Sqrt[b]*x*Sqrt[(
a - b*x^4)/x^2])
```

IntegrateAlgebraic [C] time = 3.66, size = 66, normalized size = 2.00

$$-\frac{ix\sqrt{\frac{a-bx^4}{x^2}}\log\left(\sqrt{a-bx^4}+i\sqrt{b}x^2\right)}{2\sqrt{b}\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[(a - b*x^4)/x^2],x]
```

```
[Out] ((-1/2*I)*x*Sqrt[(a - b*x^4)/x^2]*Log[I*Sqrt[b]*x^2 + Sqrt[a - b*x^4]])/(Sq
rt[b]*Sqrt[a - b*x^4])
```

fricas [A] time = 0.42, size = 88, normalized size = 2.67

$$\left[-\frac{\sqrt{-b}\log\left(2bx^4-2\sqrt{-b}x^3\sqrt{-\frac{bx^4-a}{x^2}}-a\right)}{4b}, -\frac{\arctan\left(\frac{\sqrt{b}x^3\sqrt{-\frac{bx^4-a}{x^2}}}{bx^4-a}\right)}{2\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^4+a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-b)*log(2*b*x^4 - 2*sqrt(-b)*x^3*sqrt(-(b*x^4 - a)/x^2) - a)/b,
-1/2*arctan(sqrt(b)*x^3*sqrt(-(b*x^4 - a)/x^2)/(b*x^4 - a))/sqrt(b)]
```

giac [A] time = 0.19, size = 47, normalized size = 1.42

$$\frac{\log(|a|)\operatorname{sgn}(x)}{4\sqrt{-b}}-\frac{\log\left(\left|-\sqrt{-b}x^2+\sqrt{-bx^4+a}\right|\right)}{2\sqrt{-b}\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^4+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(a))*sgn(x)/sqrt(-b) - 1/2*log(abs(-sqrt(-b)*x^2 + sqrt(-b*x^4 +
 a)))/(sqrt(-b)*sgn(x))
```

maple [B] time = 0.19, size = 53, normalized size = 1.61

$$\frac{\sqrt{-bx^4+a}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{-bx^4+a}}\right)}{2\sqrt{-\frac{bx^4-a}{x^2}}\sqrt{b}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-b*x^4+a)/x^2)^(1/2),x)
```

```
[Out] 1/2/(-(b*x^4-a)/x^2)^(1/2)/x*(-b*x^4+a)^(1/2)/b^(1/2)*arctan(1/(-b*x^4+a)^(
1/2)*b^(1/2)*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{x^5}{\left(bx^4-a\right)\sqrt{-bx^4+a}}\,dx + \frac{x^2}{2\sqrt{-bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^4+a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b*integrate(x^5/((b*x^4 - a)*sqrt(-b*x^4 + a)), x) + 1/2*x^2/sqrt(-b*x^4 +
a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{a-b\,x^4}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^4)/x^2)^(1/2),x)
```

```
[Out] int(1/((a - b*x^4)/x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x**4+a)/x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a - b*x**4)/x**2), x)
```

3.316 $\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx$

Optimal. Leaf size=33

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^3}-bx^2}}\right)}{5\sqrt{b}}$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1979, 2008, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^3}-bx^2}}\right)}{5\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a - b*x^5)/x^3],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[a/x^3 - b*x^2]])/(5*Sqrt[b])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx &= \int \frac{1}{\sqrt{\frac{a}{x^3}-bx^2}}\,dx\\ &= \frac{2}{5}\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x^3}-bx^2}}\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^3}-bx^2}}\right)}{5\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 2.00

$$\frac{2\sqrt{a-bx^5}\tan^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{a-bx^5}}\right)}{5\sqrt{b}\,x^{3/2}\sqrt{\frac{a-bx^5}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a - b*x^5)/x^3],x]
```

```
[Out] (2*Sqrt[a - b*x^5]*ArcTan[(Sqrt[b]*x^(5/2))/Sqrt[a - b*x^5]])/(5*Sqrt[b]*x^
(3/2)*Sqrt[(a - b*x^5)/x^3])
```

IntegrateAlgebraic [C] time = 74.85, size = 72, normalized size = 2.18

$$-\frac{2ix^{3/2}\sqrt{\frac{a-bx^5}{x^3}}\log\left(\sqrt{a-bx^5}+i\sqrt{b}\,x^{5/2}\right)}{5\sqrt{b}\sqrt{a-bx^5}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[(a - b*x^5)/x^3],x]
```

```
[Out] (((-2*I)/5)*x^(3/2)*Sqrt[(a - b*x^5)/x^3]*Log[I*Sqrt[b]*x^(5/2) + Sqrt[a -
b*x^5]])/(Sqrt[b]*Sqrt[a - b*x^5])
```

fricas [A] time = 0.93, size = 111, normalized size = 3.36

$$\left[-\frac{\sqrt{-b}\log\left(-8\,b^2x^{10}+8\,abx^5-a^2+4\left(2\,bx^9-ax^4\right)\sqrt{-b}\sqrt{-\frac{bx^5-a}{x^3}}\right)}{10\,b}, -\frac{\arctan\left(\frac{2\,\sqrt{b}\,x^4\sqrt{-\frac{bx^5-a}{x^3}}}{2\,bx^5-a}\right)}{5\,\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^5+a)/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/10*sqrt(-b)*log(-8*b^2*x^10 + 8*a*b*x^5 - a^2 + 4*(2*b*x^9 - a*x^4)*sqr
t(-b)*sqrt(-(b*x^5 - a)/x^3))/b, -1/5*arctan(2*sqrt(b)*x^4*sqrt(-(b*x^5 - a
)/x^3)/(2*b*x^5 - a))/sqrt(b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^5+a)/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(t_nostep)]Warning, integration of
abs or sign assumes constant sign by intervals (correct if the argument is
real):Check [abs(x)]Undef/Unsigned Inf encountered in limitLimit: Max order
 reached or unable to make series expansion Error: Bad Argument Value
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{-b\,x^5+a}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-b*x^5+a)/x^3)^(1/2),x)
[Out] int(1/((-b*x^5+a)/x^3)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\frac{bx^5-a}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^5+a)/x^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-(b*x^5 - a)/x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{a-b\,x^5}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^5)/x^3)^(1/2),x)
[Out] int(1/((a - b*x^5)/x^3)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x**5+a)/x**3)**(1/2),x)
[Out] Timed out
```

3.317 $\int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}}\,dx$

Optimal. Leaf size=38

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^{2-n}-bx^2}}\right)}{\sqrt{b}\,n}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1979, 2008, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^{2-n}-bx^2}}\right)}{\sqrt{b}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^(2 - n)*(a - b*x^n)],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^(2 - n)]])/(Sqrt[b]*n)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x^{2-n}\,(a-bx^n)}}\,dx &= \int \frac{1}{\sqrt{-bx^2+ax^{2-n}}}\,dx\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-bx^2+ax^{2-n}}}\right)}{n}\\
&= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2+ax^{2-n}}}\right)}{\sqrt{b}\,n}
\end{aligned}$$

Mathematica [B] time = 0.06, size = 78, normalized size = 2.05

$$\frac{2\sqrt{a}\,x^{1-\frac{n}{2}}\sqrt{1-\frac{bx^n}{a}}\,\sin^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a}}\right)}{\sqrt{b}\,n\sqrt{x^{2-n}\,(a-bx^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^(2 - n)*(a - b*x^n)],x]
```

```
[Out] (2*Sqrt[a]*x^(1 - n/2)*Sqrt[1 - (b*x^n)/a]*ArcSin[(Sqrt[b]*x^(n/2))/Sqrt[a]
])/(Sqrt[b]*n*Sqrt[x^(2 - n)*(a - b*x^n)])
```

IntegrateAlgebraic [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^{2-n}\left(a-bx^{n}\right)}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/Sqrt[x^(2 - n)*(a - b*x^n)],x]
```

```
[Out] Defer[IntegrateAlgebraic][1/Sqrt[x^(2 - n)*(a - b*x^n)], x]
```

fricas [A] time = 0.43, size = 106, normalized size = 2.79

$$\left[-\frac{\sqrt{-b}\log\left(-\frac{2\,bxx^{n}-ax-2\,\sqrt{-b}\,x^{n}\sqrt{-\frac{bx^{2}x^{n}-ax^{2}}{x^{n}}}}{x}\right)}{bn},\ -\frac{2\,\arctan\left(\frac{\sqrt{-\frac{bx^{2}x^{n}-ax^{2}}{x^{n}}}}{\sqrt{b}\,x}\right)}{\sqrt{b}\,n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a-b*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-b)*log(-(2*b*x*x^n - a*x - 2*sqrt(-b)*x^n*sqrt(-(b*x^2*x^n - a*x^2)
/x^n))/x)/(b*n), -2*arctan(sqrt(-(b*x^2*x^n - a*x^2)/x^n)/(sqrt(b)*x))/(sqr
t(b)*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(bx^{n}-a)x^{-n+2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a-b*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-(b*x^n - a)*x^(-n + 2)), x)
```

maple [F] time = 1.38, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(-b\,x^{n}+a\right)x^{-n+2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(-n+2)*(-b*x^n+a))^(1/2),x)
```

```
[Out] int(1/(x^(-n+2)*(-b*x^n+a))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(bx^{n}-a)x^{-n+2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a-b*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-(b*x^n - a)*x^(-n + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{x^{2-n}\,\left(a-b\,x^n\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2 - n)*(a - b*x^n))^(1/2),x)
[Out] int(1/(x^(2 - n)*(a - b*x^n))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^{2-n}\left(a-bx^{n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(2-n)*(a-b*x**n))**(1/2),x)
[Out] Integral(1/sqrt(x**(2 - n)*(a - b*x**n)), x)
```

3.318 $\int \frac{1}{\sqrt{x^n\left(a+bx^{2-n}\right)}}\,dx$

Optimal. Leaf size=37

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1979, 2008, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^n*(a + b*x^(2 - n))],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x^n\left(a+bx^{2-n}\right)}}\,dx&=\int \frac{1}{\sqrt{bx^2+ax^n}}\,dx\\&=\frac{2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+ax^n}}\right)}{2-n}\\&=\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}\,(2-n)}\end{aligned}$$

Mathematica [B] time = 0.10, size = 78, normalized size = 2.11

$$-\frac{2\sqrt{a}\,x^{n/2}\sqrt{\frac{bx^{2-n}}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}\,(n-2)\sqrt{ax^n+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^n*(a + b*x^(2 - n))],x]
```

```
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 + (b*x^(2 - n))/a]*ArcSinh[(Sqrt[b]*x^(1 - n/2))
/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[b*x^2 + a*x^n])
```

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^n \left(a + bx^{2-n}\right)}}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/Sqrt[x^n*(a + b*x^(2 - n))],x]
```

```
[Out] Defer[IntegrateAlgebraic][1/Sqrt[x^n*(a + b*x^(2 - n))], x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a+b*x^(2-n)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(bx^{-n+2} + a\right)x^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a+b*x^(2-n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((b*x^(-n + 2) + a)*x^n), x)
```

maple [F] time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(b\, x^{-n+2} + a\right) x^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^n*(a+b*x^(-n+2)))^(1/2),x)
```

```
[Out] int(1/(x^n*(a+b*x^(-n+2)))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(bx^{-n+2} + a\right)x^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a+b*x^(2-n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((b*x^(-n + 2) + a)*x^n), x)
```

mupad [B] time = 5.21, size = 67, normalized size = 1.81

$$\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{\frac{b\,x^{2-n}}{a}+1}\,1\mathrm{i}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n+b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^n*(a + b*x^(2 - n)))^(1/2),x)
```

```
[Out] (a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2)*1i)/a^(1/2))*((b*x^(2 - n))/a +
1)^(1/2)*1i)/(b^(1/2)*(n/2 - 1)*(a*x^n + b*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x^n\left(a+bx^{2-n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**n*(a+b*x**(2-n)))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**n*(a + b*x**(2 - n))), x)
```

3.319 $\int \frac{1}{\sqrt{x^2\left(b+ax^{-2+n}\right)}}\,dx$

Optimal. Leaf size=37

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1979, 2008, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^2*(b + a*x^(-2 + n))],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x^2\left(b+ax^{-2+n}\right)}}\,dx &= \int \frac{1}{\sqrt{bx^2+ax^n}}\,dx\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+ax^n}}\right)}{2-n}\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}\,(2-n)}
\end{aligned}$$

Mathematica [B] time = 0.04, size = 78, normalized size = 2.11

$$-\frac{2\sqrt{a}\,x^{n/2}\sqrt{\frac{bx^{2-n}}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}\,(n-2)\sqrt{ax^n+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^2*(b + a*x^(-2 + n))],x]
```

```
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 + (b*x^(2 - n))/a]*ArcSinh[(Sqrt[b]*x^(1 - n/2))
/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[b*x^2 + a*x^n])
```

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2 \left(b + a x^{-2+n}\right)}} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/Sqrt[x^2*(b + a*x^(-2 + n))],x]
```

```
[Out] Defer[IntegrateAlgebraic][1/Sqrt[x^2*(b + a*x^(-2 + n))], x]
```

fricas [A] time = 0.43, size = 109, normalized size = 2.95

$$\left[\frac{\sqrt{b} \log \left(\frac{a x x^{n-2}+2 b x-2 \sqrt{a x^2 x^{n-2}+b x^2} \sqrt{b}}{x x^{n-2}}\right)}{b n - 2 b}, \frac{2 \sqrt{-b} \arctan \left(\frac{\sqrt{a x^2 x^{n-2}+b x^2} \sqrt{-b}}{b x}\right)}{b n - 2 b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(b+a*x^(-2+n)))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(b)*log((a*x*x^(n - 2) + 2*b*x - 2*sqrt(a*x^2*x^(n - 2) + b*x^2)*sqrt(
b))/(x*x^(n - 2)))/(b*n - 2*b), 2*sqrt(-b)*arctan(sqrt(a*x^2*x^(n - 2) + b*
x^2)*sqrt(-b)/(b*x))/(b*n - 2*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(a x^{n-2} + b\right) x^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(b+a*x^(-2+n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((a*x^(n - 2) + b)*x^2), x)
```

maple [F] time = 2.10, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(a x^{n-2} + b\right) x^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b+a*x^(n-2)))^(1/2),x)
```

```
[Out] int(1/(x^2*(b+a*x^(n-2)))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(a x^{n-2} + b\right) x^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(b+a*x^(-2+n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((a*x^(n - 2) + b)*x^2), x)
```

mupad [B] time = 5.32, size = 67, normalized size = 1.81

$$\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}\,1\mathrm{i}}{\sqrt{a}}\right)\sqrt{\frac{b\,x^{2-n}}{a}+1}\,1\mathrm{i}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\sqrt{a\,x^n+b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b + a*x^(n - 2)))^(1/2),x)
```

```
[Out] (a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2)*1i)/a^(1/2))*((b*x^(2 - n))/a +
1)^(1/2)*1i)/(b^(1/2)*(n/2 - 1)*(a*x^n + b*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2\left(ax^{n-2}+b\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2*(b+a*x**(-2+n)))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2*(a*x**(n - 2) + b)), x)
```

3.320 $\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx$

Optimal. Leaf size=37

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1979, 2008, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x*(b*x + a*x^(-1 + n))],x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx &= \int \frac{1}{\sqrt{bx^2+ax^n}}\,dx\\ &= \frac{2\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+ax^n}}\right)}{2-n}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}\,(2-n)}\end{aligned}$$

Mathematica [B] time = 0.03, size = 78, normalized size = 2.11

$$-\frac{2\sqrt{a}\,x^{n/2}\sqrt{\frac{bx^{2-n}}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}\,(n-2)\sqrt{ax^n+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x*(b*x + a*x^(-1 + n))],x]
```

```
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 + (b*x^(2 - n))/a]*ArcSinh[(Sqrt[b]*x^(1 - n/2))
/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[b*x^2 + a*x^n])
```

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x\left(bx + ax^{-1+n}\right)}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/Sqrt[x*(b*x + a*x^(-1 + n))],x]
```

```
[Out] Defer[IntegrateAlgebraic][1/Sqrt[x*(b*x + a*x^(-1 + n))], x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a*x^(-1+n)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(ax^{n-1} + bx\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a*x^(-1+n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((a*x^(n - 1) + b*x)*x), x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(a\,x^{n-1} + bx\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x+a*x^(n-1)))^(1/2),x)
```

```
[Out] int(1/(x*(b*x+a*x^(n-1)))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(ax^{n-1} + bx\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a*x^(-1+n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((a*x^(n - 1) + b*x)*x), x)
```

mupad [B] time = 5.13, size = 67, normalized size = 1.81

$$\frac{\sqrt{a}\, x^{n/2}\, \mathrm{asin}\left(\frac{\sqrt{b}\, x^{1-\frac{n}{2}}\, 1\mathrm{i}}{\sqrt{a}}\right) \sqrt{\frac{b\, x^{2-n}}{a}+1}\, 1\mathrm{i}}{\sqrt{b}\, \left(\frac{n}{2}-1\right) \sqrt{a\, x^n + b\, x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x + a*x^(n - 1))))^(1/2),x)
```

```
[Out] (a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2)*1i)/a^(1/2))*((b*x^(2 - n))/a +
1)^(1/2)*1i)/(b^(1/2)*(n/2 - 1)*(a*x^n + b*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x\left(ax^{n-1}+bx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a*x**(-1+n)))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x*(a*x**(n - 1) + b*x)), x)
```

3.321 $\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx$

Optimal. Leaf size=38

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1979, 2008, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^n*(a - b*x^(2 - n))],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx &= \int \frac{1}{\sqrt{-bx^2+ax^n}}\,dx\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-bx^2+ax^n}}\right)}{2-n}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}\,(2-n)}\end{aligned}$$

Mathematica [B] time = 0.11, size = 80, normalized size = 2.11

$$-\frac{2\sqrt{a}\,x^{n/2}\sqrt{1-\frac{bx^{2-n}}{a}}\,\sin^{-1}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}\,(n-2)\sqrt{ax^n-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^n*(a - b*x^(2 - n))],x]
```

```
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x^(2 - n))/a]*ArcSin[(Sqrt[b]*x^(1 - n/2))/
Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[-(b*x^2) + a*x^n])
```

IntegrateAlgebraic [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^n \left(a - bx^{2-n}\right)}}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/Sqrt[x^n*(a - b*x^(2 - n))],x]
```

```
[Out] Defer[IntegrateAlgebraic][1/Sqrt[x^n*(a - b*x^(2 - n))], x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a-b*x^(2-n)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(bx^{-n+2} - a\right)x^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a-b*x^(2-n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-(b*x^(-n + 2) - a)*x^n), x)
```

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(-b\,x^{-n+2} + a\right)x^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^n*(a-b*x^(-n+2)))^(1/2),x)
```

```
[Out] int(1/(x^n*(a-b*x^(-n+2)))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(bx^{-n+2} - a\right)x^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a-b*x^(2-n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-(b*x^(-n + 2) - a)*x^n), x)
```

mupad [B] time = 5.17, size = 66, normalized size = 1.74

$$-\frac{\sqrt{a}\;x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\;x^{1-\frac{n}{2}}}{\sqrt{a}}\right)\,\sqrt{1-\frac{b\,x^{2-n}}{a}}}{\sqrt{b}\;\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n-b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^n*(a - b*x^(2 - n)))^(1/2),x)
```

```
[Out] -(a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2))/a^(1/2))*(1 - (b*x^(2 - n))/a)
^(1/2))/(b^(1/2)*(n/2 - 1)*(a*x^n - b*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**n*(a-b*x**(2-n)))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**n*(a - b*x**(2 - n))), x)
```

3.322 $\int \frac{1}{\sqrt{x^2(-b+ax^{-2+n})}}\,dx$

Optimal. Leaf size=38

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1979, 2008, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^2*(-b + a*x^(-2 + n))],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx &= \int \frac{1}{\sqrt{-bx^2+ax^n}}\,dx\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-bx^2+ax^n}}\right)}{2-n}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}\,(2-n)}\end{aligned}$$

Mathematica [B] time = 0.02, size = 80, normalized size = 2.11

$$-\frac{2\sqrt{a}\,x^{n/2}\sqrt{1-\frac{bx^{2-n}}{a}}\,\sin^{-1}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}\,(n-2)\sqrt{ax^n-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^2*(-b + a*x^(-2 + n))],x]
```

```
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x^(2 - n))/a]*ArcSin[(Sqrt[b]*x^(1 - n/2))/
Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[-(b*x^2) + a*x^n])
```

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/Sqrt[x^2*(-b + a*x^(-2 + n))],x]
```

```
[Out] Defer[IntegrateAlgebraic][1/Sqrt[x^2*(-b + a*x^(-2 + n))], x]
```

fricas [A] time = 0.44, size = 109, normalized size = 2.87

$$\left[-\frac{\sqrt{-b}\log\left(\frac{axx^{n-2}-2\,bx-2\,\sqrt{ax^2x^{n-2}-bx^2}\,\sqrt{-b}}{xx^{n-2}}\right)}{bn-2\,b},\ \frac{2\,\sqrt{b}\arctan\left(\frac{\sqrt{ax^2x^{n-2}-bx^2}}{\sqrt{b}\,x}\right)}{bn-2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-b)*log((a*x*x^(n - 2) - 2*b*x - 2*sqrt(a*x^2*x^(n - 2) - b*x^2)*sqr
t(-b))/(x*x^(n - 2)))/(b*n - 2*b), 2*sqrt(b)*arctan(sqrt(a*x^2*x^(n - 2) -
b*x^2)/(sqrt(b)*x))/(b*n - 2*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(ax^{n-2}-b\right)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((a*x^(n - 2) - b)*x^2), x)
```

maple [F] time = 2.15, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(a\,x^{n-2}-b\right)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(-b+a*x^(n-2)))^(1/2),x)
```

```
[Out] int(1/(x^2*(-b+a*x^(n-2)))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(ax^{n-2}-b\right)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((a*x^(n - 2) - b)*x^2), x)
```

mupad [B] time = 5.10, size = 66, normalized size = 1.74

$$-\frac{\sqrt{a}\ x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\ x^{1-\frac{n}{2}}}{\sqrt{a}}\right)\sqrt{1-\frac{b\,x^{2-n}}{a}}}{\sqrt{b}\ \left(\frac{n}{2}-1\right)\sqrt{a\,x^n-b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2*(b - a*x^(n - 2)))^(1/2),x)
```

```
[Out] -(a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2))/a^(1/2))*(1 - (b*x^(2 - n))/a)
^(1/2))/(b^(1/2)*(n/2 - 1)*(a*x^n - b*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x^2\left(ax^{n-2}-b\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2*(-b+a*x**(-2+n)))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2*(a*x**(n - 2) - b)), x)
```

3.323 $\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx$

Optimal. Leaf size=38

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1979, 2008, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x*(-(b*x) + a*x^(-1 + n))],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx &= \int \frac{1}{\sqrt{-bx^2+ax^n}}\,dx\\ &= \frac{2\,\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-bx^2+ax^n}}\right)}{2-n}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}\,(2-n)}\end{aligned}$$

Mathematica [B] time = 0.03, size = 80, normalized size = 2.11

$$-\frac{2\sqrt{a}\,x^{n/2}\sqrt{1-\frac{bx^{2-n}}{a}}\,\sin^{-1}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}\,(n-2)\sqrt{ax^n-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x*(-(b*x) + a*x^(-1 + n))],x]
```

```
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x^(2 - n))/a]*ArcSin[(Sqrt[b]*x^(1 - n/2))/
Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[-(b*x^2) + a*x^n])
```

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/Sqrt[x*(-(b*x) + a*x^(-1 + n))],x]
```

```
[Out] Defer[IntegrateAlgebraic][1/Sqrt[x*(-(b*x) + a*x^(-1 + n))], x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(ax^{n-1}-bx\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((a*x^(n - 1) - b*x)*x), x)
```

maple [F] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(a\,x^{n-1}-bx\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(-b*x+a*x^(n-1)))^(1/2),x)
```

```
[Out] int(1/(x*(-b*x+a*x^(n-1)))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(ax^{n-1}-bx\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((a*x^(n - 1) - b*x)*x), x)
```

mupad [B] time = 5.12, size = 66, normalized size = 1.74

$$-\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)\sqrt{1-\frac{b\,x^{2-n}}{a}}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\sqrt{a\,x^n-b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x*(b*x - a*x^(n - 1)))^(1/2),x)
```

```
[Out] -(a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2))/a^(1/2))*(1 - (b*x^(2 - n))/a)
^(1/2))/(b^(1/2)*(n/2 - 1)*(a*x^n - b*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x\left(ax^{n-1}-bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(-b*x+a*x**(-1+n)))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x*(a*x**(n - 1) - b*x)), x)
```

3.324 $\int \sqrt{\frac{1+x}{x^5}}\,dx$

Optimal. Leaf size=18

$$-\frac{2}{3}\left(\frac{1}{x^5}+\frac{1}{x^4}\right)^{3/2}x^6$$

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1979, 2000}

$$-\frac{2}{3}\left(\frac{1}{x^4}+\frac{1}{x^5}\right)^{3/2}x^6$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x)/x^5],x]
[Out] (-2*(x^(-5) + x^(-4))^(3/2)*x^6)/3
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{1+x}{x^5}}\,dx &= \int \sqrt{\frac{1}{x^5}+\frac{1}{x^4}}\,dx\\ &= -\frac{2}{3}\left(\frac{1}{x^5}+\frac{1}{x^4}\right)^{3/2}x^6\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.06

$$-\frac{2}{3}x(x+1)\sqrt{\frac{x+1}{x^5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x)/x^5],x]
[Out] (-2*x*(1 + x)*Sqrt[(1 + x)/x^5])/3
```

IntegrateAlgebraic [A] time = 0.03, size = 18, normalized size = 1.00

$$-\frac{2}{3}x^6\left(\frac{x+1}{x^5}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(1 + x)/x^5],x]
```

```
[Out] (-2*x^6*((1 + x)/x^5)^(3/2))/3
```

fricas [A] time = 0.39, size = 16, normalized size = 0.89

$$-\frac{2}{3}\left(x^2+x\right)\sqrt{\frac{x+1}{x^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x^5)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(x^2 + x)*sqrt((x + 1)/x^5)
```

giac [B] time = 0.19, size = 50, normalized size = 2.78

$$\frac{2\left(3\left(x-\sqrt{x^2+x}\right)^2\operatorname{sgn}(x)+3\left(x-\sqrt{x^2+x}\right)\operatorname{sgn}(x)+\operatorname{sgn}(x)\right)}{3\left(x-\sqrt{x^2+x}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x^5)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*(x - sqrt(x^2 + x))^2*sgn(x) + 3*(x - sqrt(x^2 + x))*sgn(x) + sgn(x)
)/(x - sqrt(x^2 + x))^3
```

maple [A] time = 0.05, size = 16, normalized size = 0.89

$$-\frac{2\,(x+1)\,\sqrt{\frac{x+1}{x^5}}\;x}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x+1)/x^5)^(1/2),x)
```

```
[Out] -2/3*x*(x+1)*((x+1)/x^5)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{x+1}{x^5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x^5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((x + 1)/x^5), x)
```

mupad [B] time = 5.26, size = 15, normalized size = 0.83

$$-\frac{2\,x\,\sqrt{\frac{x+1}{x^5}}\;(x+1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)/x^5)^(1/2),x)
```

```
[Out] -(2*x*((x + 1)/x^5)^(1/2)*(x + 1))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{x+1}{x^5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x**5)**(1/2),x)
```

```
[Out] Integral(sqrt((x + 1)/x**5), x)
```

3.325 $\int \sqrt{x + x^{5/2}}\, dx$

Optimal. Leaf size=20

$$\frac{4\left(x^{5/2}+x\right)^{3/2}}{9x^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2000}

$$\frac{4\left(x^{5/2}+x\right)^{3/2}}{9x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + x^(5/2)],x]
```

```
[Out] (4*(x + x^(5/2))^(3/2))/(9*x^(3/2))
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \sqrt{x+x^{5/2}}\, dx = \frac{4\left(x+x^{5/2}\right)^{3/2}}{9x^{3/2}}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{4\left(x^{5/2}+x\right)^{3/2}}{9x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + x^(5/2)],x]
```

```
[Out] (4*(x + x^(5/2))^(3/2))/(9*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.04, size = 20, normalized size = 1.00

$$\frac{4\left(x^{5/2}+x\right)^{3/2}}{9x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x + x^(5/2)],x]
```

```
[Out] (4*(x + x^(5/2))^(3/2))/(9*x^(3/2))
```

fricas [A] time = 0.43, size = 19, normalized size = 0.95

$$\frac{4\sqrt{x^{\frac{5}{2}}+x}\left(x^2+\sqrt{x}\right)}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(5/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/9*sqrt(x^(5/2) + x)*(x^2 + sqrt(x))/x
```

giac [A] time = 0.17, size = 11, normalized size = 0.55

$$\frac{4}{9}\left(x^{\frac{3}{2}}+1\right)^{\frac{3}{2}}-\frac{4}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(5/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/9*(x^(3/2) + 1)^(3/2) - 4/9
```

maple [A] time = 0.05, size = 18, normalized size = 0.90

$$\frac{4\sqrt{x^{\frac{5}{2}}+x}\,\left(x^{\frac{3}{2}}+1\right)}{9\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+x^(5/2))^(1/2),x)
```

```
[Out] 4/9*(x+x^(5/2))^(1/2)/x^(1/2)*(x^(3/2)+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^{\frac{5}{2}}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(5/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^(5/2) + x), x)
```

mupad [B] time = 5.32, size = 27, normalized size = 1.35

$$\frac{2\,x\,\sqrt{x+x^{5/2}}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},1;\;2;\;-x^{3/2}\right)}{3\,\sqrt{x^{3/2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^(5/2))^(1/2),x)
```

```
[Out] (2*x*(x + x^(5/2))^(1/2)*hypergeom([-1/2, 1], 2, -x^(3/2)))/(3*(x^(3/2) + 1
)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^{\frac{5}{2}}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x**(5/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x**(5/2) + x), x)
```

3.326 $\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx$

Optimal. Leaf size=8

$$2\tan^{-1}\left(\sqrt{x}\right)$$

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 63, 203}

$$2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] + x^(3/2))^(-1),x]
```

```
[Out] 2*ArcTan[Sqrt[x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx &= \int \frac{1}{\sqrt{x}(1+x)}\,dx\\ &= 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)\\ &= 2\tan^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] + x^(3/2))^(-1),x]
```

```
[Out] 2*ArcTan[Sqrt[x]]
```

IntegrateAlgebraic [A] time = 0.01, size = 8, normalized size = 1.00

$$2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[x] + x^(3/2))^(-1),x]
[Out] 2*ArcTan[Sqrt[x]]
```

fricas [A] time = 0.39, size = 6, normalized size = 0.75

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(3/2)+x^(1/2)),x, algorithm="fricas")
[Out] 2*arctan(sqrt(x))
```

giac [A] time = 0.15, size = 6, normalized size = 0.75

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(3/2)+x^(1/2)),x, algorithm="giac")
[Out] 2*arctan(sqrt(x))
```

maple [A] time = 0.04, size = 7, normalized size = 0.88

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)+x^(1/2)),x)
[Out] 2*arctan(x^(1/2))
```

maxima [A] time = 3.14, size = 6, normalized size = 0.75

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(3/2)+x^(1/2)),x, algorithm="maxima")
[Out] 2*arctan(sqrt(x))
```

mupad [B] time = 5.24, size = 6, normalized size = 0.75

$$2\,\mathrm{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) + x^(3/2)),x)
[Out] 2*atan(x^(1/2))
```

sympy [A] time = 0.21, size = 7, normalized size = 0.88

$$2\,\mathrm{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(3/2)+x**(1/2)),x)
[Out] 2*atan(sqrt(x))
```

3.327 $\int x\sqrt{x^2\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=25

$$\frac{2\left(x^2\left(a+bx^3\right)\right)^{3/2}}{9bx^3}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1588}

$$\frac{2\left(x^2\left(a+bx^3\right)\right)^{3/2}}{9bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[x^2*(a + b*x^3)],x]
```

```
[Out] (2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\sqrt{x^2\left(a+bx^3\right)}\,dx=\frac{2\left(x^2\left(a+bx^3\right)\right)^{3/2}}{9bx^3}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{2\left(x^2\left(a+bx^3\right)\right)^{3/2}}{9bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[x^2*(a + b*x^3)],x]
```

```
[Out] (2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)
```

IntegrateAlgebraic [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{2\left(ax^2+bx^5\right)^{3/2}}{9bx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[x^2*(a + b*x^3)],x]
```

```
[Out] (2*(a*x^2 + b*x^5)^(3/2))/(9*b*x^3)
```

fricas [A] time = 0.38, size = 28, normalized size = 1.12

$$\frac{2\sqrt{bx^5+ax^2}\left(bx^3+a\right)}{9\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2*(b*x^3+a))^(1/2),x, algorithm="fricas")
[Out] 2/9*sqrt(b*x^5 + a*x^2)*(b*x^3 + a)/(b*x)
```

giac [A] time = 0.17, size = 27, normalized size = 1.08

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}\operatorname{sgn}(x)}{9\,b}-\frac{2\,a^{\frac{3}{2}}\operatorname{sgn}(x)}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2*(b*x^3+a))^(1/2),x, algorithm="giac")
[Out] 2/9*(b*x^3 + a)^(3/2)*sgn(x)/b - 2/9*a^(3/2)*sgn(x)/b
```

maple [A] time = 0.05, size = 29, normalized size = 1.16

$$\frac{2\left(b\,x^3+a\right)\sqrt{\left(b\,x^3+a\right)x^2}}{9bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2*(b*x^3+a))^(1/2),x)
[Out] 2/9*(b*x^3+a)*(x^2*(b*x^3+a))^(1/2)/b/x
```

maxima [A] time = 1.46, size = 14, normalized size = 0.56

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2*(b*x^3+a))^(1/2),x, algorithm="maxima")
[Out] 2/9*(b*x^3 + a)^(3/2)/b
```

mupad [B] time = 5.23, size = 22, normalized size = 0.88

$$\frac{2\left(b\,x^3+a\right)^{3/2}\,\sqrt{x^2}}{9\,b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2*(a + b*x^3))^(1/2),x)
[Out] (2*(a + b*x^3)^(3/2)*(x^2)^(1/2))/(9*b*x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2*(b*x**3+a))**(1/2),x)
[Out] Timed out
```

3.328 $\int x\sqrt{ax^2+bx^5}\,dx$

Optimal. Leaf size=25

$$\frac{2\left(ax^2+bx^5\right)^{3/2}}{9bx^3}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1588}

$$\frac{2\left(ax^2+bx^5\right)^{3/2}}{9bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*(a*x^2 + b*x^5)^(3/2))/(9*b*x^3)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\sqrt{ax^2+bx^5}\,dx=\frac{2\left(ax^2+bx^5\right)^{3/2}}{9bx^3}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{2\left(x^2\left(a+bx^3\right)\right)^{3/2}}{9bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)
```

IntegrateAlgebraic [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{2\left(ax^2+bx^5\right)^{3/2}}{9bx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*(a*x^2 + b*x^5)^(3/2))/(9*b*x^3)
```

fricas [A] time = 0.38, size = 28, normalized size = 1.12

$$\frac{2\sqrt{bx^5+ax^2}\left(bx^3+a\right)}{9\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/9*sqrt(b*x^5 + a*x^2)*(b*x^3 + a)/(b*x)
```

giac [A] time = 0.16, size = 27, normalized size = 1.08

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}\operatorname{sgn}(x)}{9\,b}-\frac{2\,a^{\frac{3}{2}}\operatorname{sgn}(x)}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] 2/9*(b*x^3 + a)^(3/2)*sgn(x)/b - 2/9*a^(3/2)*sgn(x)/b
```

maple [A] time = 0.04, size = 29, normalized size = 1.16

$$\frac{2\left(b\,x^3+a\right)\sqrt{b\,x^5+a\,x^2}}{9bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^5+a*x^2)^(1/2),x)
[Out] 2/9*(b*x^3+a)*(b*x^5+a*x^2)^(1/2)/b/x
```

maxima [A] time = 1.43, size = 14, normalized size = 0.56

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/9*(b*x^3 + a)^(3/2)/b
```

mupad [B] time = 5.26, size = 29, normalized size = 1.16

$$\frac{\left(\frac{2\,a}{9\,b}+\frac{2\,x^3}{9}\right)\sqrt{b\,x^5+a\,x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x^2 + b*x^5)^(1/2),x)
[Out] (((2*a)/(9*b) + (2*x^3)/9)*(a*x^2 + b*x^5)^(1/2))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x^2\left(a+bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(x*sqrt(x**2*(a + b*x**3)), x)
```

3.329 $\int \sqrt{x^4 (a + bx^3)} \, dx$

Optimal. Leaf size=25

$$\frac{2\left(ax^4 + bx^7\right)^{3/2}}{9bx^6}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1979, 2000}

$$\frac{2\left(ax^4 + bx^7\right)^{3/2}}{9bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^4*(a + b*x^3)],x]
[Out] (2*(a*x^4 + b*x^7)^(3/2))/(9*b*x^6)
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned} \int \sqrt{x^4\left(a+bx^3\right)}\, dx &= \int \sqrt{ax^4+bx^7}\, dx \\ &= \frac{2\left(ax^4+bx^7\right)^{3/2}}{9bx^6} \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{2\left(x^4\left(a + bx^3\right)\right)^{3/2}}{9bx^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^4*(a + b*x^3)],x]
[Out] (2*(x^4*(a + b*x^3))^(3/2))/(9*b*x^6)
```

IntegrateAlgebraic [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{2\left(ax^4 + bx^7\right)^{3/2}}{9bx^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x^4*(a + b*x^3)],x]
```

```
[Out] (2*(a*x^4 + b*x^7)^(3/2))/(9*b*x^6)
```

fricas [A] time = 0.39, size = 28, normalized size = 1.12

$$\frac{2\,\sqrt{bx^7+ax^4}\left(bx^3+a\right)}{9\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4*(b*x^3+a))^(1/2),x, algorithm="fricas")
[Out] 2/9*sqrt(b*x^7 + a*x^4)*(b*x^3 + a)/(b*x^2)
```

giac [A] time = 0.19, size = 14, normalized size = 0.56

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4*(b*x^3+a))^(1/2),x, algorithm="giac")
[Out] 2/9*(b*x^3 + a)^(3/2)/b
```

maple [A] time = 0.05, size = 29, normalized size = 1.16

$$\frac{2\left(b\,x^3+a\right)\sqrt{\left(b\,x^3+a\right)x^4}}{9b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(b*x^3+a))^(1/2),x)
[Out] 2/9*(b*x^3+a)*(x^4*(b*x^3+a))^(1/2)/b/x^2
```

maxima [A] time = 1.47, size = 14, normalized size = 0.56

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4*(b*x^3+a))^(1/2),x, algorithm="maxima")
[Out] 2/9*(b*x^3 + a)^(3/2)/b
```

mupad [B] time = 5.21, size = 22, normalized size = 0.88

$$\frac{2\left(b\,x^3+a\right)^{3/2}\,\sqrt{x^4}}{9\,b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^3))^(1/2),x)
[Out] (2*(a + b*x^3)^(3/2)*(x^4)^(1/2))/(9*b*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4\left(a+bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4*(b*x**3+a))**(1/2),x)
[Out] Integral(sqrt(x**4*(a + b*x**3)), x)
```

3.330 $\int \left(ax^m + bx^{1+m+mp}\right)^p \, dx$

Optimal. Leaf size=44

$$\frac{x^{-m(p+1)} \left(ax^m + bx^{mp+m+1}\right)^{p+1}}{b(p+1)(mp+1)}$$

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2000}

$$\frac{x^{-m(p+1)} \left(ax^m + bx^{mp+m+1}\right)^{p+1}}{b(p+1)(mp+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m + b*x^(1 + m + m*p))^p,x]
[Out] (a*x^m + b*x^(1 + m + m*p))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \left(ax^m + bx^{1+m+mp}\right)^p \, dx = \frac{x^{-m(1+p)} \left(ax^m + bx^{1+m+mp}\right)^{1+p}}{b(1+p)(1+mp)}$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.98

$$\frac{x^{-m(p+1)} \left(x^m \left(a + bx^{mp+1}\right)\right)^{p+1}}{b(p+1)(mp+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m + b*x^(1 + m + m*p))^p,x]
[Out] (x^m*(a + b*x^(1 + m*p)))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \left(ax^m + bx^{1+m+mp}\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x^m + b*x^(1 + m + m*p))^p,x]
[Out] Defer[IntegrateAlgebraic][(a*x^m + b*x^(1 + m + m*p))^p, x]
```

fricas [A] time = 0.65, size = 64, normalized size = 1.45

$$\frac{\left(bxx^{mp+m+1} + axx^m\right)\left(bx^{mp+m+1} + ax^m\right)^p}{\left(bmp^2 + (bm+b)p + b\right)x^{mp+m+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(m*p+m+1))^p,x, algorithm="fricas")
```

```
[Out] (b*x*x^(m*p + m + 1) + a*x*x^m)*(b*x^(m*p + m + 1) + a*x^m)^p/((b*m*p^2 + (
b*m + b)*p + b)*x^(m*p + m + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{mp+m+1} + ax^{m}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(m*p+m+1))^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^(m*p + m + 1) + a*x^m)^p, x)
```

maple [F] time = 0.86, size = 0, normalized size = 0.00

$$\int \left(a\,x^{m}+b\,x^{mp+m+1}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m+b*x^(m*p+m+1))^p,x)
```

```
[Out] int((a*x^m+b*x^(m*p+m+1))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{mp+m+1} + ax^{m}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(m*p+m+1))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^(m*p + m + 1) + a*x^m)^p, x)
```

mupad [B] time = 5.30, size = 76, normalized size = 1.73

$$\frac{a\,{\left(a\,x^m+b\,x^{m+m\,p+1}\right)}^p\,\left(\frac{b\,x^{m\,p+1}}{a}-\frac{1}{{\left(\frac{b\,x^{m\,p+1}}{a}+1\right)}^p}+1\right)}{b\,x^{m\,p}\,\left(m\,p+1\right)\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m + b*x^(m + m*p + 1))^p,x)
```

```
[Out] (a*(a*x^m + b*x^(m + m*p + 1))^p*((b*x^(m*p + 1))/a - 1/((b*x^(m*p + 1))/a
+ 1)^p + 1))/(b*x^(m*p)*(m*p + 1)*(p + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^{m} + bx^{mp+m+1}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m+b*x**(m*p+m+1))**p,x)
```

```
[Out] Integral((a*x**m + b*x**(m*p + m + 1))**p, x)
```

3.331 $\int \left(x^m \left(a+bx^{1+mp}\right)\right)^p \, dx$

Optimal. Leaf size=44

$$\frac{x^{-m(p+1)}\left(ax^m+bx^{mp+m+1}\right)^{p+1}}{b(p+1)(mp+1)}$$

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1979, 2000}

$$\frac{x^{-m(p+1)}\left(ax^m+bx^{mp+m+1}\right)^{p+1}}{b(p+1)(mp+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(a + b*x^(1 + m*p)))^p,x]
```

```
[Out] (a*x^m + b*x^(1 + m + m*p))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \left(x^m \left(a+bx^{1+mp}\right)\right)^p \, dx &= \int \left(ax^m+bx^{1+m+mp}\right)^p \, dx\\ &= \frac{x^{-m(1+p)}\left(ax^m+bx^{1+m+mp}\right)^{1+p}}{b(1+p)(1+mp)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 0.98

$$\frac{x^{-m(p+1)}\left(x^m\left(a+bx^{mp+1}\right)\right)^{p+1}}{b(p+1)(mp+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(a + b*x^(1 + m*p)))^p,x]
```

```
[Out] (x^m*(a + b*x^(1 + m*p)))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

IntegrateAlgebraic [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \left(x^m \left(a+bx^{1+mp}\right)\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^m*(a + b*x^(1 + m*p)))^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^m*(a + b*x^(1 + m*p)))^p, x]
```

fricas [A] time = 0.41, size = 61, normalized size = 1.39

$$\frac{\left(bxx^{mp+1}+ax\right)\left(bx^{mp+1}x^m+ax^m\right)^p}{\left(bmp^2+(bm+b)p+b\right)x^{mp+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^m*(a+b*x^(m*p+1)))^p,x, algorithm="fricas")
[Out] (b*x*x^(m*p + 1) + a*x)*(b*x^(m*p + 1)*x^m + a*x^m)^p/((b*m*p^2 + (b*m + b)
*p + b)*x^(m*p + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(bx^{mp+1}+a\right)x^m\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^m*(a+b*x^(m*p+1)))^p,x, algorithm="giac")
[Out] integrate(((b*x^(m*p + 1) + a)*x^m)^p, x)
```

maple [F] time = 0.85, size = 0, normalized size = 0.00

$$\int\left(\left(b\,x^{mp+1}+a\right)x^m\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a+b*x^(m*p+1)))^p,x)
[Out] int((x^m*(a+b*x^(m*p+1)))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(bx^{mp+1}+a\right)x^m\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^m*(a+b*x^(m*p+1)))^p,x, algorithm="maxima")
[Out] integrate(((b*x^(m*p + 1) + a)*x^m)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\left(x^m\,\left(a+b\,x^{m\,p+1}\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*x^(m*p + 1)))^p,x)
[Out] int((x^m*(a + b*x^(m*p + 1)))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^m\left(a+bx^{mp+1}\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**m*(a+b*x**(m*p+1)))**p,x)
[Out] Integral((x**m*(a + b*x**(m*p + 1)))**p, x)
```

3.332 $\int x^n \left(x^m \left(a+bx^{1+n+mp}\right)\right)^p \, dx$

Optimal. Leaf size=46

$$\frac{x^{-m(p+1)} \left(ax^m+bx^{mp+m+n+1}\right)^{p+1}}{b(p+1)(mp+n+1)}$$

Rubi [A] time = 0.08, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1980, 2014}

$$\frac{x^{-m(p+1)} \left(ax^m+bx^{mp+m+n+1}\right)^{p+1}}{b(p+1)(mp+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^n*(x^m*(a + b*x^(1 + n + m*p)))^p,x]
```

```
[Out] (a*x^m + b*x^(1 + m + n + m*p))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 +
p)))
```

Rule 1980

```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] &&  !Generali
zedBinomialMatchQ[u, x]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x^n \left(x^m \left(a+bx^{1+n+mp}\right)\right)^p \, dx &= \int x^n \left(ax^m+bx^{1+m+n+mp}\right)^p \, dx\\ &= \frac{x^{-m(1+p)} \left(ax^m+bx^{1+m+n+mp}\right)^{1+p}}{b(1+p)(1+n+mp)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 45, normalized size = 0.98

$$\frac{x^{-m(p+1)} \left(x^m \left(a+bx^{mp+n+1}\right)\right)^{p+1}}{b(p+1)(mp+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n*(x^m*(a + b*x^(1 + n + m*p)))^p,x]
```

```
[Out] (x^m*(a + b*x^(1 + n + m*p)))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 + p)
))
```

IntegrateAlgebraic [F] time = 0.11, size = 0, normalized size = 0.00

$$\int x^n \left(x^m \left(a+bx^{1+n+mp}\right)\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^n*(x^m*(a + b*x^(1 + n + m*p)))^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^n*(x^m*(a + b*x^(1 + n + m*p)))^p, x]
```

fricas [A] time = 0.43, size = 76, normalized size = 1.65

$$\frac{\left(bxx^{mp+n+1}x^n + axx^n\right)\left(bx^{mp+n+1}x^m + ax^m\right)^p}{\left(bmp^2 + bn + (bm + bn + b)p + b\right)x^{mp+n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x, algorithm="fricas")
```

```
[Out] (b*x*x^(m*p + n + 1)*x^n + a*x*x^n)*(b*x^(m*p + n + 1)*x^m + a*x^m)^p/((b*m
*p^2 + b*n + (b*m + b*n + b)*p + b)*x^(m*p + n + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(bx^{mp+n+1} + a\right)x^m\right)^p x^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x, algorithm="giac")
```

```
[Out] integrate(((b*x^(m*p + n + 1) + a)*x^m)^p*x^n, x)
```

maple [F] time = 0.91, size = 0, normalized size = 0.00

$$\int x^n \left(\left(b\,x^{mp+n+1} + a\right)x^m\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x)
```

```
[Out] int(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(bx^{mp+n+1} + a\right)x^m\right)^p x^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^(m*p + n + 1) + a)*x^m)^p*x^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^n \left(x^m\,\left(a + b\,x^{n+m\,p+1}\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(x^m*(a + b*x^(n + m*p + 1)))^p,x)
```

```
[Out] int(x^n*(x^m*(a + b*x^(n + m*p + 1)))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n*(x**m*(a+b*x**(m*p+n+1)))**p,x)
```

```
[Out] Timed out
```

3.333 $\int x^n \left(ax^m + bx^{1+m+n+mp}\right)^p \, dx$

Optimal. Leaf size=46

$$\frac{x^{-m(p+1)} \left(ax^m + bx^{mp+m+n+1}\right)^{p+1}}{b(p+1)(mp+n+1)}$$

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2014}

$$\frac{x^{-m(p+1)} \left(ax^m + bx^{mp+m+n+1}\right)^{p+1}}{b(p+1)(mp+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^n*(a*x^m + b*x^(1 + m + n + m*p))^p,x]
```

```
[Out] (a*x^m + b*x^(1 + m + n + m*p))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 +
p)))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^n \left(ax^m + bx^{1+m+n+mp}\right)^p \, dx = \frac{x^{-m(1+p)} \left(ax^m + bx^{1+m+n+mp}\right)^{1+p}}{b(1+p)(1+n+mp)}$$

Mathematica [A] time = 0.00, size = 45, normalized size = 0.98

$$\frac{x^{-m(p+1)} \left(x^m \left(a + bx^{mp+n+1}\right)\right)^{p+1}}{b(p+1)(mp+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n*(a*x^m + b*x^(1 + m + n + m*p))^p,x]
```

```
[Out] (x^m*(a + b*x^(1 + n + m*p)))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 + p)
))
```

IntegrateAlgebraic [F] time = 0.11, size = 0, normalized size = 0.00

$$\int x^n \left(ax^m + bx^{1+m+n+mp}\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^n*(a*x^m + b*x^(1 + m + n + m*p))^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^n*(a*x^m + b*x^(1 + m + n + m*p))^p, x]
```

fricas [**A**] time = 0.43, size = 79, normalized size = 1.72

$$\frac{\left(bxx^{mp+m+n+1}x^{n}+axx^{m}x^{n}\right)\left(bx^{mp+m+n+1}+ax^{m}\right)^{p}}{\left(bmp^{2}+bn+(bm+bn+b)p+b\right)x^{mp+m+n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x, algorithm="fricas")
```

```
[Out] (b*x*x^(m*p + m + n + 1)*x^n + a*x*x^m*x^n)*(b*x^(m*p + m + n + 1) + a*x^m)
^p/((b*m*p^2 + b*n + (b*m + b*n + b)*p + b)*x^(m*p + m + n + 1))
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{mp+m+n+1}+ax^{m}\right)^{p}x^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^(m*p + m + n + 1) + a*x^m)^p*x^n, x)
```

maple [**F**] time = 0.88, size = 0, normalized size = 0.00

$$\int x^{n}\left(a\,x^{m}+b\,x^{mp+m+n+1}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x)
```

```
[Out] int(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{mp+m+n+1}+ax^{m}\right)^{p}x^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^(m*p + m + n + 1) + a*x^m)^p*x^n, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{n}\left(a\,x^{m}+b\,x^{m+n+m\,p+1}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(a*x^m + b*x^(m + n + m*p + 1))^p,x)
```

```
[Out] int(x^n*(a*x^m + b*x^(m + n + m*p + 1))^p, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n*(a*x**m+b*x**(m*p+m+n+1))**p,x)
```

```
[Out] Timed out
```

3.334 $\int \sqrt{x^{2(-1+n)}(a+bx^n)}\,dx$

Optimal. Leaf size=44

$$\frac{2x^{3(1-n)}\left(ax^{-2(1-n)}+bx^{3n-2}\right)^{3/2}}{3bn}$$

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1979, 2000}

$$\frac{2x^{3(1-n)}\left(ax^{-2(1-n)}+bx^{3n-2}\right)^{3/2}}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^(2*(-1 + n))*(a + b*x^n)],x]
```

```
[Out] (2*x^(3*(1 - n))*(a/x^(2*(1 - n)) + b*x^(-2 + 3*n))^(3/2))/(3*b*n)
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x^{2(-1+n)}(a+bx^n)}\,dx &= \int \sqrt{ax^{2(-1+n)}+bx^{2(-1+n)+n}}\,dx\\ &= \frac{2x^{3(1-n)}\left(ax^{-2(1-n)}+bx^{-2+3n}\right)^{3/2}}{3bn}\end{aligned}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 0.82

$$\frac{2x^{3-3n}\left(x^{2n-2}(a+bx^n)\right)^{3/2}}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^(2*(-1 + n))*(a + b*x^n)],x]
```

```
[Out] (2*x^(3 - 3*n)*(x^(-2 + 2*n)*(a + b*x^n))^(3/2))/(3*b*n)
```

IntegrateAlgebraic [A] time = 0.07, size = 43, normalized size = 0.98

$$\frac{2x^{n-1}(a+bx^n)^2}{3bn\sqrt{x^{2n-2}(a+bx^n)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x^(2*(-1 + n))*(a + b*x^n)],x]
```

```
[Out] (2*x^(-1 + n)*(a + b*x^n)^2)/(3*b*n*Sqrt[x^(-2 + 2*n)*(a + b*x^n)])
```

fricas [A] time = 0.41, size = 44, normalized size = 1.00

$$\frac{2\left(bxx^{n}+ax\right)\sqrt{\frac{bx^{3\,n}+ax^{2\,n}}{x^{2}}}}{3\,bnx^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-2+2*n)*(a+b*x^n))^(1/2),x, algorithm="fricas")
[Out] 2/3*(b*x*x^n + a*x)*sqrt((b*x^(3*n) + a*x^(2*n))/x^2)/(b*n*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{(bx^{n}+a)x^{2\,n-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-2+2*n)*(a+b*x^n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt((b*x^n + a)*x^(2*n - 2)), x)
```

maple [A] time = 0.19, size = 40, normalized size = 0.91

$$\frac{2\sqrt{\frac{(b\,x^{n}+a)x^{2n}}{x^{2}}}\;(b\,x^{n}+a)\,x\,x^{-n}}{3bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2*n-2)*(b*x^n+a))^(1/2),x)
[Out] 2/3*(1/x^2*(x^n)^2*(b*x^n+a))^(1/2)*(b*x^n+a)/(x^n)*x/b/n
```

maxima [A] time = 1.55, size = 17, normalized size = 0.39

$$\frac{2\left(bx^{n}+a\right)^{\frac{3}{2}}}{3\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-2+2*n)*(a+b*x^n))^(1/2),x, algorithm="maxima")
[Out] 2/3*(b*x^n + a)^(3/2)/(b*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{x^{2\,n-2}\,\left(a+b\,x^{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2*n - 2)*(a + b*x^n))^(1/2),x)
[Out] int((x^(2*n - 2)*(a + b*x^n))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(-2+2*n)*(a+b*x**n))**(1/2),x)
[Out] Timed out
```

3.335 $\int \sqrt[3]{x^{3(-1+n)}(a+bx^n)}\,dx$

Optimal. Leaf size=44

$$\frac{3x^{4(1-n)}\left(ax^{-3(1-n)}+bx^{4n-3}\right)^{4/3}}{4bn}$$

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1979, 2000}

$$\frac{3x^{4(1-n)}\left(ax^{-3(1-n)}+bx^{4n-3}\right)^{4/3}}{4bn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3*(-1 + n))*(a + b*x^n))^(1/3),x]
```

```
[Out] (3*x^(4*(1 - n))*(a/x^(3*(1 - n)) + b*x^(-3 + 4*n))^(4/3))/(4*b*n)
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{x^{3(-1+n)}(a+bx^n)}\,dx &= \int \sqrt[3]{ax^{3(-1+n)}+bx^{3(-1+n)+n}}\,dx\\ &= \frac{3x^{4(1-n)}\left(ax^{-3(1-n)}+bx^{-3+4n}\right)^{4/3}}{4bn}\end{aligned}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 0.82

$$\frac{3x^{4-4n}\left(x^{3n-3}(a+bx^n)\right)^{4/3}}{4bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3*(-1 + n))*(a + b*x^n))^(1/3),x]
```

```
[Out] (3*x^(4 - 4*n)*(x^(-3 + 3*n)*(a + b*x^n))^(4/3))/(4*b*n)
```

IntegrateAlgebraic [A] time = 0.09, size = 43, normalized size = 0.98

$$\frac{3x^{1-n}(a+bx^n)\sqrt[3]{x^{3n-3}(a+bx^n)}}{4bn}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(3*(-1 + n))*(a + b*x^n))^(1/3),x]
```

[Out] (3*x^(1 - n)*(a + b*x^n)*(x^(-3 + 3*n)*(a + b*x^n))^(1/3))/(4*b*n)

fricas [A] time = 0.40, size = 44, normalized size = 1.00

$$\frac{3\left(bxx^{n}+ax\right)\left(\frac{bx^{4n}+ax^{3n}}{x^{3}}\right)^{\frac{1}{3}}}{4\,bnx^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(-3+3*n)*(a+b*x^n))^(1/3),x, algorithm="fricas")

[Out] 3/4*(b*x*x^n + a*x)*((b*x^(4*n) + a*x^(3*n))/x^3)^(1/3)/(b*n*x^n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left((bx^{n}+a)x^{3n-3}\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(-3+3*n)*(a+b*x^n))^(1/3),x, algorithm="giac")

[Out] integrate(((b*x^n + a)*x^(3*n - 3))^(1/3), x)

maple [A] time = 0.08, size = 40, normalized size = 0.91

$$\frac{3\left(\frac{(b\,x^{n}+a)x^{3n}}{x^{3}}\right)^{\frac{1}{3}}(b\,x^{n}+a)\,x\,x^{-n}}{4bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(3*n-3)*(b*x^n+a))^(1/3),x)

[Out] 3/4*(1/x^3*(x^n)^3*(b*x^n+a))^(1/3)*x/(x^n)*(b*x^n+a)/b/n

maxima [A] time = 1.49, size = 17, normalized size = 0.39

$$\frac{3\left(bx^{n}+a\right)^{\frac{4}{3}}}{4\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(-3+3*n)*(a+b*x^n))^(1/3),x, algorithm="maxima")

[Out] 3/4*(b*x^n + a)^(4/3)/(b*n)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\left(x^{3\,n-3}\,\left(a+b\,x^{n}\right)\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(3*n - 3)*(a + b*x^n))^(1/3),x)

[Out] int((x^(3*n - 3)*(a + b*x^n))^(1/3), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**(-3+3*n)*(a+b*x**n))**(1/3),x)

[Out] Timed out

3.336 $\int \sqrt[4]{x^{4(-1+n)}(a+bx^n)}\,dx$

Optimal. Leaf size=44

$$\frac{4x^{5(1-n)}\left(ax^{-4(1-n)}+bx^{5n-4}\right)^{5/4}}{5bn}$$

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1979, 2000}

$$\frac{4x^{5(1-n)}\left(ax^{-4(1-n)}+bx^{5n-4}\right)^{5/4}}{5bn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(4*(-1 + n))*(a + b*x^n))^(1/4),x]
```

```
[Out] (4*x^(5*(1 - n))*(a/x^(4*(1 - n)) + b*x^(-4 + 5*n))^(5/4))/(5*b*n)
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt[4]{x^{4(-1+n)}(a+bx^n)}\,dx &= \int \sqrt[4]{ax^{4(-1+n)}+bx^{4(-1+n)+n}}\,dx\\ &= \frac{4x^{5(1-n)}\left(ax^{-4(1-n)}+bx^{-4+5n}\right)^{5/4}}{5bn}\end{aligned}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 0.82

$$\frac{4x^{5-5n}\left(x^{4n-4}(a+bx^n)\right)^{5/4}}{5bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(4*(-1 + n))*(a + b*x^n))^(1/4),x]
```

```
[Out] (4*x^(5 - 5*n)*(x^(-4 + 4*n)*(a + b*x^n))^(5/4))/(5*b*n)
```

IntegrateAlgebraic [A] time = 0.08, size = 43, normalized size = 0.98

$$\frac{4x^{1-n}(a+bx^n)\sqrt[4]{x^{4n-4}(a+bx^n)}}{5bn}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(4*(-1 + n))*(a + b*x^n))^(1/4),x]
```

```
[Out] (4*x^(1 - n)*(a + b*x^n)*(x^(-4 + 4*n)*(a + b*x^n))^(1/4))/(5*b*n)
```

fricas [A] time = 0.41, size = 44, normalized size = 1.00

$$\frac{4\,(bxx^{n}+ax)\left(\frac{bx^{5\,n}+ax^{4\,n}}{x^{4}}\right)^{\frac{1}{4}}}{5\,bnx^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-4+4*n)*(a+b*x^n))^(1/4),x, algorithm="fricas")
[Out] 4/5*(b*x*x^n + a*x)*((b*x^(5*n) + a*x^(4*n))/x^4)^(1/4)/(b*n*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left((bx^{n}+a)x^{4\,n-4}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-4+4*n)*(a+b*x^n))^(1/4),x, algorithm="giac")
[Out] integrate(((b*x^n + a)*x^(4*n - 4))^(1/4), x)
```

maple [A] time = 0.07, size = 40, normalized size = 0.91

$$\frac{4\left(\frac{(b\,x^{n}+a)x^{4n}}{x^{4}}\right)^{\frac{1}{4}}(b\,x^{n}+a)\,x\,x^{-n}}{5bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(4*n-4)*(b*x^n+a))^(1/4),x)
[Out] 4/5*(1/x^4*(x^n)^4*(b*x^n+a))^(1/4)*x/(x^n)*(b*x^n+a)/b/n
```

maxima [A] time = 1.58, size = 17, normalized size = 0.39

$$\frac{4\,(bx^{n}+a)^{\frac{5}{4}}}{5\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-4+4*n)*(a+b*x^n))^(1/4),x, algorithm="maxima")
[Out] 4/5*(b*x^n + a)^(5/4)/(b*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\left(x^{4\,n-4}\,\left(a+b\,x^{n}\right)\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(4*n - 4)*(a + b*x^n))^(1/4),x)
[Out] int((x^(4*n - 4)*(a + b*x^n))^(1/4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(-4+4*n)*(a+b*x**n))**(1/4),x)
[Out] Timed out
```

3.337 $\int \left(x^{(-1+n)p}\left(a+bx^n\right)\right)^{\frac{1}{p}}\,dx$

Optimal. Leaf size=57

$$\frac{px^{(1-n)(p+1)}\left(ax^{-((1-n)p)}+bx^{n-(1-n)p}\right)^{\frac{1}{p}+1}}{bn(p+1)}$$

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1979, 2000}

$$\frac{px^{(1-n)(p+1)}\left(ax^{-(1-n)p}+bx^{n-(1-n)p}\right)^{\frac{1}{p}+1}}{bn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^((-1 + n)*p)*(a + b*x^n))^p^(-1),x]
```

```
[Out] (p*x^((1 - n)*(1 + p))*(a/x^((1 - n)*p) + b*x^(n - (1 - n)*p))^(1 + p^(-1))
)/(b*n*(1 + p))
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \left(x^{(-1+n)p}\left(a+bx^n\right)\right)^{\frac{1}{p}}\,dx &= \int \left(ax^{(-1+n)p}+bx^{n+(-1+n)p}\right)^{\frac{1}{p}}\,dx\\ &= \frac{px^{(1-n)(1+p)}\left(ax^{-((1-n)p)}+bx^{n-(1-n)p}\right)^{1+\frac{1}{p}}}{bn(1+p)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 0.82

$$\frac{x^{1-n}\left(a+bx^n\right)\left(x^{(n-1)p}\left(a+bx^n\right)\right)^{\frac{1}{p}}}{bn\left(\frac{1}{p}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^((-1 + n)*p)*(a + b*x^n))^p^(-1),x]
```

```
[Out] (x^(1 - n)*(a + b*x^n)*(x^((-1 + n)*p)*(a + b*x^n))^p^(-1))/(b*n*(1 + p^(-1
)))
```

IntegrateAlgebraic [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \left(x^{(-1+n)p}\left(a+bx^n\right)\right)^{\frac{1}{p}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^((-1 + n)*p)*(a + b*x^n))^p^(-1),x]
[Out] Defer[IntegrateAlgebraic][(x^((-1 + n)*p)*(a + b*x^n))^p^(-1), x]
```

fricas [A] time = 0.42, size = 47, normalized size = 0.82

$$\frac{\left(bpxx^{n} + apx\right)\left((bx^{n} + a)x^{(n-1)p}\right)^{\left(\frac{1}{p}\right)}}{\left(bnp + bn\right)x^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^((-1+n)*p)*(a+b*x^n))^(1/p),x, algorithm="fricas")
[Out] (b*p*x*x^n + a*p*x)*((b*x^n + a)*x^((n - 1)*p))^(1/p)/((b*n*p + b*n)*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((bx^{n} + a)x^{(n-1)p}\right)^{\left(\frac{1}{p}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^((-1+n)*p)*(a+b*x^n))^(1/p),x, algorithm="giac")
[Out] integrate(((b*x^n + a)*x^((n - 1)*p))^(1/p), x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \left((b\,x^{n} + a)\,x^{(n-1)p}\right)^{\frac{1}{p}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^((n-1)*p)*(b*x^n+a))^(1/p),x)
[Out] int((x^((n-1)*p)*(b*x^n+a))^(1/p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((bx^{n} + a)x^{(n-1)p}\right)^{\left(\frac{1}{p}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^((-1+n)*p)*(a+b*x^n))^(1/p),x, algorithm="maxima")
[Out] integrate(((b*x^n + a)*x^((n - 1)*p))^(1/p), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(x^{p\,(n-1)}\,\left(a + b\,x^{n}\right)\right)^{1/p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(p*(n - 1))*(a + b*x^n))^(1/p),x)
[Out] int((x^(p*(n - 1))*(a + b*x^n))^(1/p), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^{p(n-1)}\left(a + bx^{n}\right)\right)^{\frac{1}{p}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**((-1+n)*p)*(a+b*x**n))**(1/p),x)
```

```
[Out] Integral((x**(p*(n - 1))*(a + b*x**n))**(1/p), x)
```

3.338 $\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p\,dx$

Optimal. Leaf size=61

$$\frac{x^{\frac{(1-n)(p+1)}{p}}\left(ax^{-\frac{1-n}{p}}+bx^{n-\frac{1-n}{p}}\right)^{p+1}}{bn(p+1)}$$

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1979, 2000}

$$\frac{x^{\frac{(1-n)(p+1)}{p}}\left(ax^{-\frac{1-n}{p}}+bx^{n-\frac{1-n}{p}}\right)^{p+1}}{bn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^((-1 + n)/p)*(a + b*x^n))^p,x]
```

```
[Out] (x^(((1 - n)*(1 + p))/p)*(b*x^(n - (1 - n)/p) + a/x^((1 - n)/p))^(1 + p))/(
b*n*(1 + p))
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p\,dx &= \int \left(bx^{n+\frac{-1+n}{p}}+ax^{\frac{-1+n}{p}}\right)^p\,dx\\ &= \frac{x^{\frac{(1-n)(1+p)}{p}}\left(bx^{n-\frac{1-n}{p}}+ax^{-\frac{1-n}{p}}\right)^{1+p}}{bn(1+p)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 0.74

$$\frac{x^{1-n}(a+bx^n)\left(x^{\frac{n-1}{p}}(a+bx^n)\right)^p}{bn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^((-1 + n)/p)*(a + b*x^n))^p,x]
```

```
[Out] (x^(1 - n)*(a + b*x^n)*(x^((-1 + n)/p)*(a + b*x^n))^p)/(b*n*(1 + p))
```

IntegrateAlgebraic [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(x^{\frac{-1+n}{p}} (a+bx^n)\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^((-1 + n)/p)*(a + b*x^n))^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^((-1 + n)/p)*(a + b*x^n))^p, x]
```

fricas [A] time = 0.43, size = 54, normalized size = 0.89

$$\frac{(bxx^n + ax)\left(bx^n x^{\frac{n-1}{p}} + ax^{\frac{n-1}{p}}\right)^p}{\left(bnp + bn\right)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^((-1+n)/p)*(a+b*x^n))^p,x, algorithm="fricas")
```

```
[Out] (b*x*x^n + a*x)*(b*x^n*x^((n - 1)/p) + a*x^((n - 1)/p))^p/((b*n*p + b*n)*x^
n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((bx^n + a)x^{\frac{n-1}{p}}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^((-1+n)/p)*(a+b*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate(((b*x^n + a)*x^((n - 1)/p))^p, x)
```

maple [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \left((b\,x^n + a)\,x^{\frac{n-1}{p}}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^((n-1)/p)*(b*x^n+a))^p,x)
```

```
[Out] int((x^((n-1)/p)*(b*x^n+a))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((bx^n + a)x^{\frac{n-1}{p}}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^((-1+n)/p)*(a+b*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^n + a)*x^((n - 1)/p))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(x^{\frac{n-1}{p}}\,(a + b\,x^n)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^((n - 1)/p)*(a + b*x^n))^p,x)
```

```
[Out] int((x^((n - 1)/p)*(a + b*x^n))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^{\frac{n-1}{p}}\left(a+bx^n\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**((-1+n)/p)*(a+b*x**n))**p,x)
```

```
[Out] Integral((x**((n - 1)/p)*(a + b*x**n))**p, x)
```

3.339 $\int x^{-1+n-p(1+q)} (ax^n + bx^p)^q \, dx$

Optimal. Leaf size=39

$$\frac{x^{-p(q+1)} (ax^n + bx^p)^{q+1}}{a(q+1)(n-p)}$$

Rubi [A] time = 0.05, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2014}

$$\frac{x^{-p(q+1)} (ax^n + bx^p)^{q+1}}{a(q+1)(n-p)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n - p*(1 + q))*(a*x^n + b*x^p)^q,x]
```

```
[Out] (a*x^n + b*x^p)^(1 + q)/(a*(n - p)*(1 + q)*x^(p*(1 + q)))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^{-1+n-p(1+q)} (ax^n + bx^p)^q \, dx = \frac{x^{-p(1+q)} (ax^n + bx^p)^{1+q}}{a(n-p)(1+q)}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 1.03

$$-\frac{x^{-p(q+1)} (ax^n + bx^p)^{q+1}}{a(q+1)(p-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n - p*(1 + q))*(a*x^n + b*x^p)^q,x]
```

```
[Out] -((a*x^n + b*x^p)^(1 + q)/(a*(-n + p)*(1 + q)*x^(p*(1 + q))))
```

IntegrateAlgebraic [F] time = 0.14, size = 0, normalized size = 0.00

$$\int x^{-1+n-p(1+q)} (ax^n + bx^p)^q \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 + n - p*(1 + q))*(a*x^n + b*x^p)^q,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 + n - p*(1 + q))*(a*x^n + b*x^p)^q, x]
```

fricas [A] time = 0.44, size = 76, normalized size = 1.95

$$\frac{\left(axx^{-pq+n-p-1}x^n + bxx^{-pq+n-p-1}x^p\right)(ax^n + bx^p)^q}{\left(an - ap + \left(an - ap\right)q\right)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x, algorithm="fricas")
```

```
[Out] (a*x*x^(-p*q + n - p - 1)*x^n + b*x*x^(-p*q + n - p - 1)*x^p)*(a*x^n + b*x^
p)^q/((a*n - a*p + (a*n - a*p)*q)*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ax^n + bx^p)^q x^{-p(q+1)+n-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x, algorithm="giac")
```

```
[Out] integrate((a*x^n + b*x^p)^q*x^(-p*(q + 1) + n - 1), x)
```

maple [F] time = 1.27, size = 0, normalized size = 0.00

$$\int x^{n-(q+1)p-1} \left(a\, x^n + b\, x^p\right)^q\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x)
```

```
[Out] int(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ax^n + bx^p)^q x^{-p(q+1)+n-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x, algorithm="maxima")
```

```
[Out] integrate((a*x^n + b*x^p)^q*x^(-p*(q + 1) + n - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^{n-p\,(q+1)-1}\, \left(a\, x^n + b\, x^p\right)^q\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - p*(q + 1) - 1)*(a*x^n + b*x^p)^q,x)
```

```
[Out] int(x^(n - p*(q + 1) - 1)*(a*x^n + b*x^p)^q, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n-p*(1+q))*(a*x**n+b*x**p)**q,x)
```

```
[Out] Timed out
```

3.340 $\int x^{-1-nq-p(1+q)} \left(x^n \left(a+bx^p\right)\right)^q \, dx$

Optimal. Leaf size=40

$$-\frac{x^{-((q+1)(n+p))} \left(ax^n+bx^{n+p}\right)^{q+1}}{ap(q+1)}$$

Rubi [A] time = 0.07, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1980, 2014}

$$-\frac{x^{(q+1)(-(n+p))} \left(ax^n+bx^{n+p}\right)^{q+1}}{ap(q+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n*q - p*(1 + q))*(x^n*(a + b*x^p))^q,x]
```

```
[Out] -((a*x^n + b*x^(n + p))^(1 + q)/(a*p*(1 + q)*x^((n + p)*(1 + q))))
```

Rule 1980

```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] &&  !Generali
zedBinomialMatchQ[u, x]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-1-nq-p(1+q)} \left(x^n \left(a+bx^p\right)\right)^q \, dx &= \int x^{-1-nq-p(1+q)} \left(ax^n+bx^{n+p}\right)^q \, dx\\ &= -\frac{x^{-((n+p)(1+q))} \left(ax^n+bx^{n+p}\right)^{1+q}}{ap(1+q)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 0.95

$$-\frac{x^{-((q+1)(n+p))} \left(x^n \left(a+bx^p\right)\right)^{q+1}}{ap(q+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n*q - p*(1 + q))*(x^n*(a + b*x^p))^q,x]
```

```
[Out] -((x^n*(a + b*x^p))^(1 + q)/(a*p*(1 + q)*x^((n + p)*(1 + q))))
```

IntegrateAlgebraic [F] time = 0.13, size = 0, normalized size = 0.00

$$\int x^{-1-nq-p(1+q)} \left(x^n \left(a+bx^p\right)\right)^q \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 - n*q - p*(1 + q))*(x^n*(a + b*x^p))^q,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 - n*q - p*(1 + q))*(x^n*(a + b*x^p))^q, x]
```

fricas [A] time = 0.41, size = 64, normalized size = 1.60

$$-\frac{\left(bxx^{-(n+p)q-p-1}x^p + axx^{-(n+p)q-p-1}\right)(bx^nx^p + ax^n)^q}{apq + ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x, algorithm="fricas")
[Out] -(b*x*x^(-(n + p)*q - p - 1)*x^p + a*x*x^(-(n + p)*q - p - 1))*(b*x^n*x^p +
 a*x^n)^q/(a*p*q + a*p)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int ((bx^p + a)x^n)^q\, x^{-p(q+1)-nq-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x, algorithm="giac")
[Out] integrate(((b*x^p + a)*x^n)^q*x^(-p*(q + 1) - n*q - 1), x)
```

maple [F] time = 0.87, size = 0, normalized size = 0.00

$$\int x^{-nq-(q+1)p-1}\left((b\,x^p + a)\,x^n\right)^q\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n*q-(q+1)*p)*(x^n*(a+b*x^p))^q,x)
[Out] int(x^(-1-n*q-(q+1)*p)*(x^n*(a+b*x^p))^q,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int ((bx^p + a)x^n)^q\, x^{-p(q+1)-nq-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x, algorithm="maxima")
[Out] integrate(((b*x^p + a)*x^n)^q*x^(-p*(q + 1) - n*q - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x^n\,\left(a+b\,x^p\right)\right)^q}{x^{n\,q+p\,\left(q+1\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^n*(a + b*x^p))^q/x^(n*q + p*(q + 1) + 1),x)
[Out] int((x^n*(a + b*x^p))^q/x^(n*q + p*(q + 1) + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n*q-p*(1+q))*(x**n*(a+b*x**p))**q,x)
[Out] Timed out
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
```

```
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
```

```
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
```

```
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
```

```
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
```

```
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
```

```
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```